

A spatial-temporal random process for geotechnical design based on observational method

時間—空間確率過程を用いた観測法に基づく地盤設計法

by

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Abstract

This research proposes a methodology for geotechnical design based on spatial-temporal random process using the observational method. Temporal soil behavior is introduced based on geotechnical models which describe ground behavior at an arbitrary point with time through the unknown parameters employed in the model, while the spatial correlation of the soil behaviors is introduced through the spatial correlation of these unknown parameters. The method is based on Bayesian estimation by considering both prior information of the unknown parameters, including spatial correlation, and observation data of the ground behavior to search for the best estimates of the parameters at any arbitrary points on the ground. The optimized selection of auto-correlation distance and observation error by Akaike's Bayesian Information Criterion (ABIC) is also proposed. It is proved in this study that, for the static problem without process noise, the Kalman filter yields identical results with those from the Bayesian estimation. The use of Kalman filter formulation to introduce process noise into the proposed spatial-temporal process is described. The ordinary kriging technique is presented as an alternative approach for spatial interpolation of the unknown parameters, which in fact is included in the proposed formulation in the form of simple kriging, and the difference between the approaches is also discussed.

The application in prediction of primary consolidation by Asaoka's method and secondary compression settlement by $S \sim \log(t)$ method is presented as the application examples of the proposed method. Several case studies were carried out using both simulated settlement data and actual field observation data to investigate the performance of the proposed approach. It is concluded that the accuracy of the settlement prediction can be improved by taking into account the spatial

correlation structure and the proposed approach gives the rational prediction of the settlement at any location at any time with quantified uncertainty. The sensitivity of this improvement to variations in the auto-correlation distance, observation spacing, and number of observation points is investigated. The accuracy of the estimation of auto-correlation distance, observation error, and the final settlement at an arbitrary location is also discussed. In addition, it was found that, by including process noise in the calculation, one can introduce a kind of forgetting factor which actually gives more weight to the more recent observation and improve prediction. However, care should be taken in choosing this forgetting factor in order to optimize the prediction.

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Notations

a	constant parameter, representing the level of process noise
c	normalizing constant for Bayesian updating formulation
C_j^i	constant coefficient, formulating the trend components for the unknown parameters z_i ($i = 1, 2, \dots, P; j = 1, 2, \dots$)
E_r	error ratio
$E[X]$	mean of random variable X
$E[g(X)]$	mathematical expectation of a given function g of random variable X
$I_{n,n}$	$n \times n$ unit matrix
K	total number of observation time step
K_k	Kalman gain at time step k
k	observation time step
k/l	subscript, representing the k th step estimation conditioned on processing observation data up to the l th datum
L	the total width of the group of observation points
M_k	coefficient matrix, relating unknown parameters to observation data at observation time step k (Y_k)
$M_{n,n}^i$	$n \times n$ coefficient matrix, relating unknown parameter z_i to observation data Y_k
m	total number of observation points (n), including unobserved points at which the unknown parameters are to be estimated ($m-n$)
m_0, m_1	constant parameters for secondary compression model ($S \sim \log(t)$ method)
$\hat{m}_0(x_j), \hat{m}_1(x_j)$	estimates of m_1 and m_0 at point x_j

$\hat{m}_{1,0}(x_j), \hat{m}_{0,0}(x_j)$	prior means of m_1 and m_0 at point x_j
N_{sim}	total number of simulations
n	total number of observation points
P	total number of unknown parameters
Q_k	covariance matrix of process noise at time step k
S	ground settlement
$S(\omega), S(\omega_1, \omega_2)$	spectral density function
S_k	settlement vector at k th step of observation
$S_k(x_j)$	observed settlement at an observation point x_j at observation time step k
S_f	final settlement
s	spacing of observation points
T_v	time factor
t	time
t_0	time at a beginning stage
t_f	time at a spot of time in the future
t_k	time at observation time step k
$t(x)$	value of the trend at location x
$u(x)$	random component at location x
V_C	auto-covariance matrix
V_ε	covariance matrix of observation error
V_Z	covariance matrix of unknown parameter
w_{ij}	kriging weights attached to the data at each of the observation points
X_{est}, X_{true}	estimated value and true value

\underline{x}_j	spatial vector coordinate at point x_j , i.e. (x_j', y_j')
x_j'	x coordinate at point x_j
x_1, x_2, \dots, x_n	observation points
$x_{n+1}, x_{n+2}, \dots, x_m$	any arbitrary points at which the unknown parameters are to be estimated
Y	set of all observation data for the concerned soil behavior
Y_k	observation vector at observation time step k
y_j'	y coordinate at point x_j
$y_k(x_j)$	observation data at an observation point x_j at observation time step k
Z	unknown parameter vector
\hat{Z}	best estimator of unknown parameter vector for a discrete spatial point field
Zo_i	estimate of the unknown parameter z_i at observation points
Zr_i	estimate of the unknown parameter z_i at unobserved arbitrary points
Z_0	prior mean vector of unknown parameter
$z_i(x)$	random function of an unknown parameter z_i at an arbitrary point x
$\hat{z}_i(x_j)$	estimate of unknown parameter z_i at point x_j
$z_{0,i}(x_j)$	prior mean of unknown parameter z_i at point x_j
β_1, β_0	constant parameters for the Asaoka's model
$\hat{\beta}_1(x_j), \hat{\beta}_0(x_j)$	estimates of β_1 and β_0 at point x_j
$\hat{\beta}_{1,0}(x_j), \hat{\beta}_{0,0}(x_j)$	prior means of β_1 and β_0 at point x_j
δ	vector representing uncertainties of unknown parameters
δ_u	scale of fluctuation
ε	observation error vector

η	auto-correlation distance
μ_j	Lagrange multiplier
$\rho(/x_i - x_j/)$	auto-correlation function
$\sigma_{m1,0}^2, \sigma_{m0,0}^2$	prior variance of m_1 and m_0
σ_{or}^2	variance of interpolation by ordinary kriging
σ_{zi}^2	prior variance of the an unknown parameters z_i
$\sigma_{\beta1,0}^2, \sigma_{\beta0,0}^2$	prior variance of β_1 and β_0
σ_ε^2	variance of the observation error
ϕ_{jk}	random phase angles, uniformly and independently distributed in the interval (0,2 π)
$\omega, \omega_1, \omega_2$	circular frequency domain
$0_{n,m}$	$n \times m$ zero matrix

1 Introduction

1.1 Background

Behavior of geotechnical structure, such as settlement and movement of soil mass, possesses variability in both space and time. The spatial variation of the behavior mainly stems from the natural variability of soil properties. The temporal variation, on the other hand, is due to the complexity of the time-dependent behavior of soil, e.g. consolidation settlement and undrained creep. The influence of these variations can be significant for several design problems in geotechnical engineering practice—for example, the evaluations of differential settlements of structures.

To cope with these variations, a number of approaches have been proposed in several past literatures. For modeling spatial variability, soil property is known to be correlated spatially, for which the property tends to similar in value at closely neighboring points than at widely spaced points (Lumb 1974, Vanmarcke 1977a, DeGroot and Baecher 1993, Baecher and Christian 2003). This naturally induces the spatial correlated characteristic of the resulting soil behavior. By recognizing this fact, the spatial correlation of soil properties has been proposed to be considered in various kinds of geotechnical analysis, such as stability of earth slopes (e.g. Vanmarcke 1977b), foundation settlement (e.g. Fenton and Griffiths 2005), and liquefaction risk (e.g. Fenton and Vanmarcke 1998). As for the temporal variability, the observational method has been considered as an efficient and reliable approach to deal with the uncertainty originated from the time-dependent behavior of soil, by which the design can be calibrated dynamically using the periodically observed data. Since the essence of the method has been emphasized by Peck (1969), the observational

method has been proved to be a practical way to control the uncertainties in designing geotechnical structure, such as excavation (e.g. Young & Ho 1994), tunnel construction (e.g. Iwasaki *et al.* 1994), and embankment construction (e.g. Wakita and Matsuo 1994). Matsuo and Asaoka (1978) also presented a probabilistic approach for updating the design based on the observation data and named it as dynamic design procedure.

Unfortunately, most of the proposed approaches in past literatures provided only the means to deal with spatial and temporal uncertainties separately. To predict behavior of soil at a particular location and at a spot of time in the future, an approach which incorporates both spatial and temporal variability in the estimation process is needed. The researches in spatial-temporal modeling have been relatively active in such field of study as environmental science, meteorology, hydrology, and reservoir engineering. In geotechnical engineering, however, this research topic has still seldom been concerned. The particular characteristic of the geotechnical problem is that the temporal soil behavior is generally possible to be predicted based on the physical model which directly relates to soil properties. By introducing spatial correlation of soil properties into this temporal model, it is therefore natural to expect that the accuracy of the soil behavior prediction should be improved because, in this case, the data observed from all observation points can be rationally utilized. Furthermore, the rigorously prediction of the ground behavior at any arbitrary points at any time can be possible through this spatial-temporal estimation process. This study is actually an attempt to search for such an approach.

1.2 Objective and scope

The objective of this research is to propose a practical approach which takes into account spatial correlation of soil parameters in the prediction of temporal behavior of geotechnical structure using the observation data. Two main advantages are expected to be achieved:

1. The accuracy of the prediction is improved by systematically utilizing observation data from all observation points.
2. The rigorous prediction at any point and any time can be done with quantified uncertainty.

The proposed approach is established based on the assumption that the soil behaviors are observed at discrete points and discrete time, and this observation data is used to update the unknown parameters employed in the temporal soil model (hereinafter called the ‘unknown parameter’ or ‘model parameters’), by which the soil behavior at a specific time can be predicted through the linear relationship between these unknown parameters and behavior. The spatial correlation structure of the soil behavior is also introduced through these unknown parameters. It should be noted that because the future behavior is to be predicted based on the model parameters which are derived from the observed behavior, the uncertainty of the miscellaneous effects which influence the geotechnical behavior such as external loading uncertainties have already been included in the model.

The proposed spatial-temporal estimation method can be illustrated as shown in Figure 1-1. At the beginning stage ($t = t_0$), the statistical parameters of the prior probability distribution function (pdf) of the unknown parameters at discrete points on the ground, including the observation points and the points to be estimated, are required to be input. This can be obtained from the past experience or inferred from the soil testing results. The spatial correlation of the unknown parameters is also introduced in the variation components of prior pdf. Based on this prior pdf, the concerned behavior of soil at each observation time step ($t = t_1, t_2, \dots, t_K$) can be predicted through

the selected temporal soil model (also referred as ‘observation model’ in this thesis). Comparing this prediction with the observation data, the statistics of the unknown parameters at each point are updated through the proposed formulation based on the Bayesian estimation. To emphasize, Bayesian approach is chosen to introduce the spatial correlation into the temporal soil model in this research due to its ability to systematically combine the subjective information, i.e. the prior information including the spatial correlation of the unknown parameters, and objective information, i.e. the observation data. Finally, based on these updated estimates of the unknown parameters, the soil behavior at a spot of time in the future ($t = t_f$) can be predicted using the temporal soil model.

Due to the fact that spatial correlation structure is controlled by auto-correlation distance (η), the estimation of this key parameter is necessary. By considering this as a model selection problem, it will be shown that, within the Bayesian framework, auto-correlation distance (η) and the standard deviation of observation error (σ_e) can be appropriately selected based on Akaike’s Bayesian Information Criterion (Akaike 1980, Honjo and Kashiwagi 1999).

Two well-known techniques are also presented as alternative methods which can replace some parts of the proposed approaches. Firstly, Kalman filter (Kalman 1960, Kalman & Bucy 1961), including spatial correlation of the unknown (or model) parameters, can be use to update the statistics of the parameters based on the observation data. It is proved herein that this process is actually identical to the proposed approach when process noise in the time updating process of Kalman filter is ignored. The effect of process noise consideration is also discussed in this thesis. Secondly, the use of kriging technique (Krige 1966, Matheron 1973) for interpolating values of the unknown parameters to an arbitrary point is also presented. The fact that the interpolation process included in the proposed approach is actually equivalent to the simple kriging technique is

emphasized and the alternative use of ordinary kriging technique for the interpolation is also discussed.

In order to investigate the practical performance of the proposed approach, the applications of the proposed formulation in spatial-temporal prediction of primary consolidation and secondary compression settlement are presented in this thesis. Asaoka's method (Asaoka 1978), which is an autoregressive model representing a trend equation of time series data of settlement, is chosen as a prediction model for primary consolidation settlement. For the prediction of secondary compression settlement, the $S \sim \log(t)$ method (Bjerrum 1967, Garlanger 1972, Mesri *et al.* 1997), which represents linear relationship between logarithm of time and settlement, is used.

Both simulated data and actual observation data are used as the input for calculations under several test conditions. For the case of simulated data, the settlement data are generated based on the assigned spatial correlation structure of the unknown (or model) parameters using frequency domain technique (Shinozuka 1971, Shinozuka and Jan 1972). Based on this generated settlement data, the statistical inferences of the unknown parameters are back-calculated using the proposed approach. As for the case of actual observation data, the observed settlement of preloaded peat ground site in a suburb area of Tokyo, Japan, is used. This is treated as secondary compression settlement and modeled by $S \sim \log(t)$ method. The calculation results of all cases will be presented and discussed in detail later in this thesis.

1.3 Organization of the dissertation

The introductory chapter is followed by Chapter 2 which is devoted to a review of previous literatures associated with this study. Chapter 3 presents the basic assumptions and formulations of the proposed spatial-temporal estimation process. The uses of Kalman filter and kriging technique as the alternative approaches are also discussed in this chapter. Chapter 4 introduces the examples for

practical applications of the proposed approach in geotechnical design and the discussion of the results. The chapter is divided into 2 main sections. Section 4.1 exhibits an example of the primary consolidation settlement prediction by Asaoka's method using simulated data, while Section 4.2 presents an example of secondary compression prediction by $S \sim \log(t)$ method using both simulated and actual observation data. Finally, the summary and conclusion of the research are given in Chapter 6.

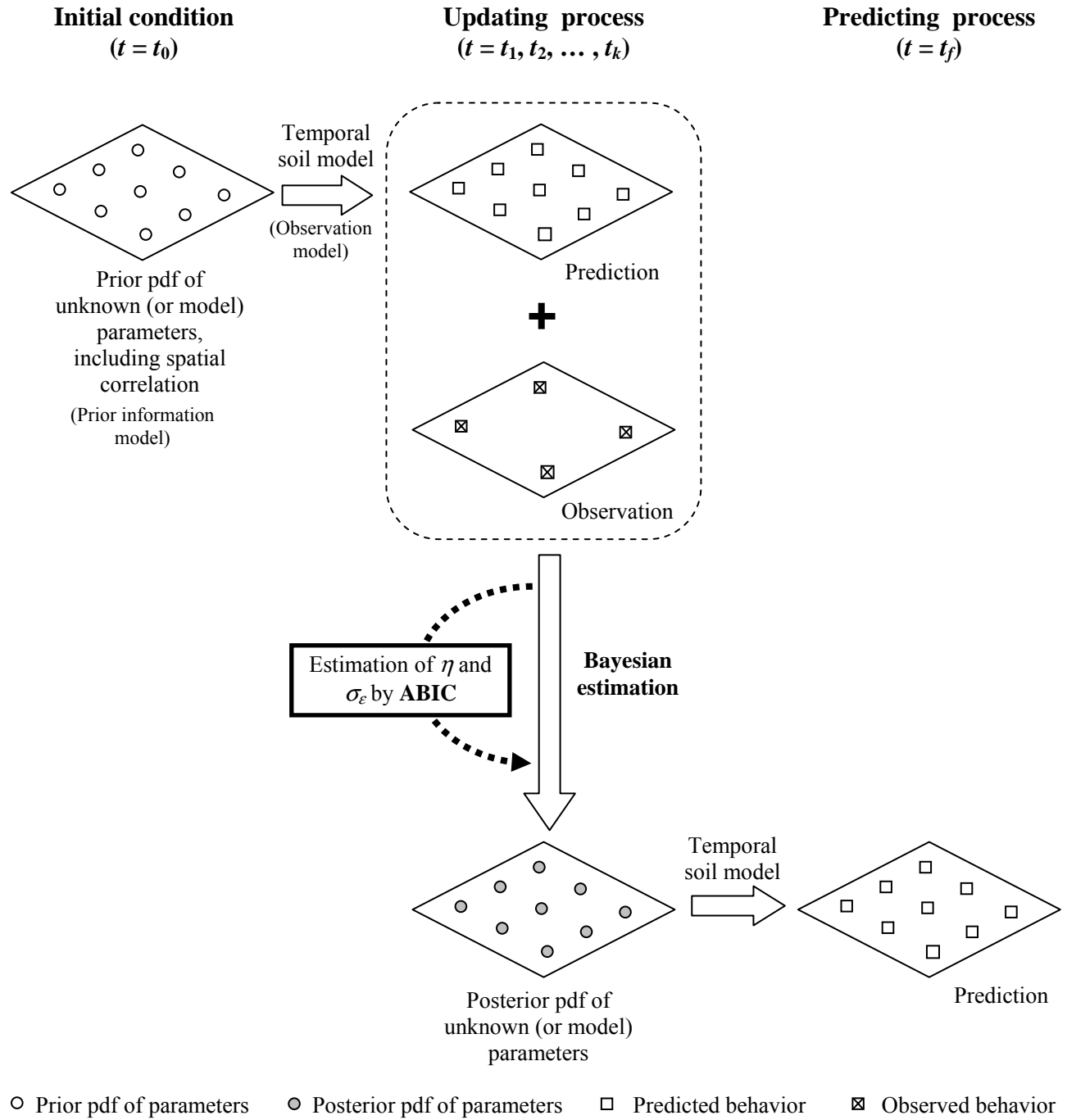


Figure 1-1: A scheme of the proposed spatial-temporal estimation method.

2 Literature review

2.1 Uncertainties in geotechnical engineering

2.1.1 Introduction

Geotechnical engineers are always dealing with significant level of uncertainties, relating to geotechnical works. Natural soils are known to be heterogeneity, for which the soil properties are different from one spatial location to another inside the soil mass. These variations, basically, cannot be completely investigated. In practice, extensive subsoil investigations are usually constrained by financial possibility. At these selected locations of soil investigations, soil properties are acquired either by field or laboratory tests. These measured properties, which may need to be transformed through empirical or theoretical model to achieve for the design parameters, are then used as the input parameters of geotechnical models or design techniques for predicting pertinent behaviors of the geotechnical structures or evaluating the stabilities of the structures.

Several sources of uncertainties can be clearly observed from the above design process. In addition to the inherent spatial variation of natural soil, the measurement, model, and statistical uncertainties can be expected. Traditional engineering practice tends to adopt a factor of safety in the design in order to cope with these unexpected uncertainties. In 1970, however, Wu and Kraft presented an approach to quantify geotechnical uncertainties and to study the effect of these uncertainties on the reliability of a slope. By this approach, various sources of uncertainty can be systematically compared, analyzed and combined using the probabilistic procedure. Despite these

attractive benefits, however, the geotechnical professions have been somewhat reluctant to use this new method in practice. This might be due to the fact that they have seen little need to adopt the more complex and untried probabilistic method in stead of using the deterministic method which they have plenty of lessons learned from years of professional practice.

However, recent years have shown the trend that the geotechnical uncertainties are required to be treated in the more quantitative way. Geotechnical engineers have been increasingly forced by regulatory agencies or project decision makers to provided quantitative answers about reliability of their design, especially for the large scale or risky structures. Moreover, Load and Resistance Factor Design (LRFD), which are based on reliability methods, are now being introduced into such areas as pile design for highway structure. Due to these facts, the demands for the studies on applications of reliability methods in geotechnical engineering have been increased over the past decades. There has been steady steam of publications on geotechnical engineering in various topics such as environmental geotechnology (e.g. Bogardi *et al.*1989 and 1990), offshore foundation (e.g. Wu *et al.*1989, Ronold and Bjerager 1992), effect of geologic anomalies (e.g. Baecher 1979, Tang 1987, Halim and Tang 1993), statistical evaluation of soil properties (e.g. Kulhawy *et al.*1991, Phoon and Kulhawy 1999a & 1999b), probabilistic slope stability analysis (e.g. Li and Lumb 1987, Chowdhury *et al.*1987, Honjo and Kuroda 1991, Griffiths and Fenton 2004), and problems related to earthquake hazards (e.g. Fenton and Vanmarcke 1998, Juang *et al.*2002).

Regardless of the types of problems to be tackled, the most important step for reliability analysis of geotechnical structure is perhaps to define and quantified the components of uncertainties. However, the characterization of geotechnical uncertainties can be a tough task due to the complexity of the sources of uncertainties and even the confusing terminologies. The evidence of these difficulties is the fact that both the classified sources of uncertainties and terms used to explain

them are usually different among the authors. For example, Wu (2009) designated that the uncertainties in geotechnical design are consisted of random and systematic uncertainties. Based on his view, soil variability is the most important source of random error, while inaccuracy or simplifications in material and analytical models are the common sources of systematic errors. It seems that his classification roughly separates inherent variability of soil from the model uncertainties, which include parameter transformation model and analytical model. On the other hands, Baecher and Christian (2003) provided more extensive classification of uncertainties by categorizing into natural variability, knowledge uncertainty, and decision model uncertainty. Natural variability, also referred as aleatory uncertainty, represents the inherent randomness of natural process both spatially and temporally. Knowledge uncertainty, also referred as epistemic uncertainty, is attributed to the lack of information about events and processes which contains site characterization uncertainty, model uncertainty, and parameter uncertainty. Finally, for implementation of the design in practice concerning the economic issues, decision model uncertainty is proposed to be concerned in the calculations.

Focusing on the uncertainties in soil parameter estimates, Tang (1993) classified these uncertainties into three main components, namely inherent variability, statistical uncertainty, and systematic uncertainty which, in this case, is contributed by the inability of a test to reproduce the in situ property due to factors such as sample disturbance, limited specimen size, etc. Phoon and Kulhawy (1999a), however, indicated that three primary sources of uncertainties in soil property estimation consist of inherent variability, measurement error, which is comparable to systematic uncertainty presented by Tang (1993), and transformation uncertainty, which results from the empirical or theoretical model used to relate measure quantities to design parameters.

2.1.2 A proposed view for uncertainty classification

Based on the above review of several past literatures about classification of geotechnical uncertainties, one found inconsistency mainly in terminologies used for defining different sources of uncertainties. In this thesis, an attempt has been made to reorganize categories of geotechnical uncertainties in the more systematic way based on the most commonly used terminology found in the reviewed literatures. This proposed classification is presented in Figure 2-1.

The uncertainties at the different design steps are clearly classified and also grouped to summarize all viewpoints of the uncertainties. During subsoil investigation, the uncertainties consist of (i) combination between *inherent soil variability*, which primarily results from the natural geologic process, and *statistical uncertainty*, which is contributed by the statistical assumption or limited number and location of samples, and (ii) *measurement uncertainty*, which is due to equipment, procedure-operator, and random testing effects. The field or laboratory measurements are then transformed into design soil parameters using empirical or other correlation models. The uncertainty arises at this step is termed *transformation uncertainty*. Finally, the transformed parameters are put into analytical models or selected design techniques for designing geotechnical structure. Clearly, uncertainty at this step, namely *design method uncertainty*, arises from the error of using the model to predict the complicated actual soil behavior. It is only the inherent soil variability which is classified as aleatory uncertainty, indicating that this is the only source of uncertainty which cannot be reduced by any observers.

It should be noted that the contribution of the inherent soil variability and statistical uncertainty is freshly proposed in the combined form. This is based on the perspective that these two uncertainties cannot be separated completely, by which one can be induced by another. This

conclusion is illustrated by an example of subsoil investigations at two different sites with the same soil sampling scheme as shown in Figure 2-2.

Figure 2-2 presents an example of subsoil investigations at two different sites, namely site A and site B, showing the measured properties and the approximated trends along with the actual properties. It is assumed that three soil samples are taken at the same depths from both sites and the measured soil properties are also identical at each corresponding depth. No doubt, one must obtain the same linear trend, with depth, of the measured properties at both sites. However, it can be clearly seen from Figure 2-2 that it is only at Site A that the approximated trend can be a good representative of the actual properties. This is because the spatial variation of the properties at site B is much greater than those at site A. It appears that the statistical uncertainty for the current soil investigation at site B is quite significant and the shorter interval of soil samplings is required to acquire for better description of the actual condition, despite the fact that the soil sampling scheme and the trend component are not different from those of site A. This significant amount of statistical uncertainty at site B is induced by the vertical variation of the pertinent soil properties. It should be noted that this conclusion is also applicable for the case of horizontal variation, i.e. when the horizontal trends of soil properties are to be observed. This example illustrates the relation between these two sources of geotechnical uncertainties. Therefore, they are proposed in the combined form as shown in Figure 2-1.

In addition to the inherent spatial variation of soil, the statistical uncertainty also depends on the design situation. This should also be mentioned here in order to provide a comprehensive description of uncertainty in soil property characterization. Honjo and Setiawan (2007) emphasised the essence of classifying the estimations of geotechnical parameters into two different types, namely local estimation and general estimation, in accordance with the corresponding design

situation. Local estimation refers to the situation that the estimation is needed to be performed at a specific spot, e.g. right at the spot where a structure is to be built. General estimation, on the other hand, refers to the case that the estimation is needed to be done generally at any arbitrary location in a certain area, i.e. a reclaimed area where a container yard is to be located. In local estimation, because the location to be design has been predefined, it is possible to reduce the statistical uncertainty by considering relative spatial location between the field exploration point and the concerned structure, e.g. performing kriging technique (Krige 1966, Matheron 1973). In general estimation, however, because targeted estimation value is expected to represent the pertinent soil property of a large area, the statistical uncertainty is inevitably greater than that of the local estimation. Moreover, the difference of the uncertainty levels between these two types of estimation also depends on the spatial correlation condition of the soil property (Honjo and Setiawan 2007). This emphasizes the strong relation between statistical uncertainty and the inherent soil variability.

2.1.3 Dealing with uncertainty

After the uncertainties in geotechnical design have been clearly classified, a problem arises that how one can perform the satisfactory design despite these underlying uncertainties. Wu (2009) states that two principle components of geotechnical design are the (i) calculated risk, i.e. reliability-based design and (ii) the observational method. The reliability-based design involves the evaluation of the failure probability of the structure, whose comprehensive formulation was firstly provided by Freudenthal (1947). The use of first-order second moment (FOSM) method is then introduced as a more convenient approach for evaluation of failure probability (Cornell 1969). An improvement over FOSM is the use of first-order reliability method (FORM) (Hasofer and Lind 1974). More elaborate are stochastic and simulation method (Fenton and Griffiths 2002). The detail explanations

of these techniques for reliability-based design in geotechnical engineering are well documented in Phoon (2008).

The important role of the observational method in solving complex design problems has been recognized since it was proposed by Terzaghi and was detailed described by Peck (1969). Through Bayesian updating, the observational method can also be used to enhance the performance of reliability analysis (Matsuo and Asaoka 1978). This thesis proposes an approach for dealing with spatial-temporal variation of geotechnical problem using the observational method. It should be emphasized here that the proposed approach focuses on providing the *local estimation* (as mentioned above) by taking into account spatial correlation of geotechnical parameters. Next section presents the review of past literatures in the application of the observational method for reliability analysis.

2.2 The observational method for reliability analysis

2.2.1 The observational method in geotechnical design

Basically, it was seldom technically or economically possible for geotechnical engineer to design for all unfavorable situations. Terzaghi and Peck (1967) states that the significant savings can be made by designing on the basis of the most probable rather than the most unfavorable possibilities, and the design can later be modified based on the observations during construction. This approach was originally termed as “observation procedure”. Peck (1969) specified and expanded the approach, which he named “the observational method”, and this approach now becomes an essential feature of geotechnical practice, especially for large or difficult project. The observational method is a practical way to deal with the uncertainty for which simple conservatism is unsatisfactory. It can be described in a simple procedure as follows:

- (1) Performing design based on available information.
- (2) Making a detailed list of all possible differences between reality and the assumptions.
- (3) Computing various quantities that can be measured in the field based on the original assumptions.
- (4) On the basis of the field measurements, gradually close the gaps in knowledge and, if necessary, modify the design during construction.

The observational method provides a way of assuring the safety while minimizing the construction costs. The major limitation of the method is that it must be possible to modify the design during the construction process.

There have been plenty of reports in several past decades which proved the success of using the observational method in geotechnical practice. The special issue of *Géotechnique* in 1994, titled as “The observational method in geotechnical engineering”, provided a rich collection of the papers in the practical use of this method in several kinds of geotechnical work, for example excavation (Young & Ho 1994, Ikuta *et al.* 1994), tunnel construction (Iwasaki *et al.* 1994, Harris *et al.* 1994), groundwater control (Roberts and Preene 1994), and embankment construction (Wakita and Matsuo 1994). Powderham (1994) proposed the concept of using more probable condition to provide an intermediate starting case instead of using the most probable design which implies lower margin of safety. The new term “progressive modification” is used for the gradual introduction of changes, based on the monitoring results, to move towards the most probable design condition.

2.2.2 Settlement prediction by the observational method

The ground settlement prediction and control is a topic which is considered to be suitable for performing the observational method due to its time-dependent behavior and possibility to modify the design. This topic is also focused in this thesis. The ground settlement observed during the

construction can be used to update the prediction model parameters and adjust the design, if needed. Asaoka (1978) proposed a trend equation of time series data of settlement for a one-dimensional consolidation process based on an autoregressive model. On the other hand, the hyperbolic method, by which the curve of consolidation settlement with time is proved to be fitted very well by a rectangular hyperbola over a specific range of data, has also been proposed as a powerful approach for predicting settlement by several researchers (Sridharan *et al.* 1987, Tan 1994, etc.). For the current research, however, Asaoka's method was chosen to present the application example of the proposed approach since it is soundly founded on the physical processes of consolidation theory.

For prediction of secondary compression settlement, which is significant in highly organic soil such as peat, the linear relationship between logarithm of time and settlement, i.e. $S \sim \log(t)$ method, is a very common empirical relationship found by several studies (Bjerrum 1967, Garlanger 1972, Mesri *et al.* 1997, etc.). This relationship also can be used for observation-based prediction of this kind of ground settlement and the example of which is presented in the current research.

2.2.3 Reliability-based design and the observational method

Application of the observational method in reliability analysis of geotechnical structure was presented in several literatures and it was proved to be a valuable method to enhance the performance of the reliability based design. Bayes's theorem provides a basis for probabilistically updating the design using the observation data. Through the process called Bayesian inference, the probability density function (pdf) of the primarily available information, e.g. prior pdf, can be updated by the observation data to come up with the posterior pdf. The early examples of this approach include Tang (1971), and Matsuo and Asaoka (1978). Matsuo and Asaoka (1978) presented the so-called "Dynamic design procedure" which can be considered as an application of the observational method by introducing Bayesian inference. The application of this procedure in

stability analysis of slope was shown and it was emphasized that both uncertainties from the natural variation of ground and the chosen design method can be improved by the updating process using the observation data during construction. Asaoka (1978) also presented Bayesian inference as an alternative approach for applying his proposed observation procedure of consolidation settlement prediction. This gives a predictive probability distribution of future settlement and provides the preliminary framework for reliability-based design of settlement problem. This approach, however, only gives general estimation for the whole area of observations. Providing the local estimations is focused in the current research and the application using the Asaoka's method will be presented later in this thesis. The examples of recent applications of the observational method in reliability analysis include Gilbert *et al.* (1998) for evaluation of mobilized shear strength of clay-geosynthetic interface, and Cheung and Tang (2005) for estimation of slope failure probability by using age and rainfall as prior information.

For the case which the observation data is successively collected with time, the sequential updating process may be expected. This can be considered as a sequential form of the observational method. Kalman filter (Kalman 1960, Kalman & Bucy 1961, Jazwinski 1976) is a sequential least mean square estimation based on a series of noisy measurements. This method consists of two different updating phases, i.e. time updating and observation updating. The errors involve in each phase include process noise and observation error, respectively. This method has then been applied to the structural identification problems which can be found in several literatures, such as Yun and Shinozuka (1980), and Shinozuka *et al.* (1982). Hoshiya and Yoshida (1996) presented a general formulation to provide the best estimator of stochastic Gaussian field when the observation is made at discrete spatial points. The extended Kalman filter was also proved as a special solution of the proposed formulation. Based on the same formulation, Hoshiya and Yoshida (1998) extended their

research to study the mathematical and mechanical roles of the process noises in the updating process. It is found that the greater the involvement of noises, the more reliability the later data is treated in the sequential updating process. The current thesis presents a modification of this general formulation by including the spatial correlation into the formulation to improve the accuracy of the estimation and enable the local estimation. The effect of process noise also has been investigated and summarized in Section 3.4.

2.3 Spatial-temporal modeling of soil behavior

2.3.1 Spatial and temporal variability of soil behavior

As previously described in Section 2.1, soil properties inherently possess spatial variation. Soil are basically formed by natural processes, such as transporting, depositing, and weathering processes, and then subjected to various stresses, pore fluids, and physical and chemical changes. The soil properties, therefore, can be regarded as being controlled by a sequence of random events (Lumb 1974). Even within nominally homogeneous soil layers, the engineering properties may exhibit considerable variation from point to point. As a result, it is hardly surprising that the behaviors of geotechnical structures, e.g. settlement of embankment or lateral movement of retaining wall, also vary from place to place.

In addition to the spatial variation, several kinds of soil behaviors, such as consolidation settlement or undrained creep, also gradually change with time. These time-dependent behaviors are mainly due to the soil-water interaction process in fine-grained soil with relatively low permeability. To predict these kinds of behaviors, one has to concern about the temporal variation of each process. The combination between this temporal variability and the spatial variation lead to the difficulties to

accurately predict at a specific point or time of such problem as the ground settlement due to the earth fill on a large area. This type of problem is focused and an approach for performing geotechnical design with the consideration of both spatial and temporal variability of soil behavior is proposed in the current thesis.

2.3.2 *Modeling the spatial variation*

Due to the fact that the spatial variation of soil behavior directly relates to spatial conditions of soil properties, modeling the stochastic characteristic of soil profiles is important in geotechnical performance prediction and reliability-based design of geotechnical structure. In principal, the spatial variation of a soil deposits can be characterized in detail, but it required a large number of tests which far exceeds what can be acquired in practice. For engineering purpose, a simplified model is therefore introduced by separating the spatial variation into two parts (Lumb 1974, Vanmarcke 1977a): (i) major trend or trend component, and (ii) the local variation or random component. This model can be written as the following equation:

$$z(x) = t(x) + u(x) \quad (2-1)$$

in which $z(x)$ is the soil property at location x , where x refers spatial coordination vector, $t(x)$ is the value of the trend at x , and $u(x)$ represents the random component. Rather than characterizing soil properties at every point, data are used to estimate a smooth trend, and remaining variations are described statistically.

Trend ($t(x)$) can be simply estimated by fitting well-defined mathematical functions to data points in space. It can be considered that this major trend, i.e. change in average properties, are basically incorporated in conventional subsoil modeling. Examples of the use of trend surfaces in the geotechnical applications can be found in Wu *et al.* (1996), and Ang and Tang (2007)

Even though soil deposits can be considered to be formulated randomly, in general there are a great tendency for the soil properties to be similar in value at closely neighboring points than at widely spaced points. This knowledge can be incorporated in modeling the random component ($u(x)$). Vanmarcke (1977a) provides an extensive analytical approach to describe the random components, to which he refers as local deviations. Given the condition by which the trend is fitted, the random component by definition has zero mean. The spatial variation of $u(x)$ is proposed to be described by two statistical parameters (u), i.e. standard deviation (\tilde{u}), and scale of fluctuation (δ_u). Standard deviation measures the degree at which $u(x)$ deviate from the mean, while scale of fluctuation represents the distance within which the soil properties show relatively strong correlation from point to point. The spatial correlation of $u(x)$ can then be described by correlation function, usually called *auto-correlation function*, as follows:

$$\rho(\Delta x) = \frac{1}{\tilde{u}} E[u(x)u(x+\Delta x)] \quad (2-2)$$

where $E[u(x)u(x+\Delta x)] = Cov[u(x)u(x+\Delta x)]$ is the covariance of $u(x)$ at the locations spaced at separation distance Δx . For very small interval Δx , the coefficient will be close to 1, and it usually decays as Δx increases. Typically, auto-correlation is assumed to be stationary, also called statistically homogeneous, for which it is assumed that

- (i) the mean and variance of $u(x)$ do not change with the spatial coordinate; and
- (ii) the correlation between the values of $u(x)$ at two different locations is a function only of their separation distance, rather than their absolute positions.

For the purpose of modeling and analysis, it is usually convenient to approximate the auto-correlation structure of the random components by a smooth function, which is usually formulated as a function of scale of fluctuation (δ_u). Lumb (1974) illustrated the appropriateness of using the

exponential function, which is derived from unilateral Markov Process, to model horizontal variability and the modified Bessel Function of the second kind and first order, which is derived from bilateral Markov Process, to model vertical variability of soils. Vanmarcke (1977a) summarized several types of auto-correlation functions and emphasized that, because no fundamental basis can be found from all of them, the one that fits most to the actual computed correlation coefficients should be chosen. Baecher and Christian (2003) also presented an extensive collection of one-dimensional auto-correlation model which consists of white noise, linear, exponential, Gaussian (squared exponential), and power models. The models are presented as the functions of δ_0 , usually called auto-correlation distance, which is different but related to the scale of fluctuation proposed by Vanmarcke (1977a) (e.g. in case of exponential model, $\delta_0 = \delta_u/2$). In this thesis, the spatial correlation of the soil behavior has been modeled through the soil, or model, parameters using the specified auto-correlation function, which basically is characterized by auto-correlation distance.

There are a number of ways to estimate the auto-correlation based on sample data. This, in fact, focuses on estimation the statistical parameters which govern the correlation structure such as auto-correlation distance. The most common method is the method of moment, by which the correlation coefficient will be calculated and plot against the separation distance (Δx). This method is conceptually and operationally simple but has poor samplings properties for small sample sizes (Baecher and Christian 2003). An example for application of this approach was presented by Christian *et al.* (1994). Also frequently used is the maximum likelihood method. This method searches for the most likely values of the statistical parameters of spatial correlation for the set of actually observed data. This approach, in fact, can be used to simultaneously estimate both the trend component and the auto-correlation function of the random component, which was illustrated in the

literatures by Mardia and Marshall (1984), and DeGroot and Baecher (1993). The Bayesian inference, which has been mentioned in Section 2.2.3, is also possible to be used for estimation of the auto-correlation. This method, however, has not been widely used in geotechnical application because the difficulties for appropriately choosing the prior distribution which may lead to an improper posterior pdf (Baecher and Christian 2003). Given this problem, Berger *et al.* (2001) suggest the reference non-informative prior which does lead to a proper posterior.

This research also proposes an approach for estimation of statistical parameter which controls the correlation structure, i.e. auto-correlation distance. The approach is based on Bayesian inference but, in contrast to the conventional approach, the observation values are not the parameters to be estimated for their spatial correlation. The pertinent soil behaviors, instead, are assumed to be observed and the relations to the concerned soil parameters are drawn through the geotechnical models. The auto-correlation distance is considered as a hyperparameter and is selected based Akaike's Bayesian Information Criterion, i.e. ABIC (Akaike 1980). In this case, the more informative prior distributions of the soil or model parameters are required instead of the non-informative priors of the spatial correlation processes.

A common problem in site characterization is interpolation among observation points to estimate soil properties at specific locations where they have not been observed. The knowledge about spatial correlation can provide a systematic approach for spatial interpolation in an assumed random field. The most common approach is to construct a simple linear unbiased estimator based on the observation, generally known as the technique of kriging (Krige 1966, Matheron 1973, Wackernagel 1998). Kriging is a local estimation technique which provided the best linear unbiased estimator (abbreviated to BLUE) of a concerned parameter. This technique was originally developed in mining industry, but has been adopt in various fields of study including geotechnical engineering.

Soulie *et al.* (1990) used kriging to interpolate shear vane strengths of a marine clay both horizontally and vertically among borings. Honjo and Kuroda (1991) also proposed the use of kriging technique to enhance the performance of reliability design of slope stability by taking into account the relative location of the samples to structure. In the current thesis, an interpolation process for estimating soil behavior at an arbitrary point also performed based a technique of kriging called simple kriging, which is indirectly included in the proposed formulation based on Bayesian estimation. The derivation of several techniques of kriging using Bayesian formulations is presented in Appendix B, while the detailed assumptions and equations can be found in Appendix C.

2.3.3 Modeling the temporal variation

The temporal variation focused in this research is the temporal variation of soil behavior which is directly related to the time-dependent properties of soil. The main problems which are encountered by geotechnical engineers in this aspect consist of movement and settlement of fine-grained soils. To predict this kind of behavior at a specific time, analytical or empirical model can be used. For sophisticated problems, numerical method, e.g. finite element method, may be used as a powerful tool to aid the prediction. However, due to the fact that behavior of concern is changed gradually with time, the observational method previously described in Section 2.2 can be an attractive approach.

By the observational method, the design can be sequentially updated using the observed behavior and the accuracy of the prediction can be successively improved. Due to this advantage, this method is adopted to model the temporal soil behavior in the current research. The temporal soil model is introduced in the form of the observation model based on the assumption that the soil behavior is observed at discrete time steps. Two different kinds of temporal models are used to illustrate the practical application of the proposed approach. These consist of Asaoka's method for

predicting primary consolidation settlement and $S \sim \log(t)$ method for predicting secondary compression. The detailed reviews of literatures about these methods can be found in Section 2.2.2.

Systematic update of the concerned parameters using the observation data can be done through the Kalman filter which also has previously review in Section 2.2.3. Appendix A shows the proof that, for static problem without process noise, the Kalman filter gives solutions that are identical to Bayesian estimation, base on which the proposed formulation is created. Therefore, it can be concluded that the proposed formulation is found on a similar theoretical basis with Kalman filter.

2.3.4 *Spatial-temporal model*

Predicting the temporal soil behavior at an arbitrary point in soil mass can be a complicated issue due to the combination between variations in space and time. The modeling of spatial-temporal, also called spatiotemporal, variability is critical in many scientific and engineering fields, such as geology, environmental science, meteorology, hydrology, and reservoir engineering. Kyriakidis and Journel (1999) provided an extensive review of literatures and theoretical background of spatial-temporal model in geostatistical study. It mostly focused on stochastic models involving extension of spatial statistics tools to include the additional time dimension. By assuming $Z(\underline{x}, t)$ as a spatial-temporal random variable, $Z(\underline{x}, t)$ can take a series of outcome values (realization) at any location in space \underline{x} and at any time t . The random variable $Z(\underline{x}, t)$ can be characterized by a joint probability distribution function of spatial coordinates (\underline{x}) and time (t). Two major conceptual approaches for modeling spatial-temporal variations were summarized which can be simply explained as follows;

- (1) A single spatial-temporal random field (RF) model is decomposed into trend component, representing average smooth variability of spatial-temporal process $Z(\underline{x}, t)$, and a stationary residual component, modeling fluctuations around the trend in both space and time. In this case, $Z(\underline{x}, t)$ can be written as

$$z(\underline{x}, t) = m(\underline{x}, t) + R(\underline{x}, t) \quad (2-3)$$

where $m(\underline{x}, t)$ is the trend component, which can be deterministic or stochastic, and $R(\underline{x}, t)$ is a zero mean stationary residual component.

- (2) A spatial-temporal process $Z(\underline{x}, t)$ is viewed as a set of temporally correlated random fields (RF), or a set of spatially correlated time series (TS) depending on which one is the more densely informed. Figure 2-3 illustrates the viewpoint underlying this approach for both cases.

To emphasize, the first approach allows continuous estimation at locations (\underline{x}) and time (t), whereas the second one gives estimation at a discrete point or a discrete time.

Water resource is also a field of study at which the researches about spatial-temporal variation are relatively active. Stein (1986) presented a mathematical model for spatial-temporal process, for which the observations are modeled as the sum of the random field fixed in time plus a second independent random field that varies both spatially and temporally. This can be considered as a modified form for the first approach presented above. Or and Hanks (1992) also proposed a spatial temporal estimation method to reduce the uncertainty in irrigation management, which basically stems from soil variability and climatic uncertainty. Temporal soil water storage estimates were obtained by the Kalman filter, while spatial estimates were obtained by conditional multivariate normal method. These spatial and temporal estimates were then combined an additional Kalman filter step that considers spatial estimates as a measurements. The separation of spatial uncertainty from temporal uncertainty indicates that this method should be classified in the second approach mentioned above.

The literatures relating to spatial-temporal modeling in the field of Geotechnical engineering are relatively rare and in general focus on the movement of the pore liquid within the soil mass. This

thesis aims to propose a practical approach to introduce spatial-temporal modeling in geotechnical design. Therefore, the temporal variation concerned in the current thesis is for the prediction of temporal soil behavior, such as ground settlement. Due to the fact that the temporal soil behavior basically can be explained by physical models of soil, it is more informative to characterize temporal variation than spatial variation, i.e. the pattern of spatial variation is less likely to be estimated. Therefore, the proposed approach separates spatial variation, which focuses on estimation and interpolation of the soil or model parameters, from the temporal variation, which mainly concerns the prediction of temporal soil behavior at a particular spot of time. In this sense, it can be concluded that the proposed approach shares the similar viewpoint with the second approach presented by Kyriakidis and Journel (1999) for the case that the random process is considered as a set of spatially correlated time series (TS) (see Fig. 2-3(b)), and also similar to the method presented by Or and Hanks (1992). Due to the fact that the proposed formulation can also be transformed to Kalman filter equations (see Appendix A), it can be viewed as a modification of Kalman filter by introducing the spatial correlation into the basic formulation of Kalman filter, which originally deals only with the time and observation updating problem.

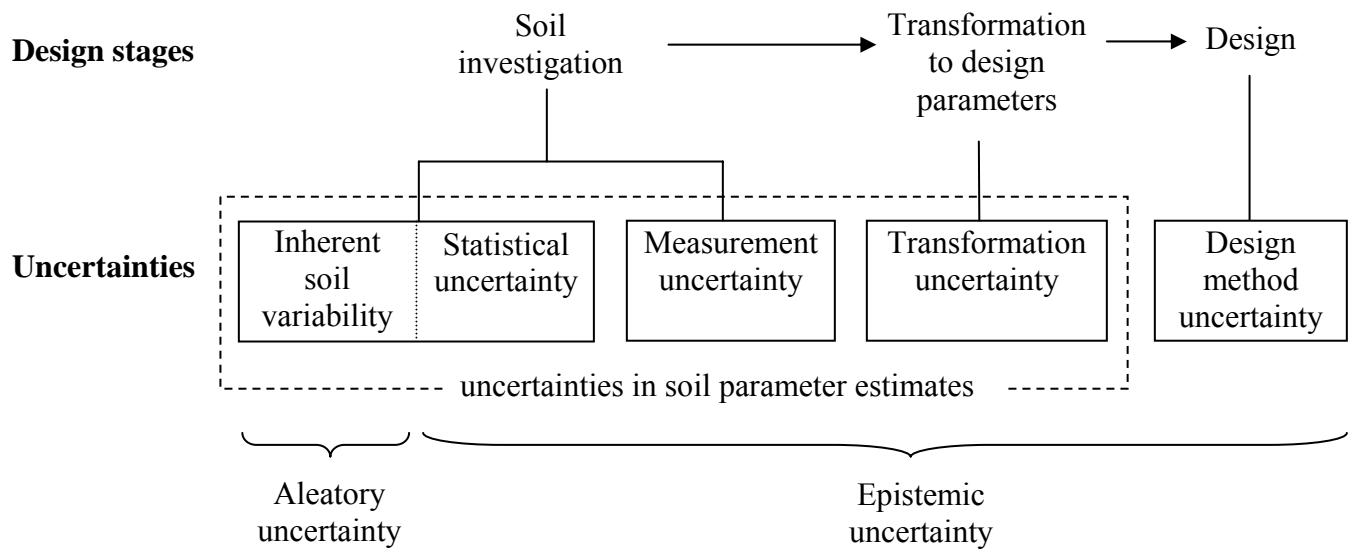


Figure 2-1: Uncertainties in geotechnical design.

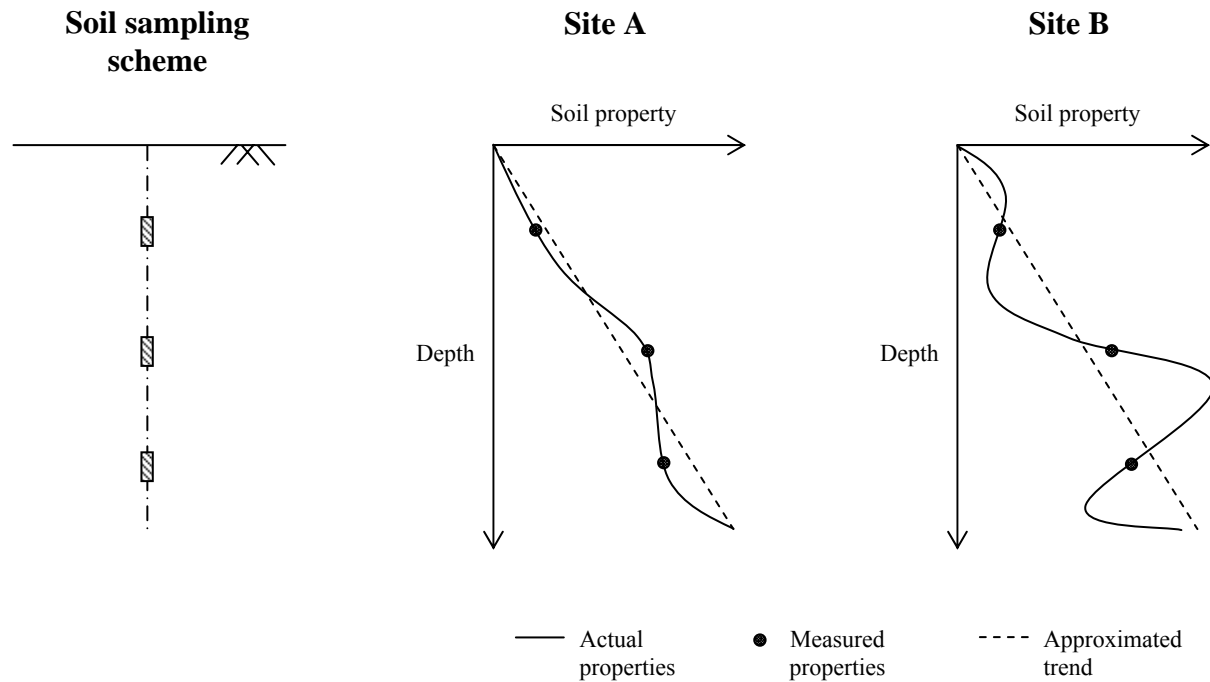


Figure 2-2: An example of subsoil investigations at two different sites.

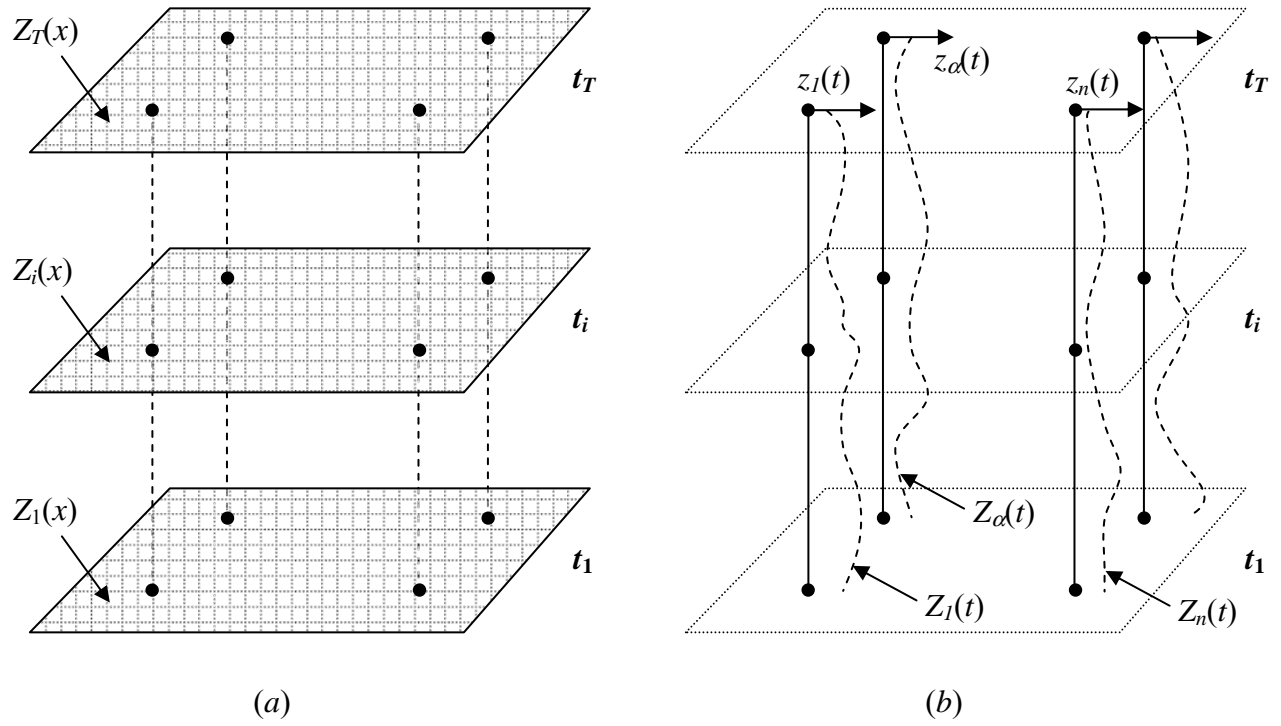


Figure 2-3: Conceptual models of spatial-temporal process when are viewed as (a) a set of random functions (RF) $Z_i(x)$, $i = 1, 2, \dots, T$; and (b) a set of vectors of time series (TS) $Z_\alpha(t)$, $\alpha = 1, 2, \dots, n$ (Kyriakidis and Journel 1999).

3 Spatial-temporal process

In this chapter, a formulation for observation-based design using spatial-temporal random process is proposed. In order to identify the best estimator of a stochastic Gaussian field of the unknown parameters employed in geotechnical model, the consecutively collected observation data at discrete spatial points and prior information are used with consideration of spatial correlation structure. The proposed formulation is derived and presented both in batch form based on Bayesian estimation and sequential form based on Kalman filter, by which the process noise can be considered. Application of the ABIC for estimation of auto-correlation distance and standard deviation of observation error is also proposed. For estimation of the unknown parameters at an arbitrary point, ordinary kriging method is also presented as an alternative for parameter interpolation. The application examples of the proposed approach for ground settlement prediction will be presented and discussed in the next chapter.

3.1 *Basic assumptions*

Prior to the detail description of the proposed method, it is important to clearly state the basic assumptions used in this approach. The assumptions are categorized as major and minor assumptions and summarized in the following subsections.

3.1.1 *Major assumptions*

The major assumptions contain the core premises of the proposed approach. Changing these assumptions require the major revise of the proposed formulation and calculation method. They,

therefore, reflect the application extent of the proposed approach. The major assumptions are listed as follows;

- (1) Temporal soil behavior (Y) is introduced based on geotechnical models, e.g. Asaoka's method, which describe ground behavior at an arbitrary point with time through the soil parameters employed in the model (Z). It is assumed in this research that the relation between soil parameters or, so called, 'unknown parameters' and the soil behavior is linear, i.e. $Y = M Z$.
- (2) The spatial correlation of the soil behaviors (Y) is introduced through the spatial correlation of the unknown parameters (Z) which characterize these behaviors. The uncertainties of the unknown parameters are assumed to be described by stochastic Gaussian field.
- (3) It is assumed that the concerning soil behaviors (Y) are observed at discrete time and discrete points on the ground. The observation errors (ε) are described by Gaussian distribution and are assumed to be both spatially and temporally independent.

3.1.2 Minor assumptions

The minor assumptions are mostly made for simplicity and applicability of the proposed formulation. However, they can be relieved in case the sufficient information is available without any major change of the basic formulation proposed. The minor assumptions are listed as follows;

- (1) The unknown parameters (Z) are independent of each other; however, they share the same spatial correlation structure.
- (2) Spatial correlation structure is characterized by exponential type auto-correlation function.
- (3) Process noises, if they are introduced, at each time step are independent. The covariance matrix of process noises (Q_k) is assumed to be proportional to the covariance matrix of the unknown parameters (V_Z).

3.2 Bayesian estimation considering spatial correlation structure

In order to improve the estimation and to enable local estimation, utilization of Bayesian estimation including spatial correlation is proposed in this research. This approach uses prior information of the unknown parameters which characterize soil behavior, e.g. model parameters or soil properties, and the observation data, e.g. observed settlement or movement, from all observation points to search for the best estimates of the unknown parameters. The formulation consists of two statistical components, namely, the observation model and the prior information model. These two models will then be combined by Bayes' theorem to obtain the solution.

3.2.1 Observation model

This model relates the observation data to the unknown parameters which are defined in a multivariate stochastic Gaussian field $Z(x) = [z_1(x), z_2(x), \dots, z_P(x)]^T$ where x is a spatial vector coordinate; P is total number of the unknown parameters; and $z_i(x)$ (for $i = 1, 2, \dots, P$) is a random function of an unknown parameter (e.g. soil or model parameter) at any location x in a specific domain. This paper proposes a method to identify the best estimator of Z for a discrete spatial point field, $x_1, x_2, \dots, x_n, x_{n+1}, \dots, x_m$, which is defined as

$$\hat{Z} = [\hat{z}_1, \hat{z}_2, \dots, \hat{z}_P]^T \quad (3-1)$$

where

$$\hat{z}_i = [\hat{z}_i(x_1), \hat{z}_i(x_2), \dots, \hat{z}_i(x_m)] \quad ; \quad i = 1, 2, \dots, P \quad (3-2)$$

Suppose that a set of observations Y_k (e.g. ground settlement) at the discrete time step k , i.e. $k = 1, 2, \dots, K$, has been obtained at n observation points x_1, x_2, \dots, x_n . Y_k is defined as

$$Y_k = [y_k(x_1), y_k(x_2), \dots, y_k(x_n)]^T \quad (3-3)$$

It should be noted that $x_{n+1}, x_{n+2}, \dots, x_m$ are defined as any arbitrary points at which the unknown parameters are to be estimated, i.e. $m-n$ interpolation points.

The general formulations of the observation model have been presented in several literatures, e.g. Hoshiya and Yoshida (1996), Honjo and Kashiwagi (1999), etc. Here it is assumed that the observation Y_k is expressed as a linear function of Z with observation error of ε as follows:

$$Y_k = M_k Z + \varepsilon \quad (3-4)$$

where ε is the Gaussian observation error vector which is assumed to follow $N(0, V_\varepsilon)$. V_ε is defined as a covariance matrix of ε where $V_\varepsilon = \sigma_\varepsilon^2 \cdot I_{n,n}$. σ_ε^2 is the variance of the observation error and $I_{n,n}$ is an $n \times n$ unit matrix. This implies that the observation errors are assumed to be spatially independent.

M_k is the $n \times (P \cdot m)$ coefficient matrix, which is defined as

$$M_k = \begin{bmatrix} M_{n,n}^1 & 0_{n,m-n} & M_{n,n}^2 & 0_{n,m-n} & \dots & M_{n,n}^P & 0_{n,m-n} \end{bmatrix} \quad (3-5)$$

where $M_{n,n}^i$ denotes $n \times n$ coefficient matrix, relating z_i to Y_k ; $0_{n,m-n}$ denotes $n \times (m-n)$ zero matrix, attaching to each $M_{n,n}^i$ to eliminate the unknown parameters at $m-n$ arbitrary points (i.e. $x_{n+1}, x_{n+2}, \dots, x_m$) from the observation model.

Given Z and σ_ε^2 , the predicted settlement distribution at any time step k can be represented by the following multivariate normal distribution

$$p(Y_k | Z, \sigma_\varepsilon^2) = (2\pi)^{-n/2} |V_\varepsilon|^{-1/2} \cdot \exp \left[-\frac{1}{2} (Y_k - M_k Z)^T V_\varepsilon^{-1} (Y_k - M_k Z) \right] \quad (3-6)$$

3.2.2 Prior information model

It is assumed that the prior information of the unknown parameters has the following structure

$$Z = Z_0 + \delta \quad (3-7)$$

where Z_0 is the prior mean vector ($P \cdot m$ dimension) at points x_1, x_2, \dots, x_m . It can be defined as

$$Z_0 = [z_{0,1}, z_{0,2}, \dots, z_{0,P}]^T \quad (3-8)$$

where

$$z_{0,i} = [z_{0,i}(x_1), z_{0,i}(x_2), \dots, z_{0,i}(x_m)] \quad ; \quad i=1,2,\dots,P \quad (3-9)$$

$z_{0,i}(x_j)$ can be generally defined as $z_{0,i}(x_j) = C_1^i + C_2^i \cdot x'_j + C_3^i \cdot y'_j + C_4^i \cdot x'_j y'_j + \dots$, depending on the shape of the trend components considered to be suitable for the specific model parameters. Note that x'_j and y'_j denote spatial coordinates at point x_j , while $C_1^i, C_2^i, C_3^i, \dots$ represent the constant coefficients of the trend for the corresponding unknown parameters z_i ($i = 1, 2, \dots, P$). These parameters can be either deterministic or unknown, depending on the assumption made. For the later case, these coefficients can be estimated as one of the hyperparameters based on ABIC which will be presented in Section 3.3.

δ represents the uncertainty of the prior mean of the unknown parameters which is assumed to follow $N(0, V_Z)$ where V_Z is a covariance matrix. By introducing the spatial correlation structure in the formulation of V_Z , we have

$$V_Z = \begin{bmatrix} \sigma_{z1}^2 V_C & & 0 \\ & \sigma_{z2}^2 V_C & \\ & & \ddots \\ 0 & & & \sigma_{zP}^2 V_C \end{bmatrix} \quad (3-10)$$

where $\sigma_{z1}^2, \sigma_{z2}^2, \dots, \sigma_{zP}^2$ represent the prior variance of the unknown parameters z_1, z_2, \dots, z_P , respectively. These variances also can be assumed to be either deterministic or unknown and, in the same way with the prior means, they can be estimated as one of the hyperparameters based on ABIC (see Section 3.3 for detail). V_C is the auto-covariance matrix which is defined as

$$V_C = \begin{bmatrix} \rho(|x_1 - x_1|) & \cdots & \rho(|x_1 - x_m|) \\ \vdots & \ddots & \vdots \\ \rho(|x_m - x_1|) & \cdots & \rho(|x_m - x_m|) \end{bmatrix} \quad (3-11)$$

$\rho(\underline{x}_i - \underline{x}_j)$ denotes the auto-correlation function where $\underline{x}_i, \underline{x}_j$ = spatial vector coordinate. Several analytical expressions have been proposed for the auto-correlation function but, in fact, none of them can claim any fundamental basis (Vanmarcke 1977a). The exponential type auto-correlation function is chosen for the current study because it is commonly used in geotechnical applications (e.g. Vanmarcke 1977a, Fenton and Griffiths 2002, Griffiths and Fenton 2004 etc.). The function is given as

$$\rho(\underline{x}_i - \underline{x}_j) = \exp[-|\underline{x}_i - \underline{x}_j|/\eta] \quad (3-12)$$

where η = auto-correlation distance. To emphasize, this parameter is assumed to be constant at any directions in the horizontal plane. This implies that the anisotropy of soil is not considered in this case. In addition, it should be kept in mind that this type of auto-correlation function is, in fact, chosen only as an example for an application of the proposed method. In practice, several types of autocorrelation functions may be tested and the one which fits most to the observation should be used.

From the above definitions, it is clear that the spatial correlation structure is included in the form of the spatial correlation of unknown parameters, which relate to soil properties, instead of that of ground behavior. The authors believe that this is the most suitable way to introduce the spatial correlation structure to the geotechnical model due to the fact that the physical correlation of the observed ground behavior actually results from the spatial correlation of soil properties.

It should also be noted that, for the sake of simplification, there are two important assumptions about the correlation structure for formulating the above covariance matrix (V_Z). Firstly, the unknown parameters, z_1, z_2, \dots, z_P , are assumed to be independent of each other. Secondly, the correlation structures of these parameters are identical, meaning that they share the same auto-correlation distance. In fact, these assumptions can be released without major change of the

formulation, if the observation data is available in the amount that the detail specification of the spatial correlation is possible.

Given η , prior means, and prior variances of the unknown parameters, the prior distribution of the unknown parameters is also assumed as a multivariate normal distribution of the following form

$$p(Z|\eta) = (2\pi)^{-(P \cdot m)/2} |V_Z|^{-1/2} \exp\left[-\frac{1}{2}(Z - Z_0)^T V_Z^{-1} (Z - Z_0)\right] \quad (3-13)$$

3.2.3 Bayesian estimation

Suppose that the set of observations Y_k at the discrete time step $k = 1, 2, \dots, K$ has already been obtained. By employing Bayes' theorem, the posterior distribution of the state vector Z can be formulated as

$$p(Z|Y, \sigma_\varepsilon^2, \eta) = c \cdot p(Z|\eta) \prod_{k=1}^K p(Y_k|Z, \sigma_\varepsilon^2) \quad (3-14)$$

where Y denotes the set of all observation data, i.e. $Y = (Y_1, Y_2, \dots, Y_K)$, and c denotes the normalizing constant. By substituting Eq. (3-6) and (3-13) into the above equation, we have

$$p(Z|Y, \sigma_\varepsilon^2, \eta) = c \cdot (2\pi)^{-(P \cdot m + K \cdot n)/2} |V_Z|^{-1/2} |V_\varepsilon|^{-K/2} \cdot \exp\left\{-\frac{1}{2}\left[(Z - Z_0)^T V_Z^{-1} (Z - Z_0) + \sum_{k=1}^K (Y_k - M_k Z)^T V_\varepsilon^{-1} (Y_k - M_k Z)\right]\right\} \quad (3-15)$$

The Bayesian estimator of Z , i.e. \hat{Z} , is the one that maximizes the above function. Therefore, it is equivalent to minimizing the following objective function

$$J(Z|\sigma_\varepsilon^2, \eta) = (Z - Z_0)^T V_Z^{-1} (Z - Z_0) + \sum_{k=1}^K (Y_k - M_k Z)^T V_\varepsilon^{-1} (Y_k - M_k Z) \quad (3-16)$$

It should be noted that σ_ε^2 and η are assumed to be given in this case. The Bayesian method, however, does not provide the rational way to determine these values. In order to choose the most

appropriate values of σ_ε^2 and η based on the information in hand, Akaike's Bayesian Information Criterion (ABIC) is introduced and presented in the next section.

3.3 *Model selection: Akaike's Bayesian Information Criterion*

Choosing appropriate values of σ_ε^2 and η can be considered as the model selection problem. Akaike's Bayesian Information Criterion (ABIC) introduced in this study is developed on the same information theory principal as Akaike's Information Criterion (AIC) which is specifically used to selected the best model from several alternative models (Akaike 1980, Honjo and Kashiwagi 1999). By considering σ_ε^2 and η as hyperparameters, the Bayesian likelihood can be formulated as follows:

$$L(\sigma_\varepsilon^2, \eta | Y, Z) = \int p(Z | \eta) \prod_{k=1}^K p(Y_k | Z, \sigma_\varepsilon^2) dZ \quad (3-17)$$

Substitute Eq. (3-6) and (3-13) into the above equation, we obtain

$$L(\sigma_\varepsilon^2, \eta | Y, Z) = \int_{-\infty}^{\infty} (2\pi)^{-(P \cdot m + K \cdot n)/2} |V_Z|^{-1/2} |V_\varepsilon|^{-K/2} \cdot \exp \left\{ -\frac{1}{2} \left[(Z - Z_0)^T V_Z^{-1} (Z - Z_0) + \sum_{k=1}^K (Y_k - M_k Z)^T V_\varepsilon^{-1} (Y_k - M_k Z) \right] \right\} dZ \quad (3-18)$$

By performing integration of the above equation, we have

$$L(\sigma_\varepsilon^2, \eta | Y, Z) = (2\pi)^{-K \cdot n/2} |V_Z|^{-1/2} |V_\varepsilon|^{-K/2} \left| \sum_{k=1}^K M_k^T V_\varepsilon^{-1} M_k + V_Z^{-1} \right|^{-1/2} \cdot \exp \left\{ -\frac{1}{2} \left[(\hat{Z} - Z_0)^T V_Z^{-1} (\hat{Z} - Z_0) + \sum_{k=1}^K (Y_k - M_k \hat{Z})^T V_\varepsilon^{-1} (Y_k - M_k \hat{Z}) \right] \right\} \quad (3-19)$$

Noted that \hat{Z} denotes Bayesian estimator of Z , which is obtained by minimizing objective function in Eq. (3-16). By substituting Eq. (3-16) into the above equation and taking natural logarithm, we have the log Bayesian likelihood

$$l(\sigma_\varepsilon^2, \eta | Y, Z) = -\frac{1}{2} \ln |V_Z| - \frac{K}{2} \ln |V_\varepsilon| - \frac{1}{2} \ln \left| \sum_{k=1}^K M_k^T V_\varepsilon^{-1} M_k + V_Z^{-1} \right| - \frac{1}{2} \left[J(\hat{Z} | \sigma_\varepsilon^2, \eta) \right] + const \quad (3-20)$$

The general definition of ABIC (Akaike 1980) is given as $ABIC = (-2) \log (\text{maximum Bayesian likelihood}) + 2 (\text{number of hyperparameters})$. In this case, the number of hyperparameters is fixed as 2. Thus, we have

$$ABIC = \ln |V_Z| + K \ln |V_\varepsilon| + \ln \left| \sum_{k=1}^K M_k^T V_\varepsilon^{-1} M_k + V_Z^{-1} \right| + \left[J(\hat{Z} | \sigma_\varepsilon^2, \eta) \right] + const \quad (3-21)$$

By optimizing the above equation, the values of σ_ε^2 , η , and the corresponding \hat{Z} that give the minimum value of the ABIC, can be obtained. This can be considered as the optimized selection of these parameters.

However, for the case that the prior mean and prior variance of the unknown parameters as defined in Section 3.2.2 is assumed to be unknown, Eq. (3-20) can be replaced by the following equation in order to include the estimation of these parameters into the calculation.

$$l(\sigma_\varepsilon^2, \eta, \sigma_{z1}^2, \sigma_{z2}^2, \dots, \sigma_{zP}^2, C_{11}^i, C_{22}^i, C_{33}^i, \dots | Y, Z) = -\frac{1}{2} \ln |V_Z| - \frac{K}{2} \ln |V_\varepsilon| - \frac{1}{2} \ln \left| \sum_{k=1}^K M_k^T V_\varepsilon^{-1} M_k + V_Z^{-1} \right| - \frac{1}{2} J(\hat{Z} | \sigma_\varepsilon^2, \eta) + const \quad ; \quad i=1, 2, \dots, P \quad (3-22)$$

It is clear that, in this case, the values of prior variance, i.e. $\sigma_{z1}^2, \sigma_{z2}^2, \dots, \sigma_{zP}^2$, and the coefficients of the trend components, i.e. $C_{11}^i, C_{22}^i, C_{33}^i, \dots$ ($i = 1, 2, \dots, P$) are also included in the calculation as the hyperparameters in addition to σ_ε^2 and η . The set of these parameters which gives the minimum value of the ABIC in Eq. (3-21) will be determined and also considered as the optimized selection of these parameters.

3.4 Process noise consideration by the Kalman filter method

In the previous section, the batch procedure by which all of the observations are treated equally for parameters updating was proposed. In practice, however, it is natural to give higher weight to the more recent observations. This can be done by considering a kind of forgetting factor, so called ‘process noise’, through a sequentially updating procedure, Kalman filter (Kalman 1960, Kalman & Bucy 1961, Jazwinski 1976). The higher process noise means the quicker the previously observed data will be ‘forgotten’ from the calculation as the observation time goes by.

The Kalman filter has two distinct phases: *time updating* and *observation updating*. The *time updating* phase uses the state estimate from the previous time step to produce the state estimate at the current time step by considering process noise. In the *observation updating* phase, measurement information at the current time step is used to refine this prediction in order to arrive at a new state estimate. In fact, it can be proved that, without process noise, the Kalman filter gives the same results as the previously proposed approach, i.e. Bayesian estimation. The details of this proof are shown in APPENDIX A.

For the estimation of geotechnical parameters, the unknown parameters is usually considered to be stationary, then the *Time updating* process can be expressed by

$$Z_{k/k-1} = Z_{k-1/k-1} \quad (3-23)$$

$$V_{Z,k/k-1} = V_{Z,k-1/k-1} + Q_k \quad (3-24)$$

where Z and V_Z represent, respectively, mean vector and covariance matrix as defined in Section 3.2.2; Q_k denotes covariance matrix of process noise; suffix k stands for the k th step of processing, e.g. $k/k-1$ represents the k th step estimation conditioned on processing observation data up to the $k-1$ th datum.

In order to systematically define the value of process noise, we assume that Q_k are given based on a priori information, as follows (Hoshiya & Yoshida, 1998):

$$Q_k = aV_{Z,k-1/k-1} \quad (3-25)$$

where a is a constant parameter representing the level of process noise. From Eq. (3-24) and (3-25), we have

$$V_{Z,k/k-1} = (1+a)V_{Z,k-1/k-1} \quad (3-26)$$

For *Observation updating* process, the updated estimates and covariance matrix of unknown parameters based on the assumption of linear relationship between Z and the observation, Y_k , as previously defined in Eq. (3-4) is given as follows;

$$Z_{k/k} = Z_{k/k-1} + K_k (Y_k - M_k Z_{k/k-1}) \quad (3-27)$$

$$V_{Z,k/k} = [I - K_k M_k] V_{Z,k/k-1} \quad (3-28)$$

By defining the Kalman gain

$$K_k = V_{Z,k/k-1} M_k^T (V_\varepsilon + M_k V_{Z,k/k-1} M_k^T)^{-1} \quad (3-29)$$

where M_k and V_ε represent, respectively, coefficient matrix of the observation model and covariance matrix of observation error as defined in Section 3.2.1; suffix k/k , similarly, represents the k th step estimation conditioned on processing observation data up to the k th datum. It should be emphasized that $Z_{0/0}$ and $V_{Z,0/0}$ are needed to be defined in the same way as Z_0 and V_Z presented in Section 3.2.2, in order to take into account the prior information of the model parameters together with the spatial correlation structure.

With its ability to sequentially update the estimation and systematically take into account process noise, this method is used in estimation of the unknown parameter (Z), while σ_ε^2 and η are estimated based on ABIC as described in Section 3.3.

3.5 Local estimation by the kriging method

Section 3.2 presents the general equations by which the estimates of the unknown parameters at any arbitrary points, together with those at the observation points, can be calculated in the same format. However, based on the calculated estimates of the unknown parameters (\hat{Z}) at the observation points and the estimated auto-correlation distance, the statistics of the parameters at any arbitrary locations can also separately be determined by the *ordinary kriging* (Krige 1966, Matheron 1973, Wackernagel 1998). This method provides an unbiased and least error estimator built on the data from a random field for which second-order stationary is assumed. In fact, this method can also be derived based on aforementioned concept of Bayesian method. This emphasizes that all of the proposed formulations of spatial-temporal process shown in this thesis are found on the same basic concept of Bayesian approach. The derivation is summarized and presented in APPENDIX B.

Based on the unknown parameters, z_i , at the n observation points, i.e. $Zo_i = [\hat{z}_i(x_1), \hat{z}_i(x_2), \dots, \hat{z}_i(x_n)]$, the value of the parameters, at $m-n$ arbitrary points, i.e. $Zr_i = [\hat{z}_i(x_{n+1}), \hat{z}_i(x_{n+2}), \dots, \hat{z}_i(x_m)]$, can be estimated by the following equations:

$$\begin{bmatrix} Zr_1 \\ Zr_2 \\ \vdots \\ Zr_p \end{bmatrix} = \begin{bmatrix} Zo_1 \\ Zo_2 \\ \vdots \\ Zo_p \end{bmatrix} \begin{bmatrix} w_{1,1} & w_{1,2} & \cdots & w_{1,m-n} \\ w_{2,1} & w_{2,2} & \cdots & w_{2,m-n} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n,1} & w_{n,2} & \cdots & w_{n,m-n} \end{bmatrix} \quad (3-30)$$

where

$$\begin{bmatrix} w_{1,1} & \cdots & w_{1,m-n} \\ \vdots & \ddots & \vdots \\ w_{n,1} & \cdots & w_{n,m-n} \\ \mu_1 & \cdots & \mu_{m-n} \end{bmatrix} = \begin{bmatrix} \rho(|x_1 - x_1|) & \cdots & \rho(|x_1 - x_n|) & -1 \\ \vdots & \ddots & \vdots & \vdots \\ \rho(|x_n - x_1|) & \cdots & \rho(|x_n - x_n|) & -1 \\ 1 & \cdots & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} \rho(|x_1 - x_{n+1}|) & \cdots & \rho(|x_1 - x_m|) \\ \vdots & \ddots & \vdots \\ \rho(|x_n - x_{n+1}|) & \cdots & \rho(|x_n - x_m|) \\ 1 & \cdots & 1 \end{bmatrix} \quad (3-31)$$

w_{ij} ($i = 1, 2, \dots, n; j = 1, 2, \dots, m-n$) are the weights attached to the data at each of the observation points. μ_j ($j = 1, 2, \dots, m-n$) is the Lagrange multiplier used for minimizing the kriging

error. $\rho(|\underline{x}_i - \underline{x}_j|)$ represents the auto-correlation function as defined in Eq. (3-12). The ordinary kriging error at any arbitrary point, x_{n+j} ($j = 1, 2, \dots, m-n$), is represented by the variance, σ_{or}^2 , as follows (Wackernagel 1998);

$$\sigma_{or}^2(x_{n+j}) = \rho(|\underline{x}_{n+j} - \underline{x}_{n+j}|) - \left[\begin{array}{c} \rho(|\underline{x}_1 - \underline{x}_{n+j}|) \\ \vdots \\ \rho(|\underline{x}_n - \underline{x}_{n+j}|) \\ -1 \end{array} \right]^T \left[\begin{array}{c} w_{1,j} \\ \vdots \\ w_{n,j} \\ \mu_j \end{array} \right] \quad ; \quad j=1,2,\dots,m-n \quad (3-32)$$

It should be emphasized that the spatial-temporal updating process previously proposed in Section 3.2 implicitly includes *simple kriging* in the formulations. Similar to ordinary kriging, the simple kriging is also a technique to interpolate the value of the random field at an unobserved location using the values at the observation points. The random field is assumed to be second-order stationary for both techniques, which means the random parameter has constant mean all over the domain. However, simple kriging is different from ordinary kriging in that this constant mean is assumed to be given prior to the calculations, while it is an unknown in ordinary kriging. Simple kriging actually is a method for estimating the residuals from this given mean value and, therefore, is sometime called ‘kriging with known mean’ (Wackernagel 1998). In addition, one can perform another kriging technique, so called ‘kriging the mean’ to estimate this constant mean and use it as the mean for simple kriging. This, in fact, gives the identical results with those of ordinary kriging. The assumptions and the equations relating to these three kriging techniques are summarized in APPENDIX C.

By performing the proposed approach for estimation of the unknown parameters at an arbitrary location, it is equivalent to applying simple kriging to interpolate the estimates at the observation points to those at unobserved point by considering the prior mean of the unknown parameters as the given mean for interpolation. It should be noted that the prior information strongly

contributes to the interpolation process in this case. On the other hand, by using ordinary kriging for interpolation, the unknown mean is implicitly evaluated using the most recently updated estimates at the observation points and the kriging estimators at the unobserved points will then be calculated based on this mean. It is clear that, in this case, the observation data is considered more in the interpolation process. To sum up, using the proposed formulation for estimation of the unknown parameters at an arbitrary point can be advantageous in case the rational prior information is available; otherwise, the use of ordinary kriging for parameter interpolation can be considered as an appropriate alternative.

4 Application examples

A formulation for introducing spatial correlation to the temporal model of geotechnical design has been proposed in previous chapter. In this chapter, the application of the proposed method in settlement prediction models will be presented. Two of the settlement-time related models are chosen, consisting of Asaoka's method for primary consolidation prediction and $S \sim \log(t)$ method for secondary compression prediction. Several parametric studies have been conducted based on the simulated data and the results are presented and discussed in this chapter. A case study using the actual observation data of the secondary compression in peat is also performed based on $S \sim \log(t)$ method in order to illustrate an practical application of the proposed approach. All of the calculations are implemented by a computer program written by FORTRAN 90, the source codes of which are presented in Appendix F.

4.1 Primary consolidation settlement prediction by Asaoka's method

4.1.1 Settlement prediction model

The settlement prediction model introduces in this section is the first-order autoregressive model proposed by Asaoka (1978). The model is applicable for one-dimensional consolidation and is used for predicting the primary consolidation settlement based on the previously observed settlement data, as follows:

$$S_k = \beta_0 + \beta_1 S_{k-1} + \varepsilon_k \quad (4-1)$$

where S_k = observed settlement at k th step of observation, β_0 and β_1 = constant parameters for the Asaoka's model, and ε_k = observation error. It is assumed that ε_k follows $N(0, \sigma_\varepsilon)$, where σ_ε denotes standard deviation of the observation error.

In practice, one of the most required information is the final settlement (S_f). This can be simply estimated based on the parameters of the Asaoka's model. By defining the final state as the state at which the settlement stops increasing with time (e.g. the stable state, at $k \rightarrow \infty$), it can be concluded that, at this state, $S_k = S_{k-1} = S_f$, where S_f refers to the final settlement. By substituting this to Eq. (4-1) and theoretically assuming $\varepsilon_k = 0$, we have $S_f = \beta_0 + \beta_1 S_f$. Thus, the final settlement

$$S_f = \frac{\beta_0}{1 - \beta_1} \quad (4-2)$$

Suppose that the consolidation settlement at n observation point, x_1, x_2, \dots, x_n , has been sequentially observed at discrete time step k , $k = 0, 1, \dots, K$. Based on the Asaoka's model, the components of the observation model equations given in section 3.2.1 can be defined as follows:

$$\hat{Z} = \left[\hat{\beta}_1(x_1), \hat{\beta}_1(x_2), \dots, \hat{\beta}_1(x_m), \hat{\beta}_0(x_1), \hat{\beta}_0(x_2), \dots, \hat{\beta}_0(x_m) \right]^T \quad (4-3)$$

$$Y_k = \left[S_k(x_1), S_k(x_2), \dots, S_k(x_n) \right]^T \quad (4-4)$$

$$M_k = \begin{bmatrix} S_{k-1}(x_1) & 0 & \vdots & 0_{n,m-n} & I_{n,n} & 0_{n,m-n} \\ & \ddots & & & & \\ 0 & S_{k-1}(x_n) & \vdots & & & \end{bmatrix} \quad (4-5)$$

where $I_{n,n}$ denotes $n \times n$ unit matrix. Note that, in this case, the total number of unknown parameters (P) is 2, while \hat{z}_1 and \hat{z}_2 are represented by $\hat{\beta}_1$ and $\hat{\beta}_0$, respectively. As stated previously in Section 3.2.1, it is also assumed that $V_\varepsilon = \sigma_\varepsilon^2 I_{n,n}$. This implies that the observation errors are assumed to be independent.

As for the prior information model presented in Section 3.2.2, it is assumed in this calculation that the prior mean is constant through the region. In other words, trend component of the prior information can be represented by only single value, i.e. $z_{0,i}(x_j) = C^i_1$. Therefore, the components of the prior information model equations given in section 3.2.2 can be defined as follows:

$$Z_0 = \left[\hat{\beta}_{1,0}(x_1), \hat{\beta}_{1,0}(x_2), \dots, \hat{\beta}_{1,0}(x_m), \hat{\beta}_{0,0}(x_1), \hat{\beta}_{0,0}(x_2), \dots, \hat{\beta}_{0,0}(x_m) \right]^T \quad (4-6)$$

$$V_Z = \begin{bmatrix} \sigma_{\beta_{1,0}}^2 V_C & 0_{n,n} \\ 0_{n,n} & \sigma_{\beta_{0,0}}^2 V_C \end{bmatrix} \quad (4-7)$$

where $\hat{\beta}_{1,0}(x_j)$ and $\hat{\beta}_{0,0}(x_j)$ denote, respectively, the prior mean of β_1 and β_0 at point x_j , i.e. $\hat{\beta}_{1,0}(x_j) = C^1_1$ and $\hat{\beta}_{0,0}(x_j) = C^2_1$. $\sigma_{\beta_{1,0}}^2$ and $\sigma_{\beta_{0,0}}^2$ represent the prior variance of β_1 and β_0 (i.e. σ_{z1}^2 and σ_{z2}^2 in Eq. (3-10)), respectively. $0_{n,n}$ denotes an $n \times n$ zero matrix. It should be noted that, using this model, the spatial correlation of ground settlements is introduced through the spatial correlation of the model parameters which are β_1 and β_0 . This assumption is considered to be a natural one, because it is the soil properties that control the spatial variation of the ground settlement.

4.1.2 Simulation experiments

4.1.2.1 Random field generation by frequency-domain technique

To investigate the performance of the proposed approach using the simulated data, one- and two-dimensional random values of the model parameters are generated based on the assumed mean, variance, and auto-correlation distance. The observed settlement data is then calculated by Eq. (4-1), using the generated parameters and the assumed standard deviation of the observation error, σ_ε . It should be emphasized that the generated settlement data simulates the field observation data of an area with predetermined spatial correlation structure. Performing the spatial-temporal updating

procedure previously stated in Section 3.2 based on the generated observation data, the statistical inferences of the model parameters at each point can be back-calculated. Comparisons of these inferences with the simulated ones, namely the true values, reveal the efficiency of the procedure.

Various techniques have been proposed by several authors for random field generation, e.g. the turning bands method (Matheron 1973), frequency domain technique (Shinozuka 1971, Shinozuka and Jan 1972), and local average subdivision method (Fenton and Vanmarcke 1990). The frequency domain technique is chosen for this study to avoid the streaking problem which is found in the turning bands method, and implementing difficulties which are common issues for the local average subdivision method (Fenton 1994). This technique concentrates on the spectral density function (SDF) of the process, which is defined as the Fourier transform of the auto-correlation function. For an exponential type auto-correlation function, it can be proved that the SDF for one-dimensional random process is

$$S(\omega) = \frac{1}{\pi\eta\left(\frac{1}{\eta^2} + \omega^2\right)} \quad (4-8)$$

which is a function of circular frequency domain (radian/sec), i.e. ω . The detailed derivations of Eq. (4-8) based on the formulation proposed by Shinozuka (1971) is summarized in APPENDIX D.

Assuming that the power of the employed SDF is negligible outside the interval $[-\omega_0, \omega_0]$, the simulated stationary Gaussian random field at any coordinate, x , can be expressed as the following series of cosine functions

$$X(x) = \sum_{j=1}^M \sqrt{2S(\omega_j)\Delta\omega} \cdot \cos(\omega_j x + \phi_j) \quad (4-9)$$

where $\Delta\omega = 2\omega_0/M$, $\omega_j = -\omega_0 + (j-1/2)\Delta\omega$, and ϕ_j = random phase angles which uniformly and independently distribute in the interval $(0, 2\pi)$. M is the number of equally divided intervals of the range $[-\omega_0, \omega_0]$.

Based on Eq. (4-9), a realization of a random parameter with the specified mean and standard deviation at any coordinate (x) can be calculated by considering $X(x)$ as a realization of standard normal distribution, i.e. $X(x)$ follows $N(0,1)$.

For two-dimensional random process, SDF is defined as

$$S(\omega_1, \omega_2) = \frac{1}{2\pi\eta \left(\frac{1}{\eta^2} + (\omega_1^2 + \omega_2^2) \right)^{3/2}} \quad (4-10)$$

which is a function of circular frequency domain, i.e. ω_1 , and ω_2 . The detailed derivations of above equation based on the formulation proposed by Shinozuka (1971) is also summarized in APPENDIX D.

Assuming that the power of the employed SDF is negligible outside the interval $[-\omega_{1,0}, \omega_{1,0}]$ and $[-\omega_{2,0}, \omega_{2,0}]$, the simulated stationary Gaussian random field at any coordinate (x, y) can be expressed as the following series of cosine functions

$$X(x, y) = \sum_{k=1}^{M_2} \sum_{j=1}^{M_1} \sqrt{2S(\omega_{1j}, \omega_{2k})\Delta\omega_1\Delta\omega_2} \cdot \cos(\omega_{1j}x + \omega_{2k}y + \phi_{jk}) \quad (4-11)$$

where $\Delta\omega_1 = 2\omega_{1,0}/M_1$, $\Delta\omega_2 = 2\omega_{2,0}/M_2$, $\omega_{1j} = -\omega_{1,0} + (j-1/2)\Delta\omega_1$, $\omega_{2k} = -\omega_{2,0} + (k-1/2)\Delta\omega_2$, and ϕ_{jk} = random phase angles which uniformly and independently distribute in the interval $(0, 2\pi)$. M_1 and M_2 are the number of equally divided intervals of the range $[-\omega_{1,0}, \omega_{1,0}]$ and $[-\omega_{2,0}, \omega_{2,0}]$, respectively. Care must be taken when selecting these ranges and discretization intervals to ensure that the spectral density function is adequately approximated. In this section, several values of

intervals and ranges are tested to ensure the quality of the generated random values. As a result, the number of intervals, i.e. M_1 and M_2 , are chosen to be 200 and the ranges, i.e. ω_0 , $\omega_{1,0}$ and $\omega_{2,0}$, are set as the values of ω , ω_1 and ω_2 , at which the values of SDF become 0.25% and 0.5 % of the $S(0)$ for one- and two-dimensional cases, respectively. Based on this criterion, ω_0 , $\omega_{1,0}$ and $\omega_{2,0}$ are set as $[(1-0.0025)/0.0025]^{1/2}/\eta$ and $[(1/0.005^{2/3})-1]^{1/2}/\eta$ for one- and two-dimensional random process, respectively.

Based on Eq. (4-11), a realization of a random parameter with the specified mean and standard deviation at any coordinate (x, y) can be calculated by considering $X(x, y)$ as a realization of standard normal distribution, i.e. $X(x, y)$ follows $N(0,1)$.

Three-dimensional random process is out of scope of this research. However, in order to completely illustrate the practical usages of this random field generation technique, the derivation of the three-dimensional case is also provided in APPENDIX D.

4.1.2.2 Improvement of the estimation by considering spatial correlation structure

A series of simulation experiments was performed based on the aforementioned procedure. For simulation of the model parameters, it is assumed that the mean and standard deviation of the random field of β_1 are 0.9791 and 0.0028 and those of β_0 are 6.94 (cm) and 0.59 (cm), respectively. These values are chosen from the data presented by Asaoka (1978) based on the observations of Kobe Port No. 3. It should be noted that, based on Eq. (4-2), the first-order approximate mean and standard deviation (Ang & Tang 2007) of the final settlement, S_f , are 332 (cm) and 53 (cm), respectively, assuming statistical independency between β_1 and β_0 . This level of uncertainty for final settlement estimation is considered to be common in engineering practice.

The initial settlement (S_0) is set as zero for every observation points. For the current study, the observation error (σ_e) is assumed to be 1.0 (cm). By assigning the desired values of auto-correlation distance (η), random values of the model parameters together with the observed settlements at each observation point can be generated by the procedure described in Section 4.1.2.1.

Based on the generated observation data, the procedure proposed in Section 3.2 is performed by assuming that the prior mean and standard deviation of the model parameters (see Eq. (4-6) and (4-7)) equal to those used for the data generation, i.e. $\hat{\beta}_{1,0}(x_1) = \hat{\beta}_{1,0}(x_2) = \dots = \hat{\beta}_{1,0}(x_m) = 0.9791$, $\sigma_{\beta_{1,0}} = 0.0028$, and $\hat{\beta}_{0,0}(x_1) = \hat{\beta}_{0,0}(x_2) = \dots = \hat{\beta}_{0,0}(x_m) = 6.94$ (cm), $\sigma_{\beta_{0,0}} = 0.59$ (cm). In practice, these statistics can be chosen based on field observation data, laboratory test results, or even judgment of an experienced engineer. For example, one can estimate the value of β_1 using coefficient of consolidation (C_v), which can be obtained from consolidation test, based on the relationship between these two parameters (Asaoka 1978).

In this subsection, the auto-correlation distance and the observation error are assigned the same values as those used for generating the simulated data, namely the true values, in order to focus on the effect of considering spatial correlation. In other words, the model selection process presented in Section 3.3 is not included in the calculations in this subsection.

To examine the advantages of considering spatial correlation structure, the Bayesian estimation, using the observed settlement of an observation point to estimate the model parameters at that point itself, i.e. ignoring spatial correlation structure, is also performed based on the same conditions. In fact, the results of the calculations in the case with considering spatial correlation will converge to those of this case if very short spatial correlation distance is assumed. The estimations based on these two different conditions are compared and presented later in this subsection.

To verify the conclusion of the calculations, different model parameters are randomly generated for several times (N_{sim}) and the estimation errors are calculated at selected time steps. These errors are represented by the term of mean and standard deviation (SD) of the error ratio, E_r , which is defined as

$$E_r = \ln \left(\frac{X_{est}}{X_{true}} \right) \quad (4-12)$$

where X_{est} and X_{true} denote the estimated value and true value, respectively, of the parameter to be estimated at each point for each simulation. It should be noted that $E_r = +0.1$ or -0.1 imply the X_{est}/X_{true} ratios of about 1.1 or 0.9, respectively.

In order to demonstrate the usages of the proposed method, a simple example based on one-dimensional random process will be presented prior to the more common examples of two-dimensional cases. The detail calculations and the results of both cases are presented and discussed in the following parts.

One-dimensional random process

Assuming that we have n observation points, x_1, x_2, \dots, x_n , lining up on a single line with even spacing, s , the one-dimensional random process can be generated based on frequency domain technique as presented in Section 4.1.2.1. To test the quality of the generated random values, the correlation coefficient of the values at each pair of observation points is plotted versus the ratio of the distance between the points ($|x_i - x_j|$) to the auto-correlation distance (η) as shown in Figure 4-1. The theoretical curve of the exponential type auto-correlation function (see Eq. (3-12)) is also drawn as a solid line for comparison purpose. It can be seen that the generated random values are fitted quite well to the theoretical curve. This confirms that, to some extent, the quality of the generated data is acceptable.

Based on these random values, a set of sampled values of model parameters (β_1 and β_0) for the case that the assumed number of observation points (n) = 10 and the s/η ratio = 0.2 are illustrated in Figure 4-2. By assuming the observation error (σ_e) to be 1.0 (cm), the examples of the generated settlement-time curves for these 10 observation points can be presented as shown in Figure 4-3. Note that the time is represented in the form of ‘time step’ which reflects the step of observation based on the even interval of time.

Using the generated settlement data up to each time step, the estimates of the model parameters at that time step can be calculated based on the proposed method. Figure 4-4 shows the plots these calculations for the case which spatial calculations are considered and ignored, together with the true values at an observation point. It is clear from the figure that the considering spatial correlation case gives better estimations than the ignoring case, i.e. closer to the true values.

To confirm this conclusion, different model parameters are randomly generated for 1000 times ($N_{sim} = 1000$) and the estimation errors at selected time steps are calculated and presented by the error ratio, E_r , as defined in Eq. (4-12). Figure 4-5 shows the comparison of the means and standard deviations of E_r between the estimations with and without considering spatial correlation at 50th time step at each of the observation points. From the fact that the means of E_r is close to zero, it can be concluded that the bias of the estimations is negligible. Considering the standard deviation of E_r which directly relates to the level of estimation error, the case with considering spatial correlation gives much lower SD than another case. This confirms that considering spatial correlation significantly improves the estimation by reducing the estimation error.

Two-dimensional random process

To further examine the usage of the proposed approach, several examples based on two-dimensional random process are performed. Three different layouts of observation plans with 16, 36,

and 64 observation points (n) are set in order to investigate the effect of sampling size. All of these are arranged in a square grid pattern with even spacing of s and total width of L , as shown in Figure 4-6.

Similar to those shown in one-dimensional case, Figure 4-7 demonstrates the stepwise updated estimates of the model parameters and the final settlement at point A in Figure 4-6 for $n = 36$ and $s/\eta = 0.5$, i.e. the spacing of the observation points is half of auto-correlation distance, based on the generated settlement data from only one time sampling. The observation error (σ_e) is also assumed to be 1.0 (cm). The estimated values and true values of final settlement are calculated based on Eq. (4-2) using the corresponding values of the model parameters. It can be seen that, with consideration of spatial correlation, the estimation is improved, i.e. closer to the true values.

To study in the more systematic way, the calculations with 100 times random simulation ($N_{sim} = 100$) are also performed. These errors are represented by the term of mean and standard deviation (SD) of the error ratio, E_r , as defined in Eq. (4-12). Note that, because only estimation at the observation points is considered in this case, the total number of estimated values used for calculation of mean and standard deviation of E_r is $n \times N_{sim}$, i.e. $36 \times 100 = 3600$. In this study, it is found that 100-time simulations are sufficient for obtaining the stable statistics of the results.

Figure 4-8 illustrates the plots of the mean and standard deviation of E_r of the model parameter and final settlement estimation against time factor, T_v , for $n = 36$ and $s/\eta = 0.5$. The time factor is chosen to represent the observation time in replace of the observation time step because it provides more information relating to the degree of consolidation. The time factor at a specific time step can be calculated based on the relationship between β_1 and the observation time interval derived from Asaoka's formulation (Asaoka, 1978) and the approximated one-dimensional consolidation equation. The equation is given as

$$\Delta T_v = \frac{4(1-\beta_1)}{\pi^2 \beta_1} \quad (4-13)$$

where ΔT_v denotes the time factor interval which corresponds to the constant time interval between observations. T_v can then be obtained by multiplying ΔT_v by the time step. β_1 is assumed to be the mean of the random field used in data generation process, i.e. $\beta_1 = 0.9791$ in this case.

Figure 4-8 clearly shows that the standard deviations of the error ratio, E_r , for the cases of considering the spatial correlation structure are lower than those of ignoring spatial correlation structure, regardless of the observation time. This confirms that the estimation can be improved by taking into account the spatial correlation structure in terms of reduction of the estimation error. Moreover, with insignificantly low values of the means of E_r , it can be concluded that the bias of the estimation is negligible. These trends are the same for both model parameter and the final settlement estimation.

To investigate the sensitivity of this improvement for different soil and observation conditions, the same calculations for several sampling sizes (n), and ratios of observation spacing to auto-correlation distance (s/η) are performed. Then, the mean and standard deviation of the estimation errors at the 20th time step, which corresponds to T_v of 0.164, are calculated and summarized in Table 4-1.

The ‘improvement’ columns in this table present the reduction (in percent) of the standard deviation of E_r by considering spatial correlation structure, compared with the case of ignoring spatial correlation structure. This value represents the level of improvement of parameter estimation by taking into account the spatial correlation structure.

It can be seen from Table 4-1 that the improvement values increase with the reduction of s/η ratio. This leads us to conclude that a stronger spatial correlation gives a more accurate estimation.

According to the simulations, significant improvement seems to be found when the spacing of the observation points is shorter than half of the auto-correlation distance. On the other hand, for the case that the spatial correlation distance is relatively short comparing to the observation spacing, i.e. $s/\eta \geq 0.5$, enlarging the sampling size with constant spatial correlation structure does not greatly improve the accuracy of the estimation. This result can be expected because, for the site with relatively weak spatial correlation, it is only neighboring observation which contributes to the improvement of the estimation. These conclusions are the same for both model parameter and final settlement estimations. The biases of the estimation, which are reflected by the means of E_r , are relatively small for all estimations.

The application examples of the proposed method in 2-dimensional random process will be shown more into detail in Section 4.1.2.3 and 4.1.2.4.

4.1.2.3 Estimation of auto-correlation distance and observation error

In the previous section, the true values of auto-correlation distance (η) and standard deviation of the observation error (σ_e) are assumed to be known and are used in the estimation procedure. In practice, however, these parameters are unknown and needed to be estimated based on the observation data. It was proposed in Section 3.3 that these parameters can be appropriately selected by an optimization procedure based on Akaike's Bayesian Information Criterion. By performing several numerical experiments, the statistical inferences of the error ratio, E_r , of this estimation for several conditions can also be investigated.

Table 4-2 summarizes the mean and standard deviation of the E_r of auto-correlation distance and standard deviation of the observation error estimation for sampling size (n) of 16, 36 and 64 (see Fig. 4-6) at the 20th time step ($T_v = 0.164$). L is the total width of the group of observation points as

shown in Figure 4-6. The number of simulations for each trial is 100. The other parameters, such as $\hat{\beta}_{1,0}$, $\hat{\beta}_{0,0}$, $\sigma_{\beta 1,0}$, $\sigma_{\beta 0,0}$, σ_ε , and S_0 , are assigned the same values as those stated in Section 4.1.2.2.

Table 4-2 clearly shows that the error of η estimation is much higher than that of σ_ε estimation. With the standard deviation of E_r below 0.04, it is concluded that σ_ε can be accurately estimated by the proposed approach. Judging from the high positive values of the means of E_r , significant level of bias is found in η estimation. However, the error of η estimation tends to reduce with the increase of L/η ratio. Increasing the number of observation points does not greatly improve the accuracy of η estimation in this case. In addition, it should be noted that Table 4-2 shows only the estimation errors at the 20th time step. Any estimation at a later stage can give the lower level of error due to the larger amount of observation data included in the calculation.

With the significant error of η estimation, the sensitivity of the settlement predictions to this error is of interest. This will be investigated and discussed later in the next section.

4.1.2.4 Estimation of final settlement at an arbitrary location

As mentioned previously, one of the advantages of the proposed method is its ability to estimate the settlement at any arbitrary location and at any arbitrary time. It is shown in section 3.2.1 that the component of the unknown parameters at any unobserved points, i.e. x_{n+1} , x_{n+2} , ... , x_m , are included in the formulation and the estimates of the parameters at these points will be calculated, together with those at the observation points, by the optimization process based on Bayesian approach. Then, the final settlement at these unobserved points can be predicted using Eq. (4-2). To investigate the level of error for this prediction, a series of numerical examples was performed, the results of which are shown and discussed in this section.

Figure 4-9 shows the plan of the observation points and the location of the points to be considered for settlement prediction. Several calculations are performed based on different values of s/η ratio and observation period (T_v), the results of which are summarized in Table 4-3. The number of simulations for each calculation is 100. The values of other parameters are also the same as those assigned in Section 4.1.2.2.

Table 4-3 summarizes the standard deviation of the estimation error for final settlement, S_f . Estimation error is also represented by error ratio, defined in Eq. (4-12), with the true values at an arbitrary point determined by the simulated model parameters at that points based on the same random field with those of the observation points. The means of E_r are also found to be negligible in this case; therefore, they are chosen not to be shown in this table. Concerning the error of η estimation as discussed in the previous section, Table 4-3 also shows the comparison between the estimation using the true value of both η and σ_ε (Case A) and that using the estimated values of these parameters (Case B).

The advantages of including the spatial correlation structure into the settlement estimation can clearly be seen from Table 4-3. For the site that the spatial correlation of the soil parameters is relatively strong, i.e. $s/\eta = 0.25$, the final settlement can be predicted with a similar level of accuracy at the point located within the group of observation points or within the length of auto-correlation distance around the group, i.e. at points 1, 2, and 3. This level of accuracy will be reduced with the increase of the distance from the group of observation points to the point to be considered. On the other hands, for the site that the soil parameters tend to be independent, i.e. $s/\eta = 5$, the errors of the settlement estimation by the proposed method are similar, regardless of the locations. The difference between the errors for the estimations at the earlier stage, i.e. at $T_v = 0.164$ (the 20th time step), and at the later stage, i.e. at $T_v = 0.424$ (the 50th time step), is noticeable only for the case that the spatial

correlation is strong and, especially, at the points within the range of spatial correlation distance. These results emphasize the merit of considering spatial correlation structure for the local estimation, especially, when the soil parameters are strongly correlated in space, which is quite usual. Furthermore, the estimation errors for Case A and Case B are similar in any conditions. This confirms the insensibility of the proposed approach with the value of auto-correlation distance, which makes the approach practical even though the true value of the auto-correlation distance is, in some cases, difficult to be obtained.

4.2 Secondary compression prediction by $S \sim \log(t)$ method

4.2.1 Settlement prediction model

Application of the proposed approach for spatial-temporal prediction of secondary compression is presented in this section. The basic model chosen in this study is the linear relationship between logarithm of time (t) and the settlement (S), i.e. $S \sim \log(t)$ method. This model is considered to be rational and practical for prediction of the secondary compression (Bjerrum 1967, Garlanger 1972, Mesri *et al.* 1997 etc.). The equation is given as

$$S_k = m_0 + m_1 \log(t_k) + \varepsilon_k \quad (4-14)$$

where S_k = secondary compression settlement at k th step of observation; m_0 and m_1 = model parameters; t_k = time of compression until k th step of observation; ε_k = observation error.

Suppose that the secondary compression settlement at n observation point, x_1, x_2, \dots, x_n , has been sequentially observed at discrete time t_k for $k = 1, 2, \dots, K$. In this case, the components of the observation model equations given in Section 3.2.1 can be defined based on above settlement prediction model as follows:

$$\hat{Z} = [\hat{m}_1(x_1), \hat{m}_1(x_2), \dots, \hat{m}_1(x_m), \hat{m}_0(x_1), \hat{m}_0(x_2), \dots, \hat{m}_0(x_m)]^T \quad (4-15)$$

$$Y_k = [S_k(x_1), S_k(x_2), \dots, S_k(x_n)]^T \quad (4-16)$$

$$M_k = \begin{bmatrix} \log(t_k) & 0 & \vdots \\ & \ddots & 0_{n,m-n} \\ 0 & \log(t_k) & \vdots \end{bmatrix} \quad (4-17)$$

where $I_{n,n}$ denotes $n \times n$ unit matrix. As stated previously in Section 3.2.1, it is also assumed that $V_\varepsilon = \sigma_\varepsilon^2 I_{n,n}$. Note that, in this case, the total number of unknown parameters (P) is 2, while \hat{z}_1 and \hat{z}_2 are represented by \hat{m}_1 and \hat{m}_0 , respectively.

As for the prior information model presented in Section 3.2.2, it is assumed in this calculation that the prior mean is constant through the region. In other words, trend component of the prior information can be represented by only single value, i.e. $z_{0,i}(x_j) = C_1^i$. Therefore, the components of the prior information model equations given in section 3.2.2 can be defined as follows:

$$Z_0 = [\hat{m}_{1,0}(x_1), \hat{m}_{1,0}(x_2), \dots, \hat{m}_{1,0}(x_m), \hat{m}_{0,0}(x_1), \hat{m}_{0,0}(x_2), \dots, \hat{m}_{0,0}(x_m)]^T \quad (4-18)$$

$$V_Z = \begin{bmatrix} \sigma_{m1,0}^2 V_C & 0_{n,n} \\ 0_{n,n} & \sigma_{m0,0}^2 V_C \end{bmatrix} \quad (4-19)$$

where $\hat{m}_{1,0}(x_j)$ and $\hat{m}_{0,0}(x_j)$ denote, respectively, the prior mean at observation point x_j of m_1 and m_0 , i.e. $\hat{m}_{1,0}(x_j) = C_1^1$ and $\hat{m}_{0,0}(x_j) = C_1^2$. $\sigma_{m1,0}^2$ and $\sigma_{m0,0}^2$ represent the prior variance of m_1 and m_0 (i.e. σ_{z1}^2 and σ_{z2}^2 in Eq. (3-10)), respectively. $0_{n,n}$ denotes an $n \times n$ zero matrix. It should be noted that, using this model, the spatial correlation of ground settlements is introduced through the spatial correlation of the model parameters which are m_1 and m_0 in this case.

4.2.2 Simulation experiments

4.2.2.1 Random field generation by frequency-domain technique

In order to investigate the performance of the proposed approach using the simulated data, random values of the model parameters are generated based on frequency-domain technique and the observed settlement data is then calculated by Eq. (4-14), using the generated parameters and the assumed standard deviation of the observation error, σ_ε . The detail formulations for this technique can be found in Section 4.1.2.1.

4.2.2.2 Improvement of the estimation by considering spatial correlation structure

A series of simulation experiments was performed based on the procedure previously described in Section 4.1.2.1. It is assumed that total number of the observation points is 36 ($n = 36$) and these points are arranged in a square grid pattern with even spacing of s and total width of L , as shown in Figure 4-6. It was also decided to limit the number of simulations for each experiment to 50 ($N_{sim} = 50$). For the purpose of recognizing the performance of the approach, these selections seem sufficient.

For simulation of the model parameters, it is assumed that the mean and standard deviation of the random field of m_1 are 100 (cm) and 10 (cm), and those of m_0 are -100 (cm) and 10 (cm), respectively. These imply that coefficient of variation (COV) = 0.1. It is assumed that the settlement is observed for 21 times (i.e. total observation time step, $K = 21$) from the day 10th to 1000th. Note that, by substituting $m_1 = 100$ (cm) and $m_0 = -100$ (cm) into Eq. (4-14), the estimated settlement at the day 1000th is 200 (cm). For the current study, the observation error, σ_ε , is assumed to be 10 (cm). By assigning the desired values of auto-correlation distance (η), random values of the model parameters together with the observed settlement at each observation point can be generated, as described in Section 4.1.2.1. It should be emphasized that the generated data actually represents a set

of the settlement data observed from an area which the true values of settlement model parameters and underlying spatial correlation structure are known.

Based on the generated observation data, the procedure proposed in Section 3.2 is performed to back-estimate the model parameters. The prior means of the model parameters (see Eq. (4-18)) are assumed to be equal to those used for the data generation, i.e. $\hat{m}_{1,0}(x_1) = \hat{m}_{1,0}(x_2) = \dots = \hat{m}_{1,0}(x_m) = 100$ (cm), and $\hat{m}_{0,0}(x_1) = \hat{m}_{0,0}(x_2) = \dots = \hat{m}_{0,0}(x_m) = -100$ (cm). As for $\sigma_{m1,0}$ and $\sigma_{m0,0}$ (see Eq. (4-19)), COV of 0.4 is assumed, i.e. $\sigma_{m1,0} = \sigma_{m0,0} = 40$ (cm). This relatively large value of COV is assumed in order to limit the influence of prior information, which commonly does not know in practice. The auto-correlation distance and the observation error are also assigned the same values as those used for generating the simulated data, namely the true values, in order to focus only on the effect of considering spatial correlation. In other words, the model selection process presented in Section 3.3 is not included in the calculations in this section.

In order to examine the advantages of considering spatial correlation structure, the Bayesian estimation, using the observed settlement of each observation point to estimate the model parameters of that point itself, i.e. ignoring spatial correlation structure, is also performed based on the same conditions with the considering one. This is actually equivalent to the case which $\eta = 0$ is assumed. The estimations based on these two different conditions are compared and presented in this section.

The different model parameters are randomly generated for 50 times ($N_{sim} = 50$) and the estimation errors are calculated by the terms of mean and standard deviation (SD) of the error ratio, E_r , as defined in Eq. (4-12). It should be noted that the total number of estimated values for mean and standard deviation (SD) calculations is $n \times N_{sim}$. It is clear that the true values of the model parameters are known. However, those of the settlement have to be estimated. Eq. (4-14) is used for

calculating both true values and estimated values of the settlement at any time t_k , using the true values and estimated values of the model parameters, respectively.

Figure 4-10 illustrates the plots of the mean and SD of E_r for the model parameter estimation and settlement prediction at the last observation time step (the day 1000th) against observation time, until which the observation data are used in estimation. It is assumed that ratios of auto-correlation distance to spacing, $s/\eta = 0.25$. Clearly, SD of E_r for the cases of considering the spatial correlation structure are lower than those of ignoring spatial correlation structure, regardless of the observation time. This confirms that the estimation can be improved by taking into account the spatial correlation structure. The fact that the difference is larger at the earlier stage of observation emphasizes the advantage of using the proposed method for the estimation at an early time. This trend is the same for both model parameters and the settlement estimation.

To investigate the sensitivity of this improvement to the changes of spatial correlation structure, the same calculations at different values of s/η ratio are performed. Only 11 time steps of the observations from the day 10th to 100th are selected to use in the calculations. The mean and SD of E_r for the model parameter estimation and the settlement prediction at the day 1000th are determined and illustrated in Figure 4-11.

It can be seen from Figure 4-11 that, when the spatial correlation is considered, the SD of E_r for the model parameter and settlement estimation reduces with the decrease of s/η ratio. This leads us to conclude that, by the proposed method, a stronger spatial correlation gives a better estimation. Clearly, this improvement becomes significant when observation spacing is shorter than half of the auto-correlation distance, i.e. $s/\eta \leq 0.5$. Note that, in both Figure 4-10 and 4-11, the means of E_r are close to zero at any cases. This implies that the bias of these estimations is negligible.

4.2.2.3 Estimation of settlement at an arbitrary location

This section illustrates the ability of the proposed method for settlement estimations at any arbitrary locations. As shown in section 3.2.1 that the component of the unknown parameters at any unobserved points, i.e. x_{n+1} , x_{n+2} , ... , x_m , are included in the formulation and the estimates of the parameters at these points will be calculated, at the same time with those at the observation points, by the optimization process based on Bayesian approach. Then, the future settlement at these unobserved points can be predicted using Eq. (4-14).

To investigate the level of error for this type of estimation, the similar calculations with what have been done in the previous section are performed, but, for each calculation, one of the observation points is removed from consideration. Then, the model parameters and the settlement at this removed observation point will be estimated using only the simulated data of the remaining observation points. Due to the fact that the true values of model parameters at each removed observation point are unknown in this case, the estimation errors are determined by comparing the estimated values with the parameter values which are generated from the corresponding random sampling with the other observation points, the data of which is used for the estimation. Figure 4-12 and 13 show the plots of these estimation errors against observation time and s/η ratio, respectively.

For comparison purpose, the calculations presented in Figure 4-12 and 4-13 are analogous with those shown in Figure 4-10 and 4-11 in that $s/\eta = 0.25$ is assumed in Figure 4-12 and the data from the day 10th to 100th is used in Figure 4-13. The number of simulations for each trial is also 50 ($N_{sim} = 50$). The means and SD of the error ratio, E_r , are calculated based on Eq. (4-12), taking into account the estimations at all observation points for all simulations. The values of other parameters are also the same as those assigned in Section 4.2.2.2.

It can be observed from Figure 4-12 that the estimation error, which directly relates to SD of E_r , reduces with the observation time in the case with consideration of spatial correlation. In other words, the more observation data we have, the better estimation we obtain. Figure 4-13 clearly shows that the error is significantly higher if the weaker spatial correlation structure is assumed, especially for the settlement estimation. This emphasizes the advantage of the proposed approach in case the strong spatial correlation structure of the parameters is found. Clearly, the case with considering spatial correlation structure provides the more accurate estimation than the case without considering it, in that it gives lower SD of E_r , especially for the settlement estimation. The bias of estimation, which directly relates to mean of E_r , is found in the calculations without considering spatial correlation, but this also becomes negligible when spatial correlation is considered.

4.2.3 Case study using actual observation data

4.2.3.1 Description of the case

To investigate the performance of the proposed method for the practical application, a case study, using the actual field observation data of secondary compression (Ueda *et al.* 1986), has been performed. The site is a residential land development project located in suburb area of Tokyo, Japan. This area is covered by a thick alluvial deposit which can be classified as a surface layer of peats followed by a very soft clay layer down to the thickness of about 17 meters (see Figure 4-14). Below these layers, the layers of medium dense sand and silt are found, respectively. In order to avoid the large amount of settlement due to the thick soft soil layer at the surface, the soil condition is improved by preloading prior to the construction. As shown in Figure 4-15, the preloading surcharge was filled up to the maximum thickness of about 6 m during the preloading period of, approximately, 900 days.

The settlement observations were performed at both during the preloading period by the settlement plates and after removal of the surcharge by measuring settlement of the boundary stone around the housing lots. The settlement after removal of the surcharge, which is used in this study, was observed at about every 600 m² with the total number of observation points of 42. The location plan of these observation points, together with the surcharge area, is presented in Figure 4-16, while all of the observation data are shown as semi-logarithmic plots of settlement and time in Figure 4-17. Even though several observation points are located close to the surcharge boundary, the distinction between the settlement-time relationships observed at these points and those at the points inside are found to be insignificant. Therefore, all of the observation data will be used in the calculations as one-dimensional settlement problems. The raw data of all the observation settlement are summarized and presented in Appendix E.

Various techniques have been proposed by several authors for predicting the future settlement using the observed settlement, for example, hyperbola method (Sridharan *et al.* 1987, Tan 1994), $S \sim \log(t)$ method (Bjerrum 1967, Garlanger 1972, Mesri *et al.* 1997), and Asaoka's method (Asaoka 1978). In this study, $S \sim \log(t)$ method is considered to be the most suitable approach due to the fact that the primary consolidation is expected to be completed before the surcharge removal, thus the settlement occurring afterward should result from the secondary compression process. Figure 4-18 shows an example of the $S \sim \log(t)$ plot at an observation point (point A in Fig. 4-16). It can be seen that, by excluding a part of data in the early period of observation, within which the secondary compression is considered to be influenced by the rebound effect due to surcharge removal, this semi-logarithmic relationship fits quite well with the observation data. By investigating the settlement data of all the observation point, the data before the day 103rd are excluded from the calculation by judgment.

Choosing appropriate prior statistics of the unknown parameters (m_1 and m_0) is also an important issue. What has been done in the current research is that the prior mean of m_1 and m_0 , i.e. $\hat{m}_{1,0}(x_j)$ and $\hat{m}_{0,0}(x_j)$, were assumed to be equal to the value of slope and the intercept of the trend line resulting from the linear regression analysis of the plots between settlement and logarithm of time, considering the data from all of the observation points. On the other hand, the prior variances, i.e. $\sigma_{m1,0}^2$ and $\sigma_{m0,0}^2$, are selected by trying several values of prior coefficient of variation (COV) and choosing the one which is relatively insensitive to the changes of prior means. Based on this approach, the prior means of m_1 and m_0 are assigned as $\hat{m}_{1,0}(x_1) = \hat{m}_{1,0}(x_2) = \dots = \hat{m}_{1,0}(x_m) = 109.7$ (cm), and $\hat{m}_{0,0}(x_1) = \hat{m}_{0,0}(x_2) = \dots = \hat{m}_{0,0}(x_m) = -204.1$ (cm). The prior COV of 0.4 is chosen for calculating prior variance of both parameters, i.e. $\sigma_{m1,0} = 43.9$ (cm) and $\sigma_{m0,0} = 81.6$ (cm).

4.2.3.2 Estimation of the auto-correlation distance and observation error

It was proposed in Section 3.3 that auto-correlation distance (η) and the standard deviation of the observation error (σ_ε) can be appropriately selected based on Akaike's Bayesian Information Criterion (ABIC). Considering the observation data together with the prior information of the model parameters, the *ABIC* for each pair of η and σ_ε can be determined by Eq. (3-21). The values of η and σ_ε that give the minimum value of *ABIC* will be served as optimized selections of these parameters.

Figure 4-19 shows an example of contours of *ABIC* in η and σ_ε space for the case that all of the settlement data until the last step of observation, i.e. the day 1017th, is considered. In this case, the estimated η and σ_ε are 30 (m) and 7.0 (cm), respectively. Obviously, the estimated values of the observation error are more likely to be insensitive than those of the auto-correlation distance.

In practice, the observation data is collected stepwise for a period of time. Therefore, it is natural to sequentially update the estimation once the new sets of observation are provided. Figure 4-

20 illustrates the plots of the estimated values of η and σ_ε versus observation time, until which the observation data are used in estimation. It can be observed that the estimated values of auto-correlation distance tend to decrease with the observation time, while those of the observation error tend to increase, depending on the characteristic of the observation data. Both of these estimations seem unstable at the early stage of the observation, indicating insufficiency of the observation data for the calculations. However, they become more stable as the observation data accumulates.

4.2.3.3 Settlement estimation and prediction

Based on the procedure proposed in Section 3.2, the settlement estimation at observation points with consideration of spatial correlation can be performed. To represent the estimation error, the mean and SD of error ratio (E_r) previously defined in Eq. (4-12), is also used. However, in the current case, the true value refers to the observed settlement. The total number of data to calculate the mean and SD is equal to the number of observation points (n).

Figure 4-21 shows the plots of the mean and SD of E_r for prediction of settlement at the last observation time step (the day 1017th) versus observation time. For comparison purpose, both the cases which the spatial correlation is considered and ignored are also presented. It should be noted that, for the case with considering spatial correlation, the estimated values of auto-correlation distance, as shown in Figure 4-20, are used in the calculations.

Corresponding to the results of simulation examples shown in Figure 4-10, the prediction error decreases with the increase of the available observation data. However, for the current set of observation data, considering the spatial correlation does not significantly improve the estimation in terms of reduction of E_r (both mean and SD). This may be due to the fact that the auto-correlation distance (η) is relatively short in comparison with the spacing between the observation points (s) in this case. For this set of the field observation, $\eta \approx 30$ m and $s \approx 25$ m, thus the ratio $s/\eta \approx 1.0$.

Figure 4-11 clearly shows that considering the spatial correlation does not give significant improvement at this level of the s/η ratio. It should also be noted that the bias of the prediction, which can be judge from the mean of E_r , is extremely obvious in this case. This is because the current set of observed settlement data is not perfectly fit to the $S \sim \log(t)$ model, in that the settlement-log time curves tends to be concave to the settlement axis (see Fig. 4-17). The predicted settlement, therefore, tends to lower than the observed data which leads to the negative bias of the prediction. This bias can be reduced by introducing an appropriate value of the process noises, which will be illustrated in the next section.

To further investigate the efficiency of the proposed method in dealing with the space-time problem, the observation data at each selected observation point is removed and the settlement estimations, or predictions, at this point are performed using the rest of the observations. Comparison between the settlement estimated by these parameters and the actually observed one reveals the estimation error. This calculation is actually comparable to the previously presented simulation examples shown in Section 4.2.2.3.

Figure 4-22 and 4-23 show the comparison between the observed and estimated settlement at the day 1017th. The observation data from the day 103rd to the day 696th are used in the calculation. Figure 4-22 presents the comparison in the form of bar chart to provide the detail information of the settlement values. In Figure 4-23, however, the estimated settlements are presented as a surface, while the observed settlements are plotted as points with dotted lines showing difference between them. Obviously, for the case that the spatial correlation is not included in the calculation, i.e. assuming $\eta = 0$ (Fig. 4-22(a) and 4-23(a)), the predicted settlement tends to be uniform and is not likely to be able to represent the variation of ground settlement. This is because the estimated values of model parameters at each removed point are actually equal to the initial mean values of the

parameters which are input as a part of prior information model (see Section 4.2.3.1 for detail). On the other hand, for the case that the estimated value of auto-correlation distance, $\eta = 32$ m, is used (Figure 4-22(b) and 4-23(b)), the estimation gives relatively more realistic pattern of settlement in the area.

Figure 4-24 shows the plots of mean and SD of E_r (Eq. (4-12)) of settlement estimation at the removed observation points vs. observation time. Two cases of calculations are presented: the settlement estimation at the current observation day and the settlement prediction at the day 1017th. The former is an attempt to avoid the temporal error resulting from prediction of future settlement; therefore, the estimated settlement and the observed settlement are compared at that observation time (Fig. 4-24(a)). The latter is the comparison between the predicted settlement at the last observation time step (the day 1017th) and the observed settlement at that time (Fig. 4-24(b)). It should be emphasized that, for considering spatial correlation case, the estimated values of auto-correlation distance and the observation error, which are varied with the observation time as shown in Figure 4-20, are used in the calculations, while, for ignoring spatial correlation case, it is assumed that $\eta = 0$.

It can be seen from both figures that, the case with considering spatial correlation can improve the estimation in term of reducing the SD of E_r . Depending on the characteristic of the observation data, this improvement may not be significant at the early state of the observation. The estimation errors, i.e. the SD of E_r , are also decreased with the observation time. In other words, the estimation can be improved if more observation data are given, which is corresponding with the results of the simulation examples shown in Figure 4-12. However, the significant bias is also found in this case. This is expected to result from the imperfection of using the $S \sim \log(t)$ model to describe the observed settlement data, as discussed earlier in this subsection.

4.2.3.4 Effect of process noise consideration

As mentioned in Section 3.4, dealing with the real observation data, higher weight should be given to the more recent data than to the previous one. This can be done by assigning appropriate values of process noise, i.e. forgetting factor, during the time updating process in Kalman filter procedure. According to Eq. (3-25), the level of the process noise can be controlled by the value of a constant parameter, a .

By performing the same calculation as what is shown in Figure 4-21, but with the different values of a , the effect of process noise to the settlement prediction can be investigated. Figure 4-25 presents the prediction error of settlement at the last observation time step (the day 1017th) at different observation time only for the case with considering spatial correlation. In fact, for the case that $a = 0$, i.e. no process noise, this is the same with the one plotted in Figure 4-21.

It can be concluded from Figure 4-25 that, to some extent, the settlement prediction can be improved by considering the forgetting factor, i.e. reducing both mean and SD of E_r . Especially for estimation at last time step (the day 1017th), at which all observation data until the target day for the prediction are included in the calculations, the SD of E_r reduce dramatically from about 0.079 to 0.04. However, this is not always the case. At some stage of prediction, assuming too high value of process noise may mislead the prediction and the error may become higher instead. This can be seen in Figure 4-25 when $a = 2.0$ are assigned. Therefore, the optimization of this process noise coefficient is required. This is, however, out of scope of the current research.

Table 4-1: Comparison of estimation error between the cases with considering and ignoring spatial correlation for different n and s/η ratios, using 2-D simulations of 100 times ($N_{sim} = 100$) by Asaoka's model, assuming $\sigma_\varepsilon = 1.0$ cm and $T_v = 0.164$.

(a) β_1 estimation

		E_r of β_1 estimation				
n	s/η	Ignoring spatial corr.		Considering spatial corr.		Improvement ^a (%)
		Mean	SD ⁽¹⁾	Mean	SD ⁽²⁾	
16	2	-8.65E-05	2.44E-03	-8.29E-05	2.43E-03	0.4
	1	1.41E-05	2.41E-03	-1.08E-05	2.29E-03	4.7
	0.5	1.73E-04	2.46E-03	3.58E-05	2.11E-03	14.1
	0.25	1.20E-04	2.35E-03	-4.56E-05	1.81E-03	23.1
36	2	-1.12E-05	2.42E-03	-2.57E-05	2.41E-03	0.3
	1	-4.66E-05	2.38E-03	-9.79E-05	2.30E-03	3.4
	0.5	5.31E-05	2.47E-03	-1.15E-04	2.13E-03	13.8
	0.25	1.92E-04	2.45E-03	-1.55E-04	1.86E-03	23.8
64	2	1.75E-06	2.39E-03	-1.41E-05	2.38E-03	0.5
	1	-4.28E-05	2.39E-03	-8.80E-05	2.27E-03	4.8
	0.5	-3.71E-05	2.44E-03	-1.75E-04	2.06E-03	15.2
	0.25	1.52E-04	2.47E-03	-2.15E-04	1.72E-03	30.2

^a Improvement (%) = [(1) - (2)] x 100 / (1)

(b) β_0 estimation

n	s/η	E_r of β_0 estimation				Improvement ^a (%)
		Ignoring spatial corr.		Considering spatial corr.		
		Mean	SD ⁽¹⁾	Mean	SD ⁽²⁾	
16	2	2.71E-03	3.39E-02	2.72E-03	3.38E-02	0.5
	1	2.29E-03	3.46E-02	2.33E-03	3.36E-02	3.1
	0.5	1.39E-03	3.49E-02	1.84E-03	3.09E-02	11.4
	0.25	2.00E-03	3.39E-02	2.21E-03	2.60E-02	23.2
36	2	1.04E-03	3.38E-02	1.28E-03	3.37E-02	0.1
	1	1.79E-03	3.45E-02	2.05E-03	3.36E-02	2.6
	0.5	1.22E-03	3.43E-02	1.94E-03	3.06E-02	10.7
	0.25	2.77E-04	3.40E-02	2.08E-03	2.59E-02	23.9
64	2	3.47E-04	3.42E-02	5.83E-04	3.40E-02	0.4
	1	8.56E-04	3.40E-02	1.23E-03	3.27E-02	3.7
	0.5	1.33E-03	3.46E-02	1.84E-03	3.01E-02	12.9
	0.25	1.54E-04	3.44E-02	1.93E-03	2.47E-02	28.2

^a Improvement (%) = [(1) - (2)] x 100 / (1)

(c) final settlement estimation

		E_r of S_f estimation				
n	s/η	Ignoring spatial corr.		Considering spatial corr.		Improvement ^a (%)
		Mean	SD ⁽¹⁾	Mean	SD ⁽²⁾	
16	2	-7.65E-03	1.06E-01	-7.44E-03	1.06E-01	0.4
	1	-3.37E-03	1.03E-01	-4.10E-03	9.84E-02	4.4
	0.5	2.91E-03	1.04E-01	-1.74E-03	8.94E-02	13.7
	0.25	1.69E-03	9.89E-02	-3.79E-03	7.62E-02	23.0
36	2	-5.84E-03	1.04E-01	-6.19E-03	1.03E-01	0.5
	1	-6.51E-03	1.02E-01	-8.10E-03	9.86E-02	3.6
	0.5	-2.94E-03	1.05E-01	-8.22E-03	9.05E-02	14.2
	0.25	2.81E-03	1.03E-01	-8.74E-03	7.77E-02	24.9
64	2	-5.89E-03	1.03E-01	-6.33E-03	1.02E-01	0.5
	1	-7.43E-03	1.03E-01	-8.58E-03	9.83E-02	4.8
	0.5	-6.91E-03	1.05E-01	-1.12E-02	8.93E-02	14.7
	0.25	6.00E-04	1.05E-01	-1.15E-02	7.43E-02	29.2
^a Improvement (%) = [(1) - (2)] x 100 / (1)						

Table 4-2: Estimation error of auto-correlation distance (η) and standard deviation of observation error (σ_ε) for different n , L/η ratios and s/η ratios, using 2-D simulations of 100 times ($N_{sim} = 100$) by Asaoka's model, assuming $\sigma_\varepsilon = 1.0$ cm and $T_v = 0.164$.

n	L/η	s/η	E_r of η estimation		E_r of σ_ε estimation	
			Mean	SD	Mean	SD
16	6	2	-0.042	1.105	-0.007	0.036
	3	1	0.194	0.720	-0.008	0.035
	1.5	0.5	0.772	0.644	-0.007	0.034
	0.75	0.25	0.864	0.613	-0.009	0.034
36	10	2	-0.184	0.940	-0.0069	0.028
	5	1	0.437	0.471	-0.0074	0.029
	2.5	0.5	0.628	0.410	-0.0080	0.030
	1.25	0.25	0.838	0.366	-0.0084	0.028
64	14	2	-0.054	0.693	-0.0027	0.022
	7	1	0.371	0.387	-0.0022	0.021
	3.5	0.5	0.608	0.264	-0.0042	0.021
	1.75	0.25	0.838	0.304	-0.0062	0.020

Table 4-3: Estimation error of final settlement at arbitrary points (point 1 to 5 in Fig. 4-9) for different s/η ratios, L/η ratios and T_v , using 2-D simulations of 100 times ($N_{sim} = 100$) by Asaoka's model, assuming $n = 36$, $\sigma_\varepsilon = 1.0$ cm.

s/η	L/η	Point	Standard deviation (SD) of E_r of S_f estimation			
			at $T_v = 0.164$		at $T_v = 0.424$	
			Case A ^a	Case B ^b	Case A ^a	Case B ^b
0.25	1.25	1	0.070	0.070	0.028	0.029
		2	0.075	0.075	0.030	0.031
		3	0.080	0.080	0.042	0.044
		4	0.114	0.115	0.104	0.106
		5	0.145	0.145	0.144	0.146
1	5	1	0.111	0.113	0.092	0.093
		2	0.117	0.121	0.097	0.100
		3	0.106	0.111	0.096	0.099
		4	0.148	0.150	0.147	0.148
		5	0.148	0.148	0.148	0.148
5	25	1	0.140	0.140	0.140	0.140
		2	0.139	0.139	0.138	0.140
		3	0.133	0.133	0.133	0.132
		4	0.137	0.137	0.137	0.137
		5	0.143	0.143	0.143	0.143

^a Case A refers to the case which the calculations are performed based on the true values of both η and σ_ε
^b Case B refers to the case which the calculations are performed based on the estimated values of both η and σ_ε

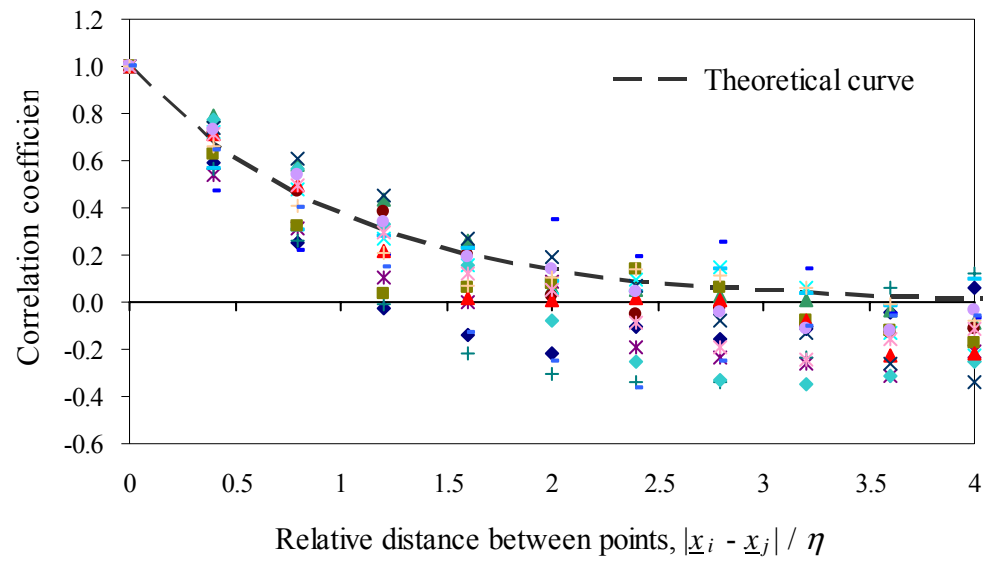
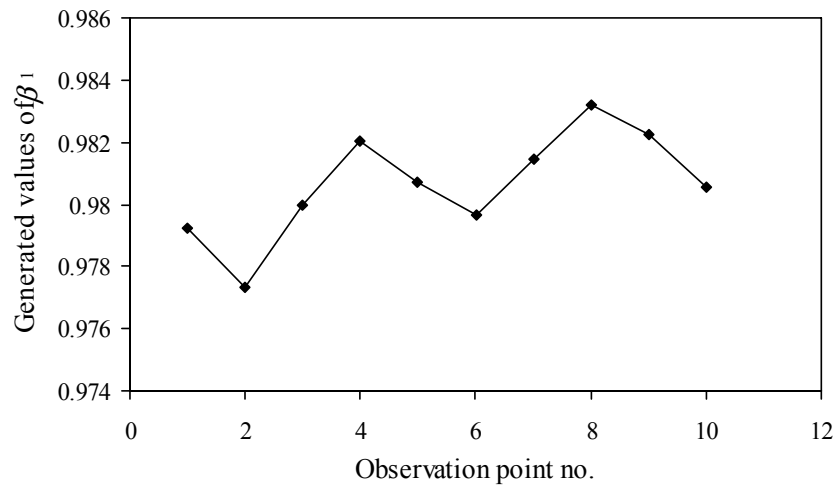
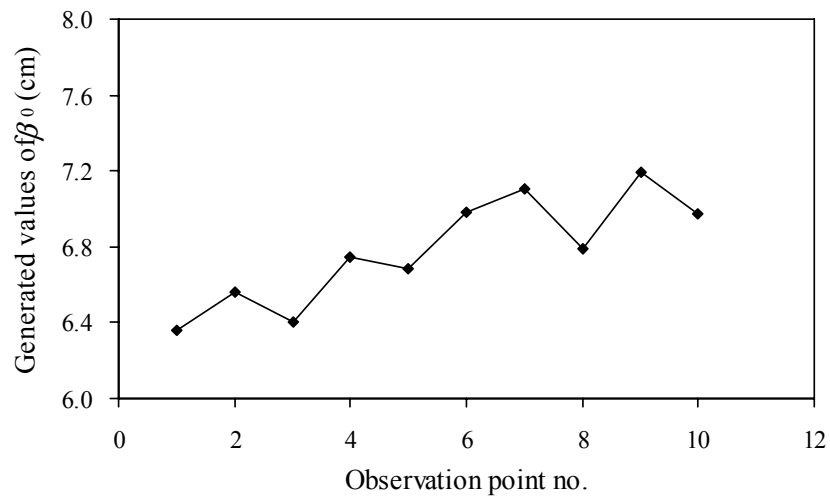


Figure 4-1: Correlation coefficient-distance relationship of the generated random values, comparing with the theoretical curve, for 1-D simulation.



(a) β_1 generation



(b) β_0 generation

Figure 4-2: Examples of the generated values of Asaoka's model parameters (β_1 , β_0) for 1-D simulation, assuming $n = 10$ and $s/\eta = 0.2$.

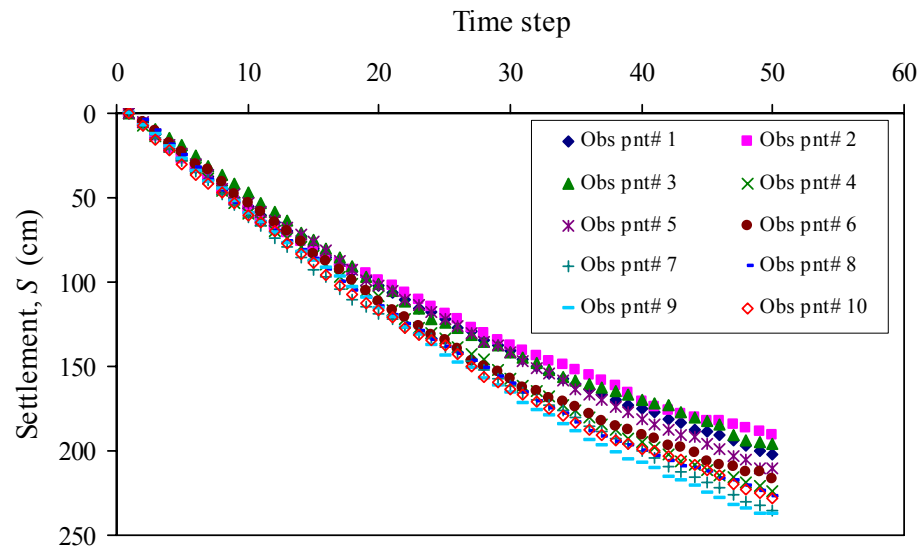
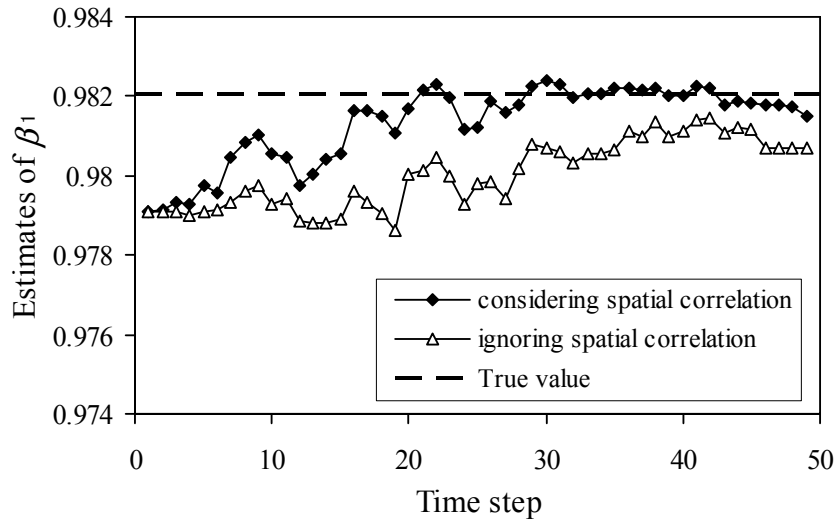
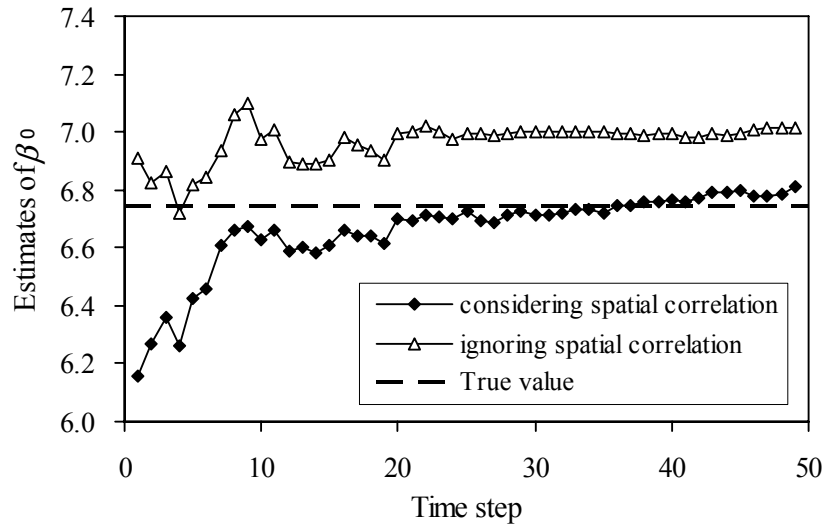


Figure 4-3: Examples of the generated settlement with time for 1-D simulation by Asaoka's model, assuming $n = 10$, $s/\eta = 0.2$ and $\sigma_\varepsilon = 1.0$ cm.

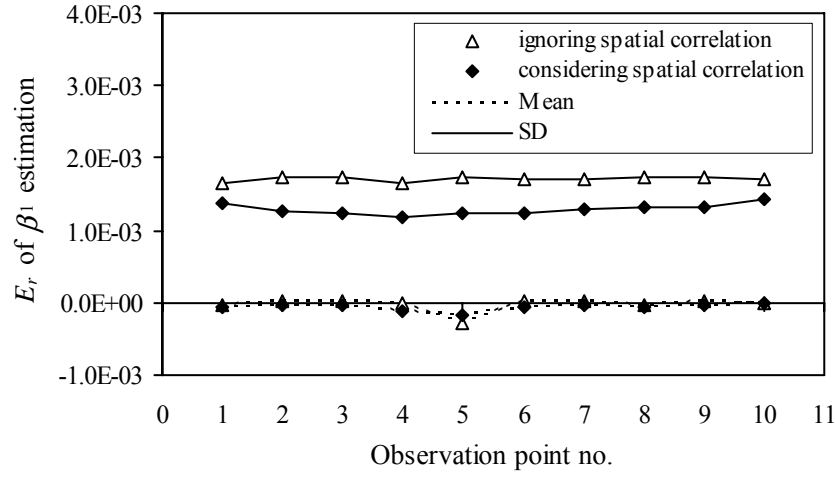


(a) β_1 estimation

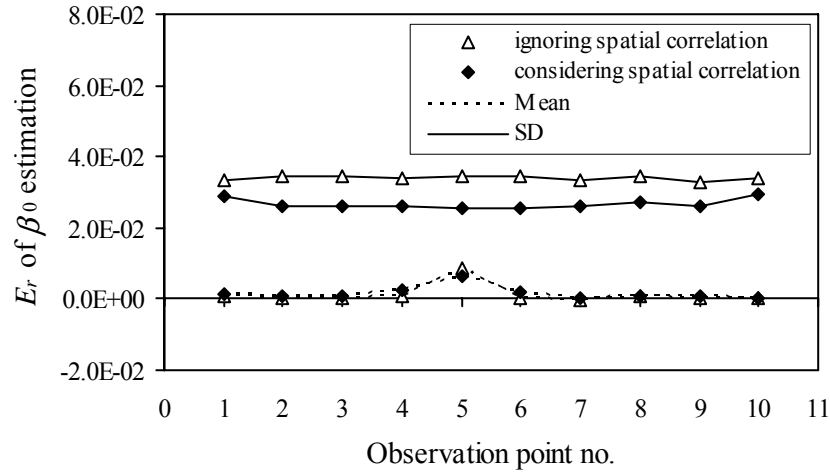


(b) β_0 estimation

Figure 4-4: Stepwise updating for estimation of the model parameters (β_1 , β_0) for 1-D simulation by Asaoka's model, assuming $n = 10$, $s/\eta = 0.2$ and $\sigma_e = 1.0$ cm.



(a) β_1 estimation



(b) β_0 estimation

Figure 4-5: Comparison of estimation error between the cases with considering and ignoring spatial correlation at each observation point at 50th time step for 1-D simulation of 1000 times ($N_{sim} = 1000$) by Asaoka's model, assuming $n = 10$,

$$s/\eta = 0.2 \text{ and } \sigma_\varepsilon = 1.0 \text{ cm.}$$

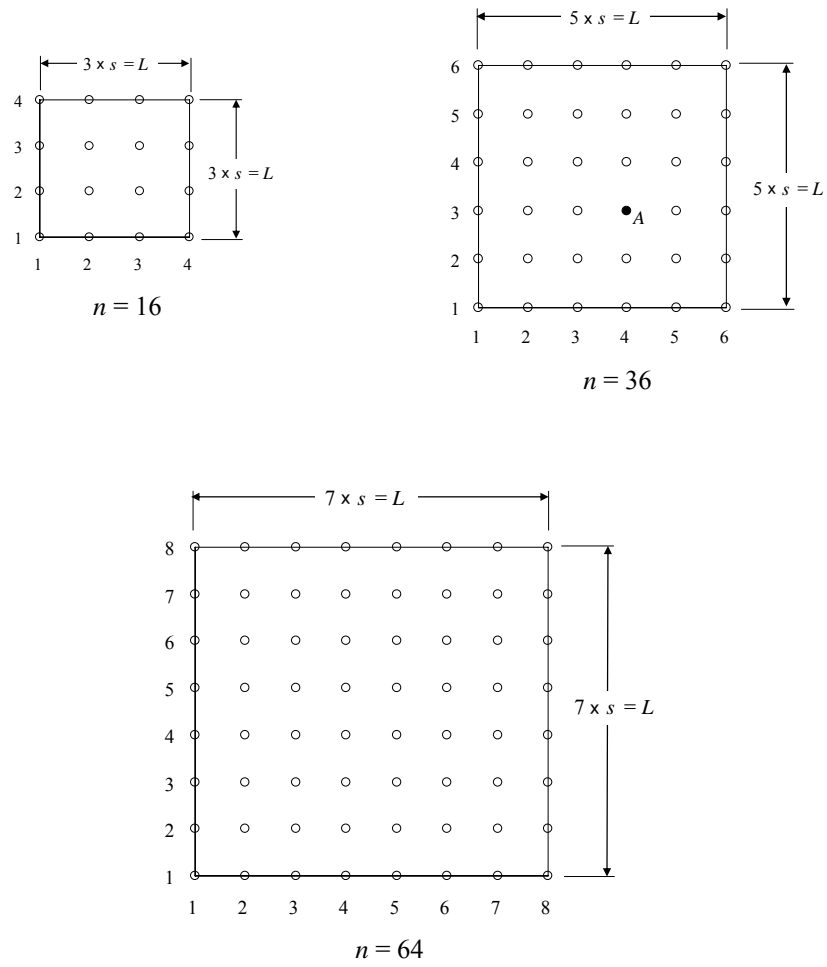
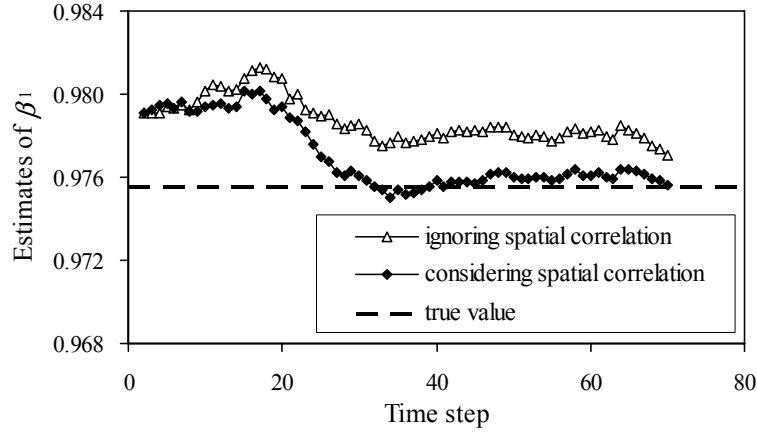
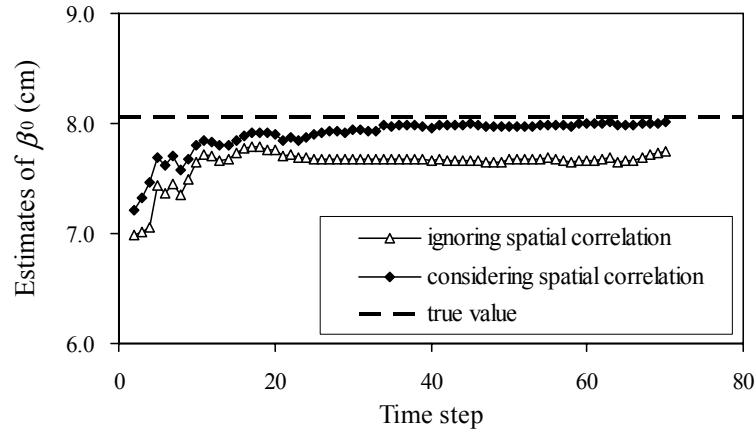


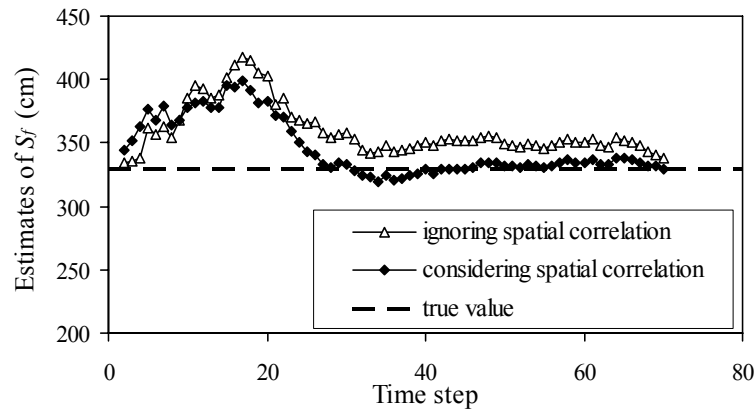
Figure 4-6: Layout of observation plans for 2-D data simulation by Asaoka's model.



(a) β_1 estimation



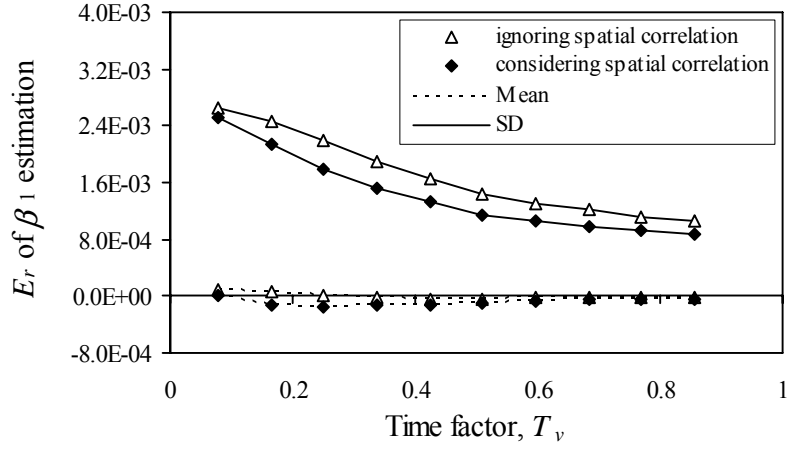
(b) β_0 estimation



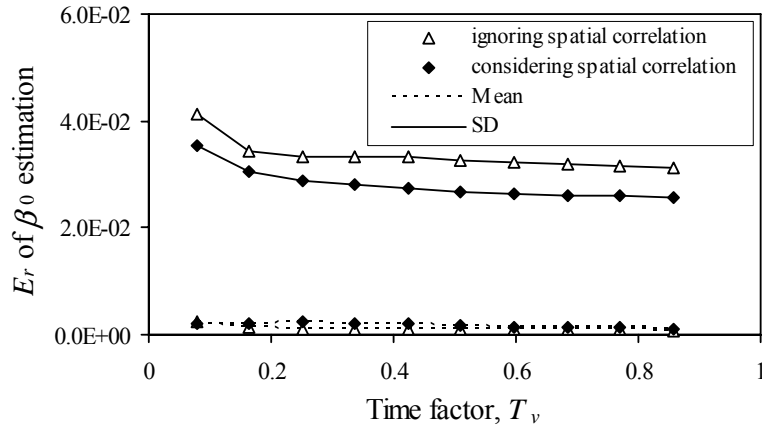
(c) final settlement estimation

Figure 4-7: Stepwise updating for estimation of the model parameters (β_1 , β_0) and final settlement at point A (see Fig. 4-

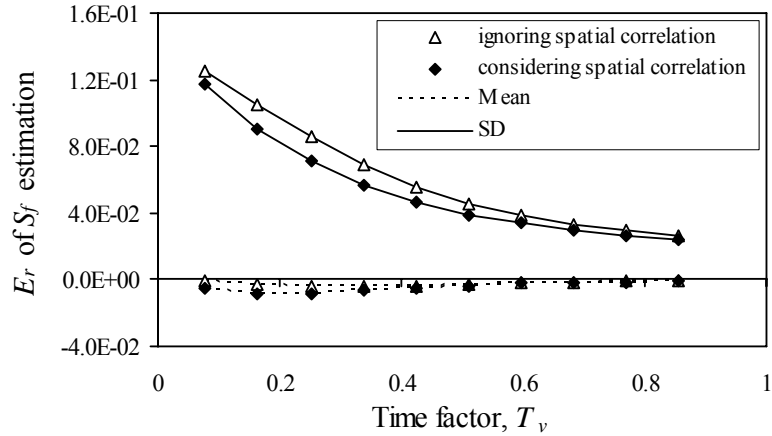
6) for 2-D simulation by Asaoka's model, assuming $n = 36$, $s/\eta = 0.5$ and $\sigma_\varepsilon = 1.0$ cm.



(a) β_1 estimation



(b) β_0 estimation



(c) final settlement estimation

Figure 4-8: Estimation error vs. time factors for 2-D simulation of 100 times ($N_{sim} = 100$) by Asaoka's model, assuming

$$n = 36, s/\eta = 0.5 \text{ and } \sigma_\varepsilon = 1.0 \text{ cm.}$$

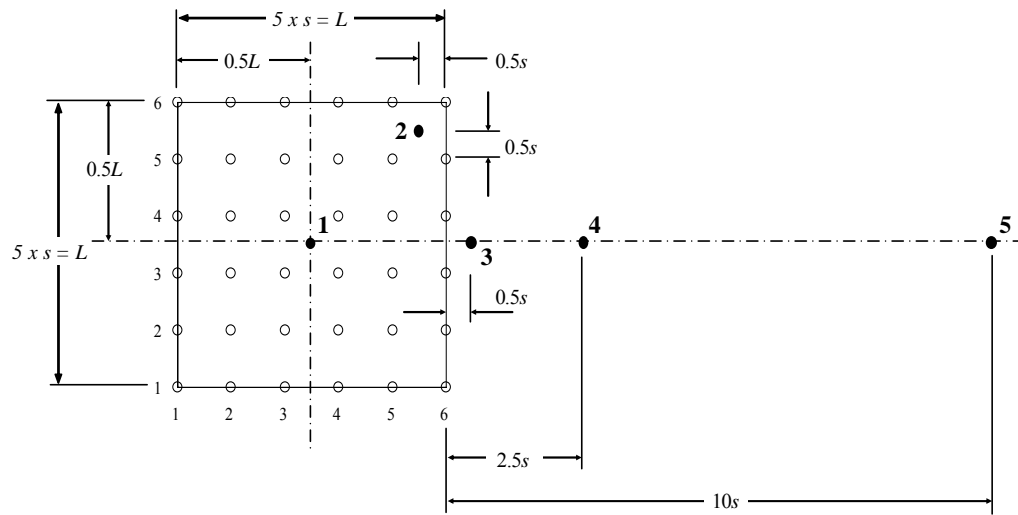
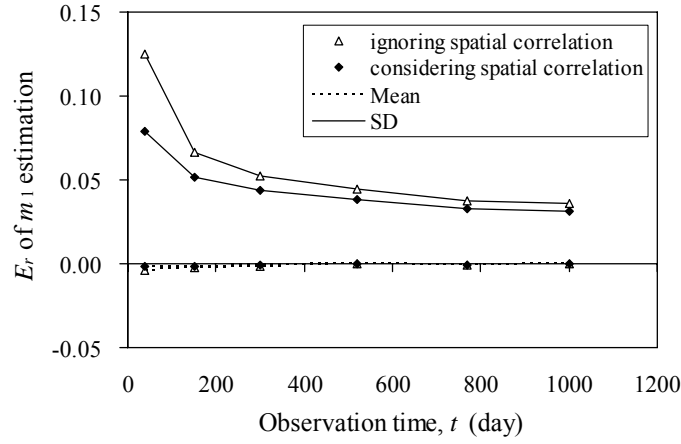
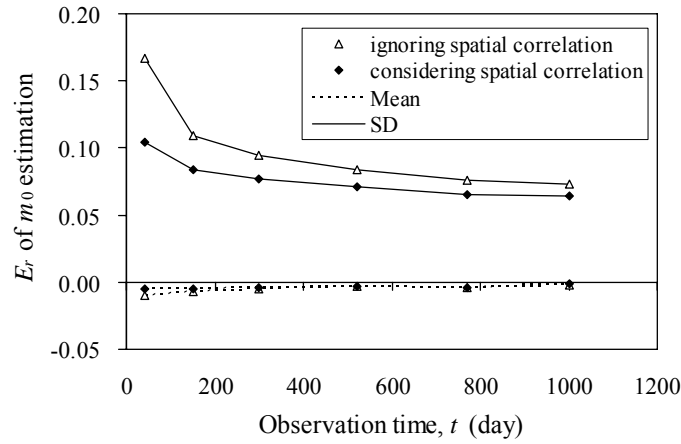


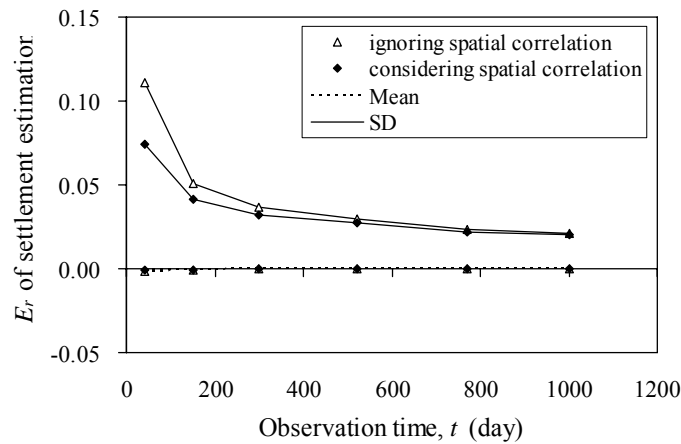
Figure 4-9: Observation plan and locations of arbitrary points to be estimated.



(a) m_1 estimation

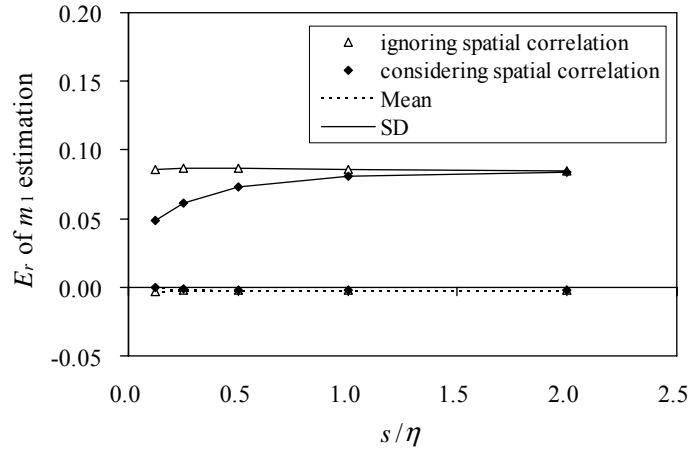


(b) m_0 estimation

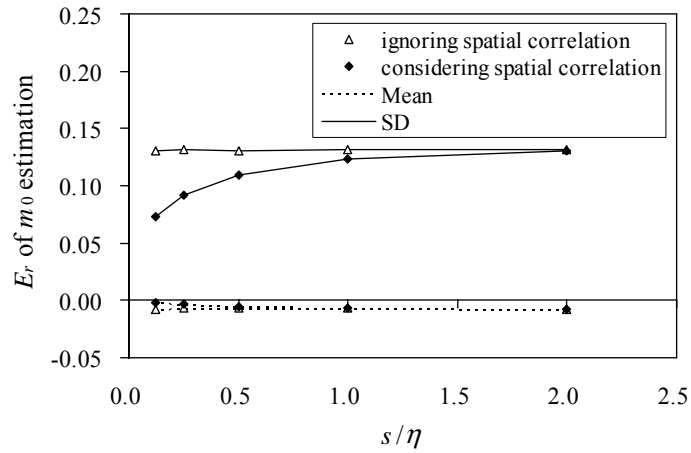


(c) settlement prediction at the day 1000th

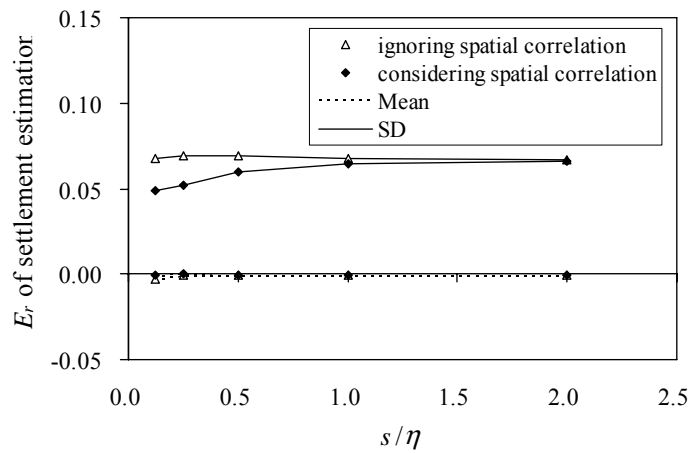
Figure 4-10: Estimation error vs. observation time for 2-D simulation of 50 times ($N_{sim} = 50$) by $S \sim \log(t)$ model, performing at the observation points, assuming $n = 36$, $s/\eta = 0.25$ and $\sigma_\varepsilon = 10$ cm.



(a) m_1 estimation

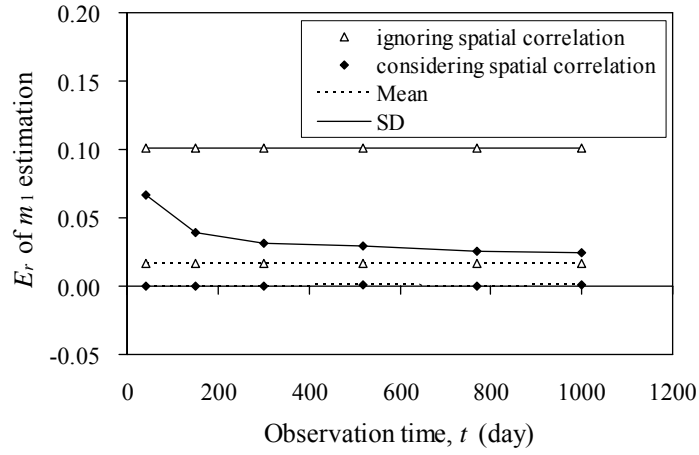


(b) m_0 estimation

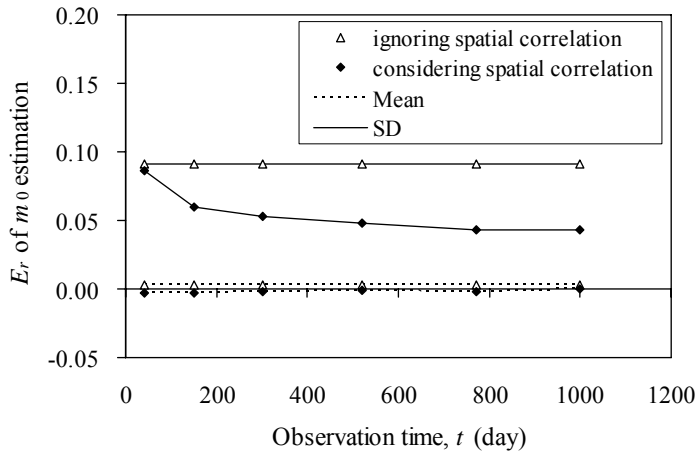


(c) settlement prediction at the day 1000th

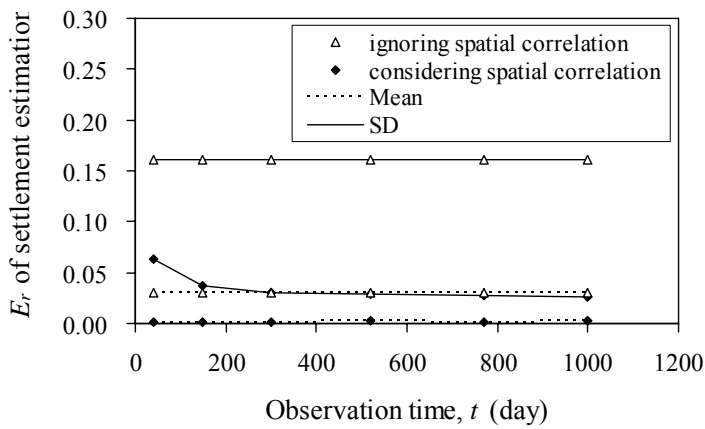
Figure 4-11: Estimation error vs. s/η ratios for 2-D simulation of 50 times ($N_{sim} = 50$) by $S \sim \log(t)$ model, performing at the observation points based on the data from the day 10th to 100th, assuming $n = 36$ and $\sigma_\varepsilon = 10$ cm.



(a) m_1 estimation

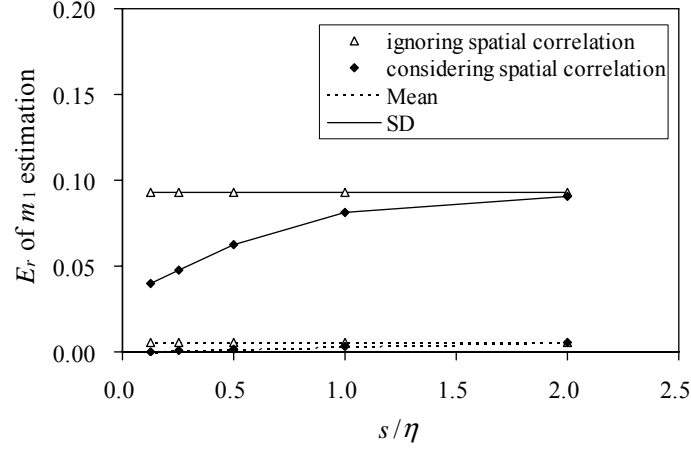


(b) m_0 estimation

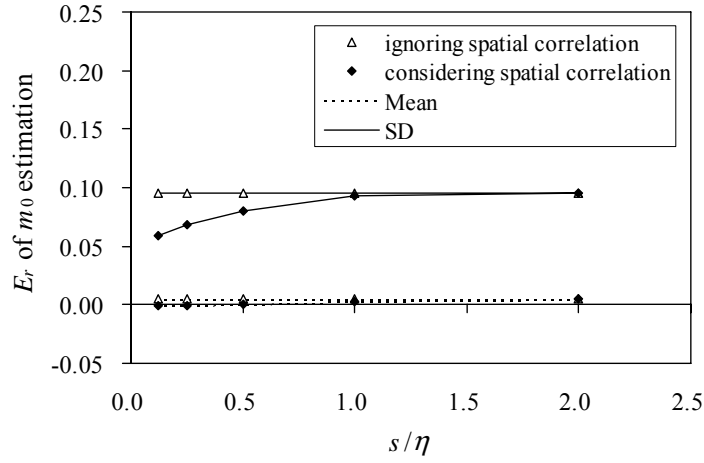


(c) settlement prediction at the day 1000th

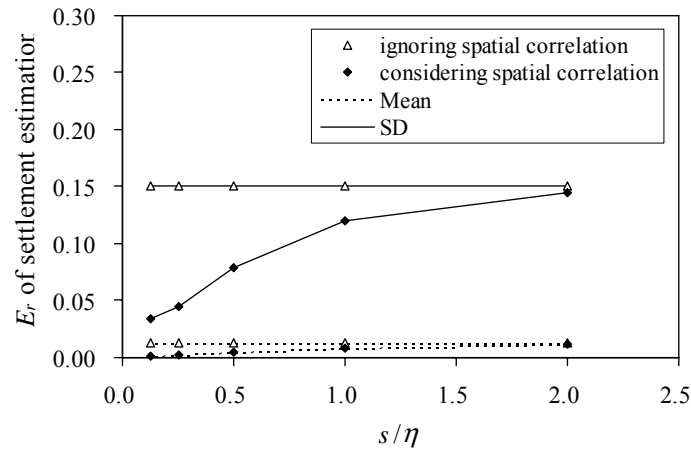
Figure 4-12: Estimation error vs. observation time for 2-D simulation of 50 times ($N_{sim} = 50$) by $S \sim \log(t)$ model, performing at the *removed* observation points, assuming $n = 36$, $s/\eta = 0.25$ and $\sigma_e = 10$ cm.



(a) m_1 estimation



(b) m_0 estimation



(c) settlement prediction at the day 1000th

Figure 4-13: Estimation error vs. s/η ratios for 2-D simulation of 50 times ($N_{sim} = 50$) by $S \sim \log(t)$ model, performing at the *removed* observation points based on the data from the day 10th to 100th, assuming $n = 36$ and $\sigma_\epsilon = 10$ cm.

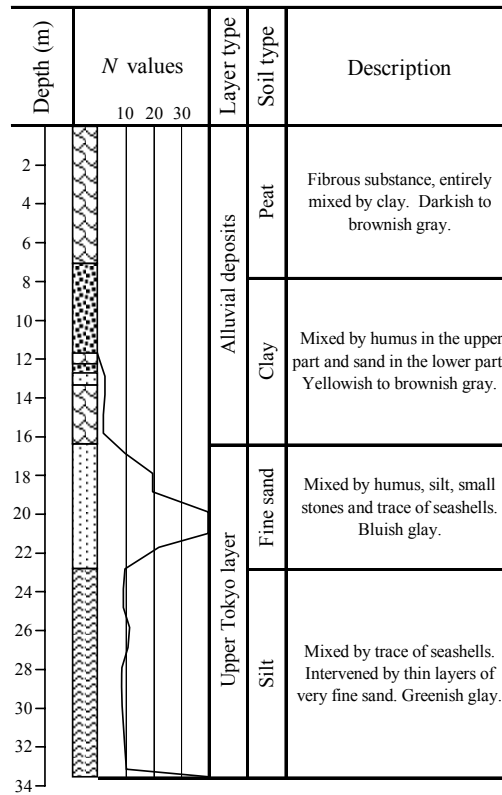


Figure 4-14: Soil condition of a land development site, selected as a case study for settlement prediction based on actual observation data.

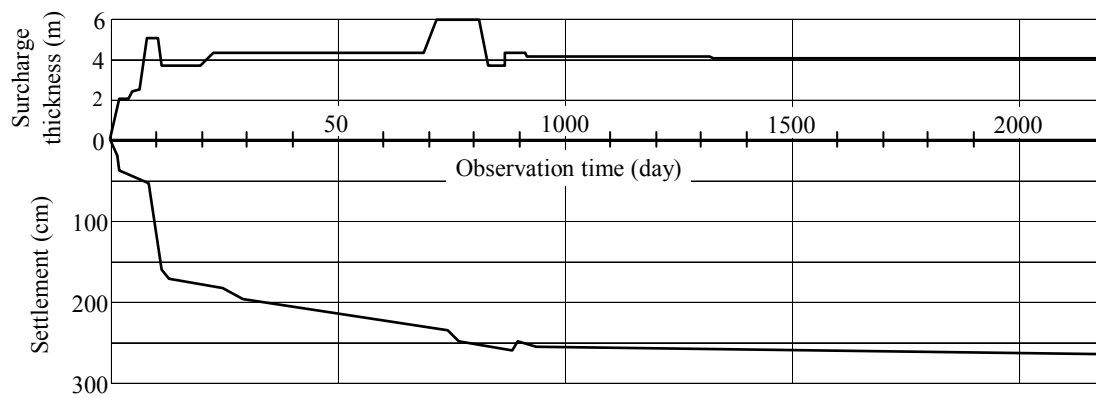


Figure 4-15: Surchage thickness and settlement versus time.

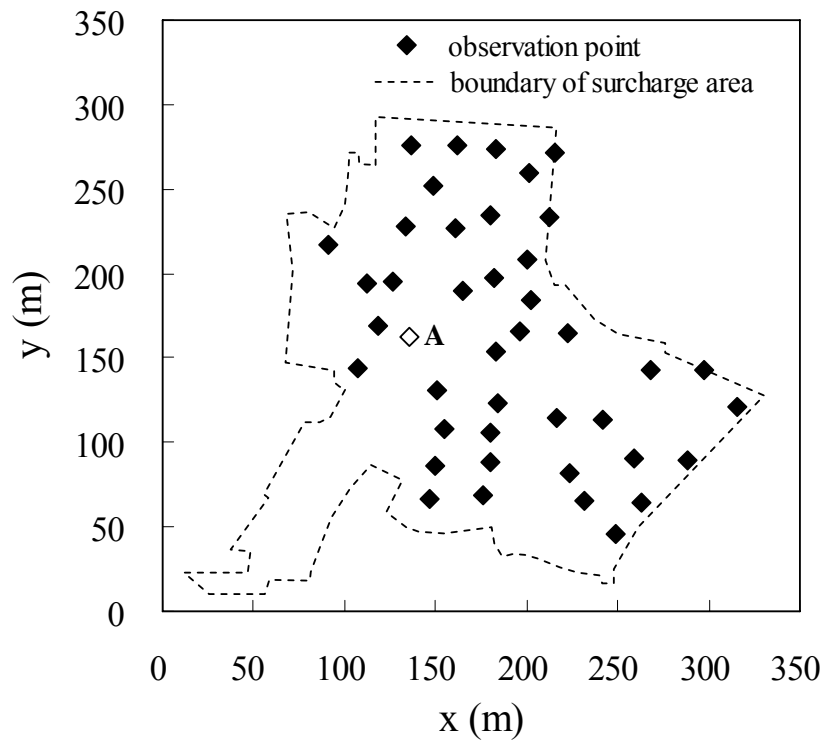


Figure 4-16: Location plan of the observation points and surcharge area.

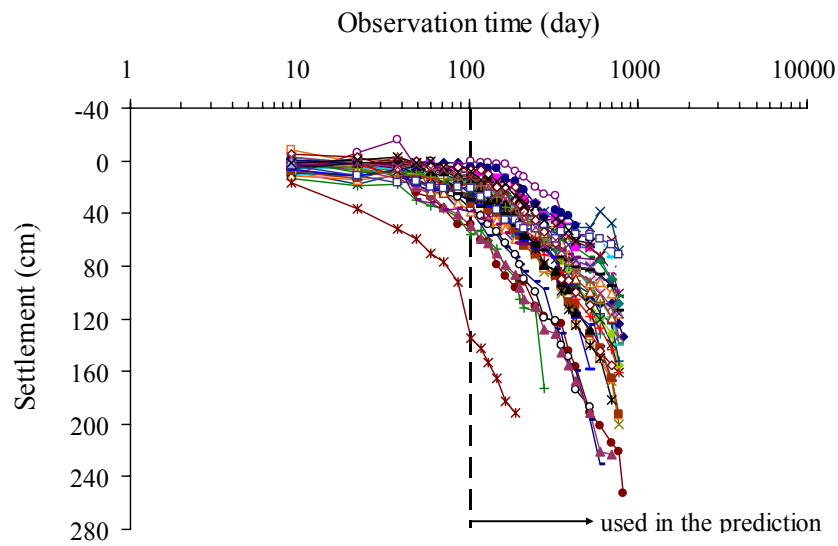


Figure 4-17: Observed settlement versus time (after surcharge removal).

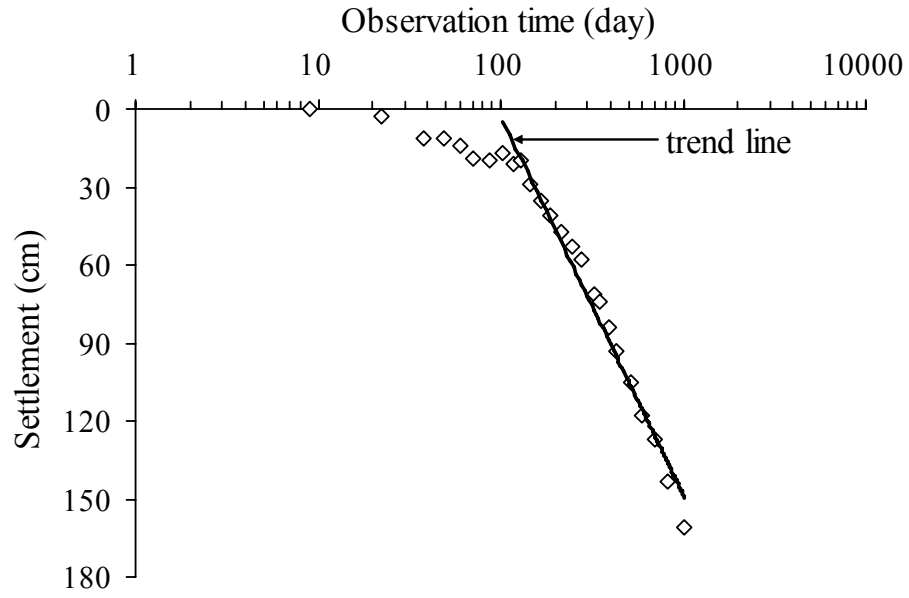


Figure 4-18: Observed settlement versus time (after surcharge removal) with trend line for the data observed at point A in Figure 4-16.

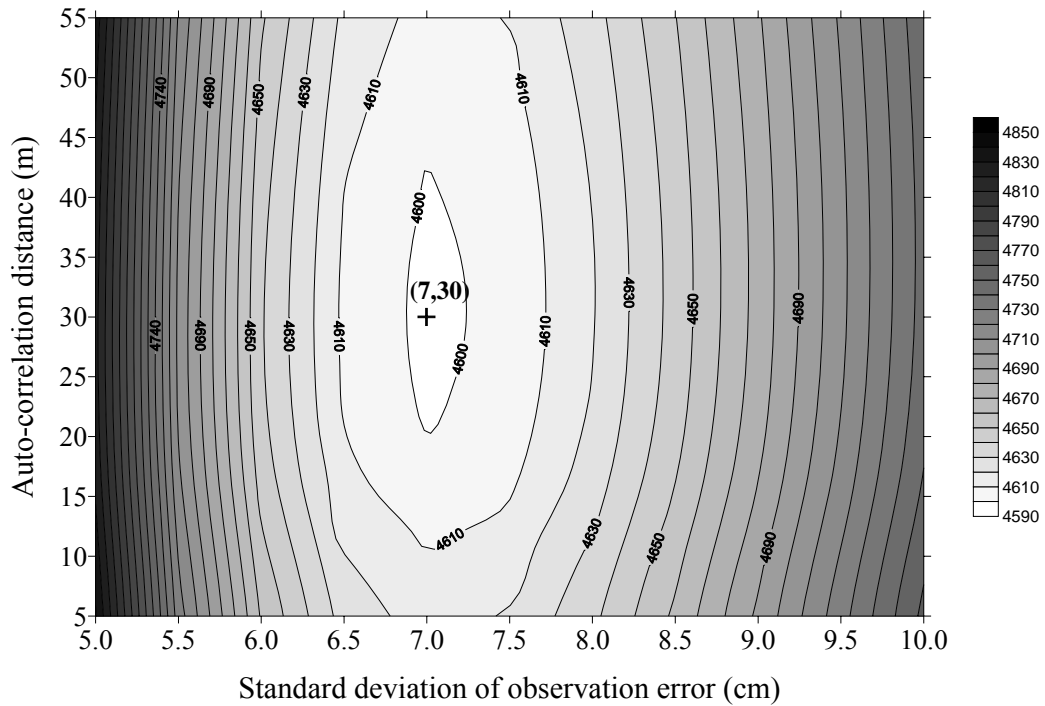
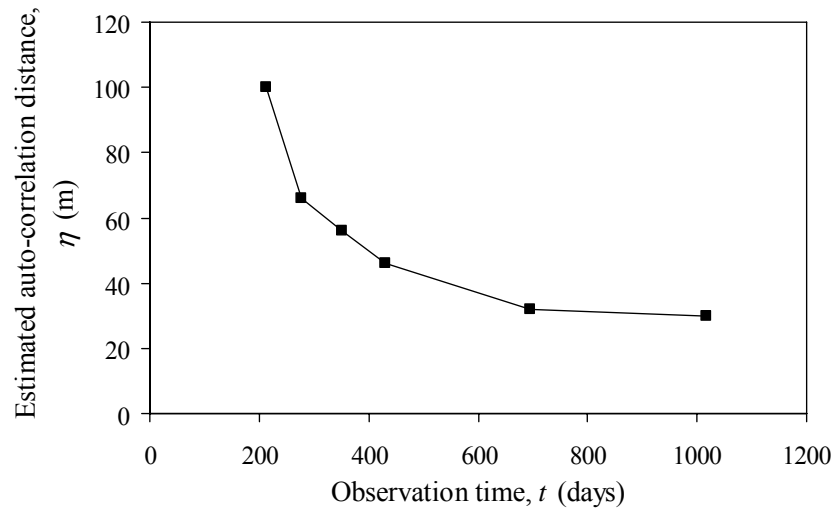
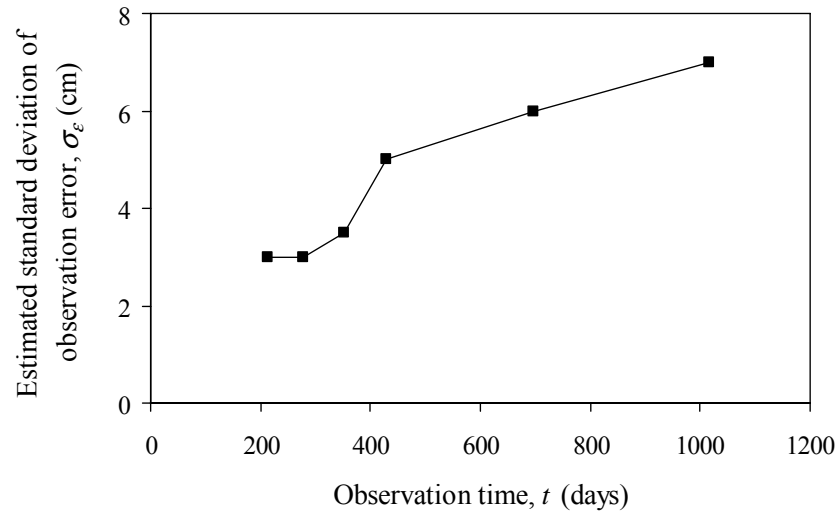


Figure 4-19: An example of contour of ABIC for estimation of auto-correlation distance (η) and the standard deviation of the observation error (σ_e), using observation data until the last step of observation (the day 1017th).



(a) auto-correlation distance (η)



(b) standard deviation of observation error (σ_e)

Figure 4-20: Plots of estimated values of auto-correlation distance (η) and the standard deviation of the observation error (σ_e) vs. observation times, for the estimation based on the actual observation data of secondary compression.

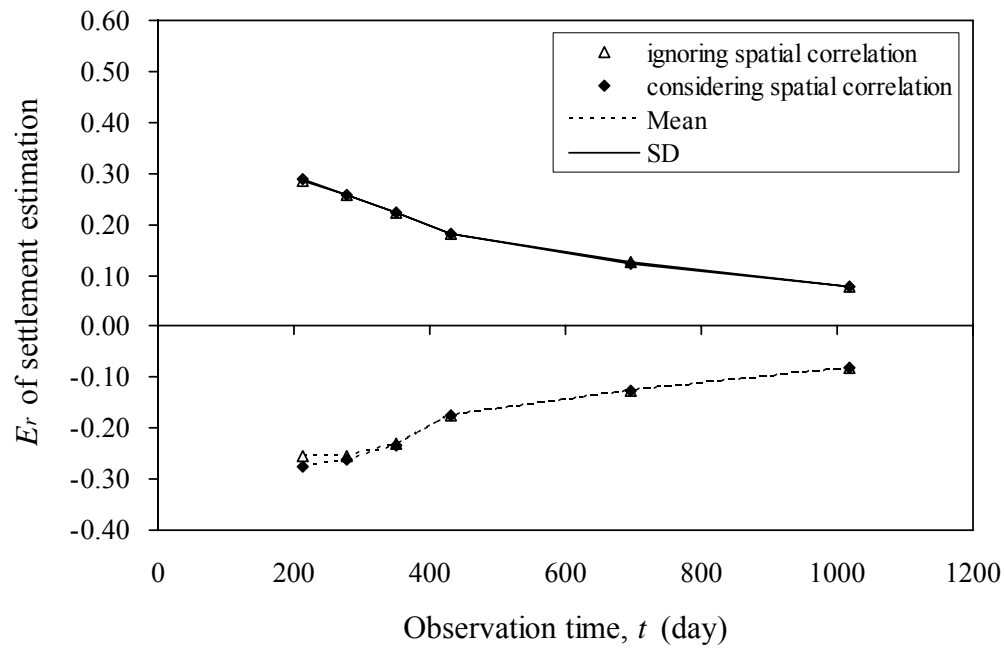
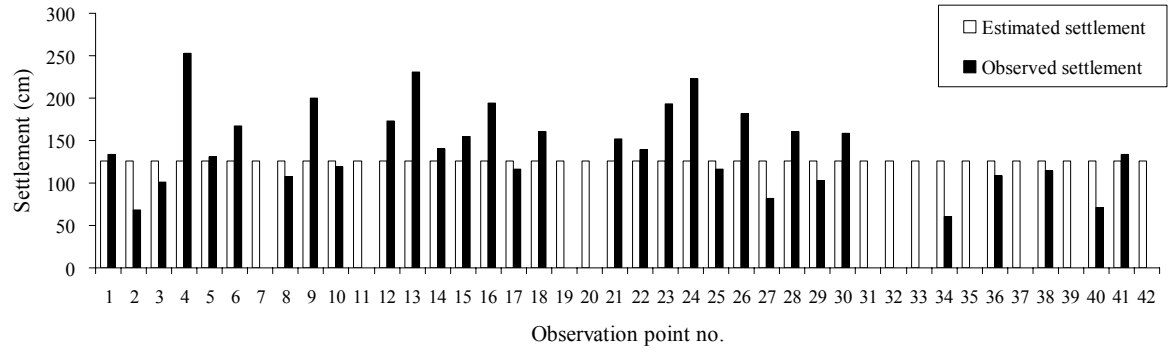
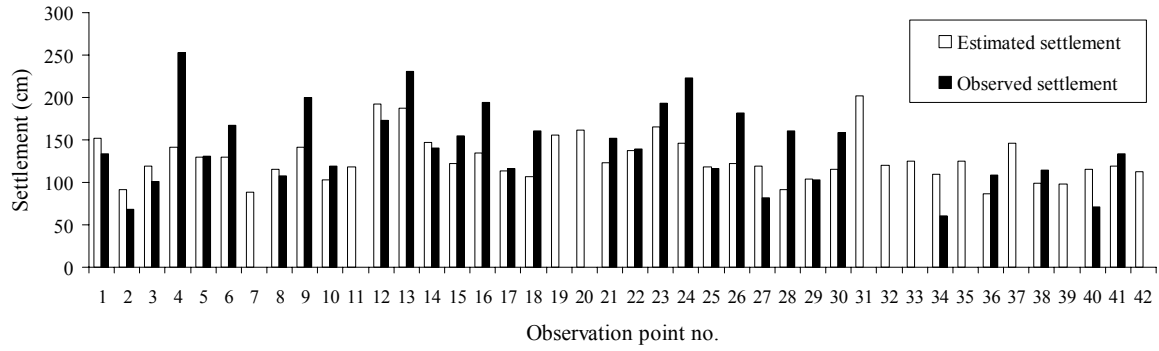


Figure 4-21: Estimation error vs. observation time for estimation of secondary compression at the last step of observation (the day 1017th) at the observation points by $S \sim \log(t)$ method, using actual observation data.

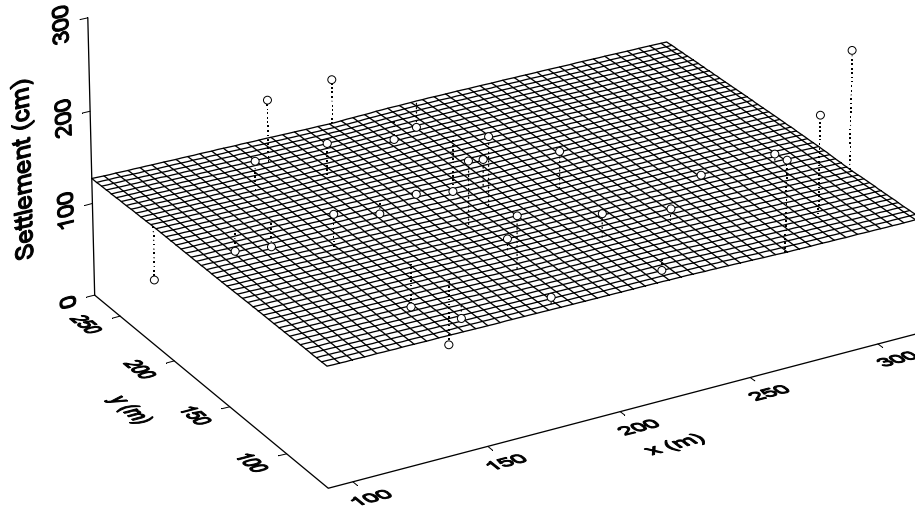


(a) ignoring spatial correlation ($\eta = 0$ m)

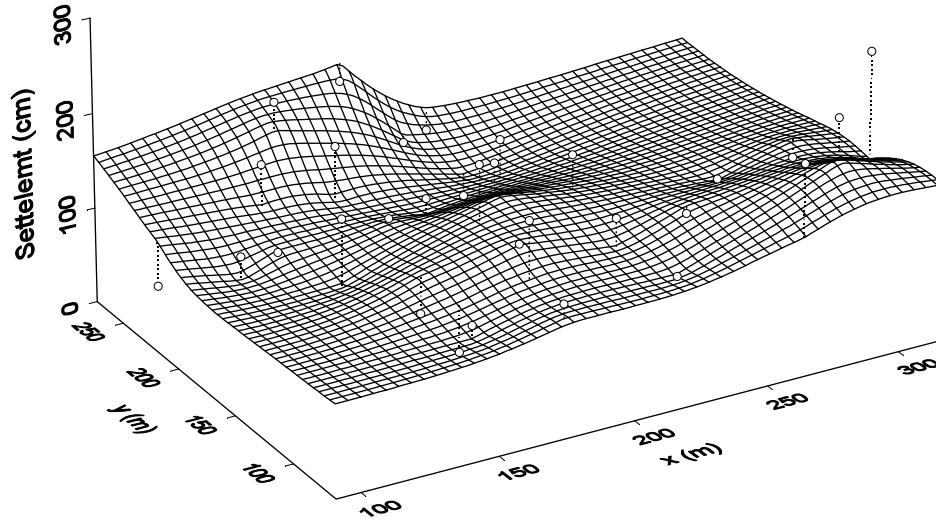


(b) considering spatial correlation ($\eta = 32$ m)

Figure 4-22: Comparison between the estimated settlement and the observed settlement for secondary compression estimation by $S \sim \log(t)$ method based on the actual observation data from the day 103rd to 696th to predict settlement at the day 1017th at the *removed* observation points.

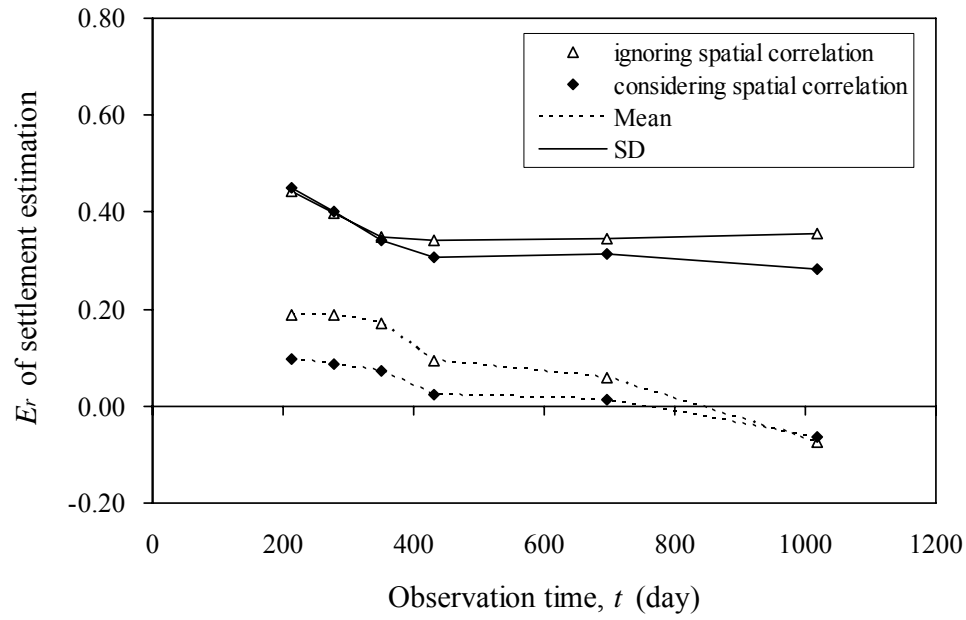


(a) ignoring spatial correlation ($\eta = 0$ m)

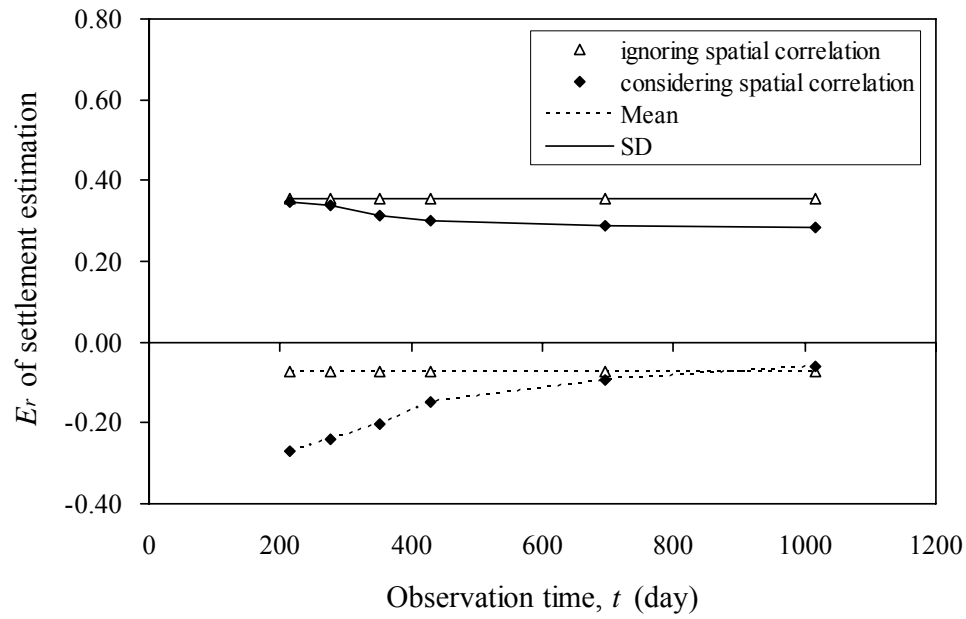


(b) considering spatial correlation ($\eta = 32$ m)

Figure 4-23: Comparison between the estimated settlement, shown as surfaces, and the observed settlement, shown as points, for secondary compression estimation by $S \sim \log(t)$ method based on observed data from the day 103rd to 696th to predict settlement at the day 1017th at the *removed* observation points.



(a) settlement estimation at the observation day



(b) settlement prediction at the day 1017th

Figure 4-24: Estimation errors vs. observation times for estimation of secondary compression at the *removed* observation points by $S \sim \log(t)$ method, using actual observation data.

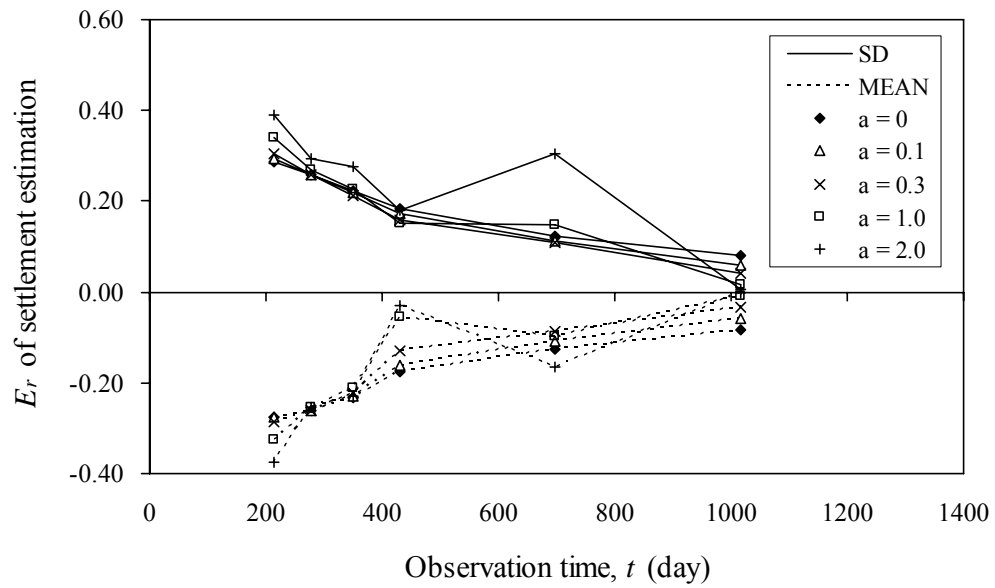


Figure 4-25: Estimation error vs. observation time with different levels of process noises, for estimation of secondary compression at the last step of observation (the day 1017th) at the observation points by $S \sim \log(t)$ method, using actual observation data.

5 Summary and conclusion

A spatial-temporal estimation process for predicting behavior of geotechnical structure based on the observation data has been proposed. A probabilistic approach is used for introducing the spatial correlation of the soil parameters (referred in this thesis as the unknown parameters) into the prediction of temporal soil behavior based on a geotechnical model, which provides the linear relationship between the parameters and the concerned soil behavior. This has been done by employing Bayesian inference to update the prior pdf of the unknown parameters, including the spatial correlation, based on observation results of the temporal soil behavior which is assumed to be measured at discrete points and discrete time. The set of parameters that maximize this posterior pdf are considered as the best estimates for the unknown parameters (e.g. Bayesian estimation) and are used to predict the future behavior through the same geotechnical model. (refer to Section 3.1 and 3.2)

Within the framework of Bayesian inference, an approach for estimating auto-correlation distance (η) and the standard deviation of the observation error (σ_ε) also has been proposed. By considering this as a model selection problem, these parameters are treated as the hyperparameters and chosen based on ABIC. (refer to Section 3.3)

The parallelism of the proposed approach and Kalman filter has been proved and the use of Kalman filter formulation to introduce process noise into the proposed spatial-temporal model also has been presented. (refer to Section 3.4 and Appendix A)

The fact that the proposed approach inherently includes the spatial interpolation process by simple kriging technique was emphasized and an alternative approach by using ordinary kriging to

perform interpolation task has been presented. The derivation of these kriging techniques using Bayesian formulation also has been provided. (refer to Section 3.5 and Appendix B, C)

Examples of using the proposed spatial-temporal process to predict primary consolidation and secondary compression have been presented using both simulated and actual field observation data. Asaoka's method is used as a model for primary consolidation prediction, while, for secondary compression, $S \sim \log(t)$ model are chosen. (refer to Section 4.1.1 and 4.2.1)

A series of simulation examples has been presented for both primary consolidation and secondary compression prediction. It is found that the trends of results from both cases seem to correspond (refer to Section 4.1.2 and 4.2.2). The settlement data are built based on the random field of the unknown parameters generated by frequency-domain technique. By these simulation examples, the proposed spatial-temporal estimation process is proved to have the following two main advantages:

- (1) The accuracy of the settlement prediction can be improved by considering the spatial correlation structure of the model parameters. According to the simulation, the improvement becomes more apparent when the spacing of the observation points is shorter than half of the auto-correlation distance.
- (2) The rational prediction of settlement at any arbitrary point at any time can be done with quantified uncertainty by introducing spatial correlation into the settlement prediction model.

In addition to these major advantages, several conclusions can be drawn from the simulation examples. These are summarized as follows:

- (1) The improvement of the prediction accuracy by taking into account spatial correlation structure tends to increase with the decrease of observation spacing to auto-correlation

- distance ratio (i.e. s/η ratio). In other words, a stronger spatial correlation gives a more accurate estimation. (refer to Section 4.1.2.2 and 4.2.2.2)
- (2) The improvement of the prediction accuracy by taking into account spatial correlation structure is larger at the earlier stage of observation. This is considered due to information provided through spatial correlation on top of temporal information which is relatively less at the early stage. This also emphasizes the advantage of using the proposed method for the estimation at an early time. (refer to Section 4.2.2.2)
 - (3) Enlarging the sampling size (n) does not greatly improve the accuracy of the estimation at a certain point. This is due to that fact that, for the site with relatively weak spatial correlation, it is only neighboring observation which contributes to the improvement of the estimation. (refer to Section 4.1.2.2)
 - (4) While the standard deviation of observation error (σ_e) can be estimated accurately, the error of auto-correlation distance (η) estimation is relatively high. The error will be even larger when the ratio of total width of the observations to the auto-correlation distance (L/η ratio) reduces. (refer to Section 4.1.2.3)
 - (5) The accuracy of settlement prediction is relatively insensitive to changes of auto-correlation distance (η). This proves the suitability of the proposed method for practical uses. (refer to Section 4.1.2.4)
 - (6) For the site that the spatial correlation of the soil parameters is relatively strong, the accuracy of settlement estimation at an arbitrary point is found to be reduced with the increase of the distance that the point is located apart from the group of observation points. On the other hands, for the site that the soil parameters tend to be independent,

the errors of the settlement estimation by the proposed method are similar, regardless of the locations. (refer to Section 4.1.2.4)

- (7) The prediction by the proposed method is found to be unbiased in all cases.

A case study on the actual observation data for the secondary compression of alluvial deposits due to preloading was carried out using the proposed approach. The prediction has been done based on $S \sim \log(t)$ model, and the examples of calculations with consideration of process noise, i.e. forgetting factor, also have been presented (refer to Section 4.2.3). It can be concluded from the results of the calculations that

- (1) The estimation of auto-correlation distance (η) is relatively unstable at the early state of observation but it becomes more stable as the observation data accumulates. (refer to Section 4.2.3.2)
- (2) It seems that the auto-correlation distance (η) of the model parameter is too short in comparison to the observation point's spacing (s) in order to provide the improvement of prediction accuracy by taking into account the spatial correlation for the estimation at the observation points. (refer to Section 4.2.3.3)
- (3) The proposed method gives the rational and more accurate estimation of the settlement at arbitrary location and time, comparing to the case which spatial correlation is ignored. (refer to Section 4.2.3.3)
- (4) Significant level of estimation bias is found. This is considered due to the imperfection of the selected model to represent the actual behavior. (refer to Section 4.2.3.3)
- (5) Considering process noises can improve the accuracy of the prediction to some extent, but the prediction error can become worst in case too high levels of process noises are assigned. (refer to Section 4.2.3.4)

It should be noted that although the application examples presented in this thesis only focus on the settlement prediction of a large preloaded area, the proposed approach is principally applicable to several types of geotechnical problems such as lateral movement of a retaining wall, and settlement of an embankment. However, in those cases, the modification of the proposed approach to deal with the more sophisticated observation model, i.e. non-linear model, may be needed and this can be considered as a potential extension of the research.

References

- Akaike, H. (1980). *Likelihood and Bayes procedure with discussion*. In Bernardo *et al.*, ed., *Bayesian Statistics*. J. M. Valencia Spain: Univ. Press, 143–166, 185–203.
- Ang, A.H-S and Tang, W.H. (2007). *Probability Concepts in Engineering: Emphasis on Applications to Civil and Environmental Engineering*, 2nd Edition, U.S.: John Wiley and Sons, Inc.
- Asaoka, A. (1978). Observational procedure for settlement prediction. *Soils and Foundations*, 18 (4), 87–101.
- Baecher, G.B. (1979). Analyzing exploration strategies. *Site Characterization and Exploration*. Dowding, C.H., ed., ASCE.
- Baecher, G.B. and Christian, J.T. (2003). Reliability and statistics in geotechnical engineering. West Sussex, UK: John Wiley & Sons.
- Baecher, G.B. and Christian, J.T. (2008). Spatial variability and geotechnical reliability. *Reliability-based design in Geotechnical Engineering*. Oxon: Taylor & Francis, 76–133.
- Berger, J.O., De Oliveira, V and Sanso, B. (2001). Objective Bayesian analysis of spatially correlated data. *Journal of American Statistical Association*, 96 (456), 1361–1374.
- Bjerrum, L. (1967). Engineering geology of Norwegian normally consolidated marine clays as related to settlement of buildings. *Géotechnique*, 17 (2), 81–118.
- Bogardi, I., Kelly, W.E. and Bardossy, A. (1989). Reliability model for soil liner: initial design. *Journal of Geotechnical Engineering, ASCE*, 115 (5), 658–669.
- Bogardi, I., Kelly, W.E. and Bardossy, A. (1990). Reliability model for soil liner: post construction. *Journal of Geotechnical Engineering, ASCE*, 116 (10), 1502–1520.

- Cheung, R.W.M and Tang, W.H. (2005). Realistic assessment of slope reliability for effective landslide hazard management. *Géotechnique*, 55 (1), 85–94.
- Chowdhury, R., Tang, W.H. and Sidi, I. (1987). Reliability model of progressive slope failure. *Géotechnique*, 37 (4), 467–481.
- Christian, J. T., Ladd, C. C. and Baecher, G. B. (1994). Reliability applied to slope stability analysis. *Journal of Geotechnical Engineering, ASCE*, 120 (12), 2180–2207.
- Cornell, C.A. (1969). A probability-based structural code. *ACI Journal*, 66 (12), 974–985.
- DeGroot, D.J. and Baecher, G.B. (1993). Estimating autocovariance of in situ soil properties. *Journal of Geotechnical Engineering Division*, 119 (GT1), 147–166.
- Fenton, G.A. (1994). Error evaluation of three random-field generators. *Journal of Engineering Mechanics, ASCE*, 120 (12), 2478–2497.
- Fenton, G.A. and Griffiths, D.V. (2002). Probabilistic foundation settlement on spatially random soil. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 128 (5), 381–390.
- Fenton, G.A. and Griffiths, D.V. (2005). Three-dimensional probabilistic foundation settlement. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 131 (2), 232–239.
- Fenton, G.A. and Vanmarcke, E.H. (1990). Simulation of random fields via local average subdivision. *Journal of Engineering Mechanics, ASCE*, 116 (8), 1733–1749.
- Fenton, G.A. and Vanmarcke, E.H. (1998). Spatial variation in liquefaction risk. *Géotechnique*, 48 (6), 819–831.
- Freudenthal, A.M. (1947). The safety of structures. *Transactions, ASCE*, 112, 125–180.
- Garlanger, J. E. (1972). The consolidation of soils exhibiting creep under constant effective stress. *Géotechnique*, 22 (1), 71–78.

- Gilbert, R.B., Wright, S.G. and Liedtke, E. (1998). Uncertainty in back analysis of slopes: Kettleman Hills case history. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 124 (12), 1167–1176.
- Griffiths, D.V. and Fenton, G.A. (2004). Probabilistic slope stability by finite elements. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 130 (5), 507–518.
- Halim, I.S. and Tang, W.H. (1993). Site exploration strategy for geologic anomaly characterization. *Journal of Geotechnical Engineering, ASCE*, 119 (2), 195–213.
- Harris, D.I., Mair, R.J., Love, J.P., Taylor, R.N. and Henderson, T.O. (1994). Observations of ground and structure movements for compensation grouting during tunnel construction at Waterloo Station. *Géotechnique*, 44 (4), 691–713.
- Hasofer, A.M. and Lind, N.C. (1974). Exact and invariant first-order reliability format. *Journal of Engineering Mechanics Division, ASCE*, 100 (EM1), 111–121.
- Honjo, Y. and Kashiwagi, N. (1999). Matching objective and subjective information in groundwater inverse analysis by Akaike's Bayesian Information Criterion. *Water Resources Research*, 35 (2), 435–447.
- Honjo, Y. and Kuroda, K. (1991). New look at fluctuating geotechnical data for reliability design. *Soils and Foundations*, 31 (1), 110–120.
- Honjo, Y. and Setiawan, B. (2007). On general and local estimation of local average and their application in geotechnical parameter estimation. *Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards*, 1 (3), 167–176.
- Hoshiya, M. and Yoshida, I. (1996). Identification of conditional stochastic Gaussian field. *Journal of Engineering Mechanics, ASCE*, 122 (2), 101–108.

- Hoshiya, M. and Yoshida, I. (1998). Process noise and optimum observation in conditional stochastic fields. *Journal of Engineering Mechanics, ASCE*, 124 (12), 1325–1330.
- Ikuta, Y., Maruoka, M., Aoki, M. and Sato, E. (1994). Application of the observational method to a deep basement excavated using the top-down method. *Géotechnique*, 44 (4), 655–664.
- Iwasaki, Y., Watanabe, H., Fukuda, M., Hirata, A. and Hori, Y. (1994). Construction control for underpinning piles and their behaviour. *Géotechnique*, 44 (4), 681–689.
- Jazwinski, A.H. (1976). *Stochastic Process and Filtering Theory*. New York, N.Y.: Academic Press, Inc.
- Juang, C.H., Jiang, T. and Andrus, R.D. (2002). Assessing probability-based methods for liquefaction potential evaluation. *Journal of Geotechnical and Geoenvironmental Engineering, ASCE*, 128 (7), 580–589.
- Kalman, R.E. (1960). A new approach to linear filtering and prediction problems. *Journal of Basic Engineering*, 82 (1), 35–45.
- Kalman, R.E. and Bucy, R.S. (1961). New results in linear filtering and prediction theory. *Journal of Basic Engineering*, 83 (1), 95–108.
- Katayama, T. (1983). *Applied Kalman filter*. Tokyo: Asakurashoten, 46. (in Japanese).
- Krige, D.G. (1966). Two dimensional weighted moving averaging trend surfaces for ore evaluation. *Proc., of Symp. on Math., Statistics and Comp. Appl. for Ore Evaluation*.
- Kulhawy, F.H., Roth, M.J.S. and Grigoriu, M.D. (1991). Some statistical evaluations of geotechnical properties. *Proceeding, Sixth ICASP*, Mexico City, 705–712.
- Kyriakidis, P. C. and Journel, A. G. (1999). Geostatistical space-time models: A review. *Mathematical Geology*, 31 (6), 651–684.

- Li, K.S. and Lumb, P. (1987). Probabilistic design of slope. *Canadian Geotechnical Journal*, 24, 520–535.
- Lumb, P. (1974). Application of statistics in soil mechanics. In Lee, I.K., ed., *Soil Mechanics: New Horizons*, London: Newnes-Butterworth, 44–111.
- Mardia, K.V. and Marshall, R.J. (1984). Maximum likelihood estimation of models for residual covariance in spatial regression. *Biometrika*, 71 (1), 135–146.
- Matheron, G. (1973). The Intrinsic Random Functions and their Applications. *Adv. in Appl. Probab.*, 5.
- Matsuo, M. and Asaoka, A. (1978). Dynamic design philosophy of soils based on the Bayesian reliability prediction. *Soils and Foundations*, 18 (4), 1–17.
- Mesri, G. *et al.* (1997). Secondary compression of peat with or without surcharging. *Journal of the Geotechnical Engineering Division, ASCE*, 123 (5), 411–421.
- Or, D. and Hanks, R. J. (1992). Spatial and temporal soil water estimation considering soil variability and evapotranspiration uncertainty, *Water Resources Research*, 28 (3), 803–814.
- Peck, R.B. (1969). Advantages and limitations of the observational method in applied soil mechanics, Ninth Rankine Lecture. *Géotechnique*, 19, 171–187.
- Phoon, K.K. (2008). *Reliability-based design in geotechnical engineering: computations and applications*, Phoon, ed., New York: Taylor & Francis.
- Phoon, K.K. and Kulhawy, F.W. (1999a). Characterization of geotechnical variability. *Canadian Geotechnical Journal*, 36, 612–624.
- Phoon, K.K. and Kulhawy, F.W. (1999b). Evaluation of geotechnical property variability. *Canadian Geotechnical Journal*, 36, 625–639.

- Powderham, A.J. (1994). An overview of the observational method: development in cut and cover and bored tunnelling projects. *Géotechnique*, 44 (4), 619–636.
- Roberts, T.O.L. and Preene, M. (1994). The design of groundwater control systems using the observational method. *Géotechnique*, 44 (4), 727–734.
- Ronold, K.O., and Bjerager, P. (1992). Model uncertainty representation in geotechnical reliability analysis. *Journal of Geotechnical Engineering*, 118 (3), 363–376.
- Shinozuka, M. (1971). Simulation of multivariate and multidimensional random processes. *Journal of the Acoustical Society of America*, 49 (1, part 2), 357–367.
- Shinozuka, M. and Jan, C.-M. (1972). Digital simulation of random processes and its applications. *Journal of Sound and Vibration*, 25 (1), 111–128.
- Shinozuka, M., Yun, C.B. and Imai, H. (1982). Identification of linear structural dynamic systems. *Journal of Engineering Mechanics Division, ASCE*, 108 (6), 1371–1390.
- Soulie, M., Montes, P. and Silverstri, V. (1990). Modeling spatial variability of soil parameters. *Canadian Geotechnical Journal*, 27 (5), 617–630.
- Sridharan, A., Murthy, N.S. and Prakash, K. (1987). Rectangular hyperbolic method of consolidation analysis. *Géotechnique*, 37 (3), 355–368.
- Stein, M. (1986). A simple model for spatial-temporal processes. *Water Resources Research*, 22 (13), 2107–2110.
- Tan, S.A. (1994). Hyperbolic method for settlements in clays with vertical drains. *Canadian Geotechnical Journal*, 31, 125–131.
- Tang, W.H. (1971). A Bayesian evaluation of information for foundation engineering design. In Lumb, P., ed., *Statistics and Probability in Civil Engineering*. Hong Kong: Hong Kong Univ. Press, 173–185.

- Tang, W.H. (1987). Updating anomaly statistics – single anomaly case. *Journal of Structural Safety*, Elsevier Science Publishers, 4 (2), 151–163.
- Tang, W.H. (1993). Recent development in geotechnical reliability. *Probabilistic Methods in Geotechnical Reliability*, Li & Lo, ed., Rotterdam: Balkema, 3–27.
- Terzaghi, K. and Peck, R.B. (1967). *Soil Mechanics in Engineering Practice*, 2nd ed., New York: John Wiley and Sons.
- Ueda, T., Honjo, Y., Hatano, T. and Sakaguchi, S. (1986). Estimation of the spatial distribution of residual settlement in a reclaimed area. *Journal of Japanese Society of Geotechnical Society (Tsuchi to Kiso)*, 34 (5), 51–58. (In Japanese)
- Vanmarcke, E.M. (1977a). Probabilistic modeling of soil profiles. *Journal of the Geotechnical Engineering Division, ASCE*, 103 (GT11), 1227–1246.
- Vanmarcke, E.M. (1977b). Reliability of earth slopes. *Journal of the Geotechnical Engineering Division, ASCE*, 103 (GT11), 1247–1265.
- Wackernagel, H. (1998). *Multivariate Geostatistics: An Introduction with Applications*, 2nd ed. Germany: Springer-Verlag Berlin Heidelberg.
- Wakita, E. and Matsuo, M. (1994). Observational design method for earth structures constructed on soft ground. *Géotechnique*, 44 (4), 747–755.
- Wu, T.H. (2009). Reliability of geotechnical predications. *Geotechnical Risk and Safety*. Honjo *et al.*, ed., 3–10.
- Wu, T.H., Abdel-Latif, M.A., Nuhfer, M.A. and Curry, B.B. (1996). Use of geologic information in site characterization. *Uncertainty in Geological Environment*, Madison, ASCE, 76–90.
- Wu, T.H. and Kraft, L.M. (1970). Safety analysis of slope. *Journal of Soil Mechanics and Foundation Engineering Division, ASCE*, 96 (SM2), 609–630.

- Wu, T.H., Tang, W.H., Sangrey, D.A. and Baecher, G.B. (1989). Reliability of offshore foundations – state of the art. *Journal of Geotechnical Engineering, ASCE*, 115 (2), 157–178.
- Young, D.K. and Ho, E.W.L. (1994). The observational approach to design of a sheet-piled retaining wall. *Géotechnique*, 44 (4), 637–654.
- Yun, C.B. and Shinozuka, M. (1980). Identification of nonlinear structural dynamic systems. *Journal of Structural Mechanics*, 8 (2), 187–203.

APPENDIX A

Derivation of Kalman filter formulation based on Bayesian approach

This section provides the proof that, for the static problem without process noise, the Kalman filter gives solutions that are identical to Bayesian estimation. The theoretical details of the relationship between these two methods are discussed in Hoshiya and Yoshida (1996 and 1998). To prove this, two assumptions are required:

- (1) The observation model is linear, and
- (2) Observation-model errors at each step of observation are independent.

Bayesian estimation

The detailed derivation of several equations for the Bayesian estimation is shown in Sections 3.2. By differentiating Eq. (3-16) with respect to the state vector, we obtain

$$\frac{\partial J(Z|\sigma_\varepsilon^2, \eta)}{\partial Z} = 2V_Z^{-1}(\hat{Z} - Z_0) - 2\sum_{k=1}^K M_k^T V_\varepsilon^{-1}(Y_k - M_k \hat{Z}) = 0 \quad (\text{A-1})$$

$$\left[V_Z^{-1} + \sum_{k=1}^K M_k^T V_\varepsilon^{-1} M_k \right] \hat{Z} = V_Z^{-1} Z_0 + \sum_{k=1}^K M_k^T V_\varepsilon^{-1} Y_k \quad (\text{A-2})$$

Including $\sum_{k=1}^K M_k^T V_\varepsilon^{-1} M_k Z_0$ into the right side of the above equation, gives

$$\left[V_Z^{-1} + \sum_{k=1}^K M_k^T V_\varepsilon^{-1} M_k \right] \hat{Z} = V_Z^{-1} Z_0 + \sum_{k=1}^K M_k^T V_\varepsilon^{-1} Y_k + \sum_{k=1}^K M_k^T V_\varepsilon^{-1} M_k Z_0 - \sum_{k=1}^K M_k^T V_\varepsilon^{-1} M_k Z_0 \quad (\text{A-3})$$

By rearranging the above equations, we obtain

$$\left[V_Z^{-1} + \sum_{k=1}^K M_k^T V_{\varepsilon}^{-1} M_k \right] \hat{Z} = \left(V_Z^{-1} + \sum_{k=1}^K M_k^T V_{\varepsilon}^{-1} M_k \right) Z_0 + \sum_{k=1}^K M_k^T V_{\varepsilon}^{-1} (Y_k - M_k Z_0) \quad (\text{A-4})$$

$$\hat{Z} = Z_0 + S \sum_{k=1}^K M_k^T V_{\varepsilon}^{-1} (Y_k - M_k Z_0) \quad (\text{A-5})$$

where

$$S = \left(V_Z^{-1} + \sum_{k=1}^K M_k^T V_{\varepsilon}^{-1} M_k \right)^{-1} \quad (\text{A-6})$$

Kalman filter

The general equations of the Kalman filter, are provided in Section 3.4. In case the process noise is neglected (e.g. $Q_k = 0$), we have from Eq. (3-23) & (3-24)

$$Z_{k/k-1} = Z_{k-1/k-1} \quad (\text{A-7})$$

$$V_{Z,k/k-1} = V_{Z,k-1/k-1} \quad (\text{A-8})$$

Therefore, the first suffix of the subscripts can be dropped from the equations. By rearranging Eq. (3-27), (3-28), and (3-29), we have

$$Z_k = Z_{k-1} + V_{Z,k} M_k^T V_{\varepsilon}^{-1} (Y_k - M_k Z_{k-1}) \quad (\text{A-9})$$

$$V_{Z,k} = \left(V_{Z,k-1}^{-1} + M_k^T V_{\varepsilon}^{-1} M_k \right)^{-1} \quad (\text{A-10})$$

Suppose that the first three steps of observation, i.e. Y_0 , Y_1 , and Y_2 , are obtained. The first step of the observation updating process can be performed as follows:

$$Z_1 = Z_0 + V_{Z,1} M_1^T V_{\varepsilon}^{-1} (Y_1 - M_1 Z_0) \quad (\text{A-11})$$

$$V_{Z,1} = \left(V_{Z,0}^{-1} + M_1^T V_{\varepsilon}^{-1} M_1 \right)^{-1} \quad (\text{A-12})$$

Similarly, for the second step of observation updating

$$Z_2 = Z_1 + V_{Z,2} M_2^T V_{\varepsilon}^{-1} (Y_2 - M_2 Z_1) \quad (\text{A-13})$$

$$V_{Z,2} = \left(V_{Z,1}^{-1} + M_2^T V_\varepsilon^{-1} M_2 \right)^{-1} \quad (\text{A-14})$$

Substituting Eq. (A-11) into (A-13), gives

$$Z_2 = Z_0 + V_{Z,1} M_1^T V_\varepsilon^{-1} (Y_1 - M_1 Z_0) + V_{Z,2} M_2^T V_\varepsilon^{-1} \left\{ Y_2 - M_2 (Z_0 + V_{Z,1} M_1^T V_\varepsilon^{-1} (Y_1 - M_1 Z_0)) \right\} \quad (\text{A-15})$$

By rearranging the above equations, we have

$$Z_2 = Z_0 + \left(V_{Z,1} - V_{Z,2} M_2^T V_\varepsilon^{-1} M_2 V_{Z,1} \right) M_1^T V_\varepsilon^{-1} (Y_1 - M_1 Z_0) + V_{Z,2} M_2^T V_\varepsilon^{-1} (Y_2 - M_2 Z_0) \quad (\text{A-16})$$

At this stage, it needs to be proved that

$$V_{Z,1} - V_{Z,2} M_2^T V_\varepsilon^{-1} M_2 V_{Z,1} = V_{Z,2} \quad (\text{A-17})$$

According to the ‘matrix inversion lemma’ concept, for the given matrixes A, B, C, and D, it can be proved that $(A^{-1} + C^T B^{-1} D)^{-1} = A - A C^T (D A C^T + B)^{-1} D A$. By substituting Eq. (A-14) into (A.17) and transforming the right side of the equation by the matrix inversion lemma, we obtain

$$V_{Z,1} - \left(V_{Z,1}^{-1} + M_2^T V_\varepsilon^{-1} M_2 \right)^{-1} M_2^T V_\varepsilon^{-1} M_2 V_{Z,1} = V_{Z,1} - V_{Z,1} M_2^T \left(M_2 V_{Z,1} M_2^T + V_\varepsilon \right)^{-1} M_2 V_{Z,1} \quad (\text{A-18})$$

Rearranging the above equation, gives

$$V_{Z,1} M_2^T V_\varepsilon^{-1} M_2 V_{Z,1} + V_{Z,1} = V_{Z,1} + V_{Z,1} M_2^T V_\varepsilon^{-1} M_2 V_{Z,1} \quad (\text{A-19})$$

This proves that Eq. (A-17) is true. By substituting Eq. (A-17) into Eq. (A-16), we obtain

$$Z_2 = Z_0 + V_{Z,2} M_1^T V_\varepsilon^{-1} (Y_1 - M_1 Z_0) + V_{Z,2} M_2^T V_\varepsilon^{-1} (Y_2 - M_2 Z_0) \quad (\text{A-20})$$

The above equation can be rewritten as

$$Z_2 = Z_0 + V_{Z,2} \sum_{k=1}^2 M_k^T V_\varepsilon^{-1} (Y_k - M_k Z_0) \quad (\text{A-21})$$

Substituting Eq. (A-12) into (A.14), gives

$$V_{Z,2} = \left(V_{Z,0}^{-1} + \sum_{k=1}^2 M_k^T V_\varepsilon^{-1} M_k \right)^{-1} \quad (\text{A-22})$$

By a similar derivation for the successively observed data from step 3 to K , Eq. (A-21) and (A-22) can be rewritten in the following forms:

$$Z_K = Z_0 + V_{Z,K} \sum_{k=1}^K M_k^T V_{\varepsilon}^{-1} (Y_k - M_k Z_0) \quad (\text{A-23})$$

and

$$V_{Z,K} = \left(V_{Z,0}^{-1} + \sum_{k=1}^K M_k^T V_{\varepsilon}^{-1} M_k \right)^{-1} \quad (\text{A-24})$$

By comparing Eq. (A-23) to (A-5) and Eq. (A-24) to (A-6), it is proved that the Kalman filter is equivalent to Bayesian estimation, when the process noise is neglected.

APPENDIX B

Derivation of kriging formulation based on Bayesian approach

This section shows the derivation of ordinary kriging (Krige 1966, Matheron 1973, Wackernagel 1998) using Bayesian approach. Hoshiya and Yoshida (1996) have proved that simple kriging can be derived based on Bayesian formulation by maximizing the conditional probability. It was assumed that observation vector (Y) and the unknown parameter (Z) are the same physical parameters, and the observation error (ε) is zero. From these assumptions, we have zero covariance of observation error

$$V_{\varepsilon} = 0_{n,n} \quad (\text{B-1})$$

and a linear function between Y and Z

$$Y = MZ = \begin{bmatrix} I_{n,n} & 0_n \end{bmatrix} \begin{bmatrix} Z_n \\ z(x_0) \end{bmatrix} \quad (\text{B-2})$$

where , at n observation points x_1, x_2, \dots, x_n , observation vector $Y = [y(x_1), y(x_2), \dots, y(x_n)]^T$, and unknown parameter $Z_n = [z(x_1), z(x_2), \dots, z(x_n)]^T$. $z(x_0)$ is the unknown parameter at an arbitrary point x_0 to be estimated. $I_{n,n}$ denotes $n \times n$ identity matrix. $0_{n,n}$ and 0_n denotes $n \times n$ and $n \times 1$ zero matrix, respectively.

The best estimator of Z conditional on Y , i.e. \hat{Z} , can be determined by maximizing the conditional probability $p(Z|Y)$ which is expressed based on Bayes' Theorems as follows:

$$p(Z|Y) \propto p(Z) p(Y|Z) \quad (\text{B-3})$$

where $p(Z)$ = a Gaussian density function with mean Z_0 and covariance V_Z ; $p(Y|Z)$ = a Gaussian density function with mean MZ and covariance V_ε . Thus, we have

$$p(Z) = (2\pi)^{-(n+1)/2} |V_Z|^{-1/2} \exp\left[-\frac{1}{2}(Z - Z_0)^T V_Z^{-1} (Z - Z_0)\right] \quad (\text{B-4})$$

$$p(Y|Z) = (2\pi)^{-n/2} |V_\varepsilon|^{-1/2} \cdot \exp\left[-\frac{1}{2}(Y - MZ)^T V_\varepsilon^{-1} (Y - MZ)\right] \quad (\text{B-5})$$

From Eq. (B-3), (B-4), and (B-5), we have

$$p(Z|Y) \propto (2\pi)^{-(2n+1)/2} |V_Z|^{-1/2} |V_\varepsilon|^{-1/2} \cdot \exp\left\{-\frac{1}{2}\left[(Z - Z_0)^T V_Z^{-1} (Z - Z_0) + (Y - MZ)^T V_\varepsilon^{-1} (Y - MZ)\right]\right\} \quad (\text{B-6})$$

By taking logarithmic function of the above equation, it is found that maximization of $p(Z|Y)$ is equivalent to minimization of the following objective function

$$J = (Z - Z_0)^T V_Z^{-1} (Z - Z_0) + (Y - MZ)^T V_\varepsilon^{-1} (Y - MZ) \quad (\text{B-7})$$

It can be proved that \hat{Z} , which gives the minimum values of J , is expressed as (e.g. Katayama 1983)

$$\hat{Z} = Z_0 + V_Z M^T (V_\varepsilon + M V_Z M^T)^{-1} (Y - M Z_0) \quad (\text{B-8})$$

Note that the above equation is actually the Kalman filter equation for observation updating (Kalman 1960, Kalman & Bucy 1961, Jazwinski 1976). Based on the simple kriging assumption of second-order stationary, the mean of the random variable, m , is assumed to be the same at any location and the value of which is known. From this assumption, we have

$$Z_0 = m \cdot u_{n+1} \quad (\text{B-9})$$

where m = the constant mean of the random variable Z ; $u_{n+1} = [1, 1, \dots, 1]_{(n+1) \times 1}^T$. Based on the assumptions previously presented in Eq. (B-1), Eq. (B-2), and Eq. (B-9), we have from Eq. (B-8) that

$$\begin{bmatrix} \hat{Z}_n \\ \hat{z}(x_0) \end{bmatrix} = m \cdot u_{n+1} + V_Z \begin{bmatrix} I_{n,n} & 0_n \end{bmatrix}^T \left(\begin{bmatrix} I_{n,n} & 0_n \end{bmatrix} V_Z \begin{bmatrix} I_{n,n} & 0_n \end{bmatrix}^T \right)^{-1} \cdot \left(\begin{bmatrix} I_{n,n} & 0_n \end{bmatrix} \begin{bmatrix} Z_n \\ z(x_0) \end{bmatrix} - \begin{bmatrix} I_{n,n} & 0_n \end{bmatrix} m \cdot u_{n+1} \right) \quad (\text{B-10})$$

It should be noted the covariance matrix V_Z can be presented in the form of covariance function as follows:

$$V_Z = \begin{bmatrix} C(\underline{x}_1 - \underline{x}_1) & \cdots & C(\underline{x}_1 - \underline{x}_n) & C(\underline{x}_1 - \underline{x}_0) \\ \vdots & \ddots & \vdots & \vdots \\ C(\underline{x}_n - \underline{x}_1) & \cdots & C(\underline{x}_n - \underline{x}_n) & C(\underline{x}_n - \underline{x}_0) \\ C(\underline{x}_0 - \underline{x}_1) & \cdots & C(\underline{x}_0 - \underline{x}_n) & C(\underline{x}_0 - \underline{x}_0) \end{bmatrix} \quad (\text{B-11})$$

where $C(\underline{x}_i - \underline{x}_j)$ denotes the covariance function where $\underline{x}_i, \underline{x}_j$ = spatial vector coordinate. By partitioning matrix V_Z , each portion is denoted by following notations:

$$V_{Z,xn} = \begin{bmatrix} C(\underline{x}_1 - \underline{x}_1) & \cdots & C(\underline{x}_1 - \underline{x}_n) \\ \vdots & \ddots & \vdots \\ C(\underline{x}_n - \underline{x}_1) & \cdots & C(\underline{x}_n - \underline{x}_n) \end{bmatrix} \quad (\text{B-12})$$

$$V_{Z,x0} = \begin{bmatrix} C(\underline{x}_1 - \underline{x}_0) \\ \vdots \\ C(\underline{x}_n - \underline{x}_0) \end{bmatrix} \quad (\text{B-13})$$

Eq. (B-10) can be rewritten in the form of the above partitioned matrix as follows:

$$\begin{bmatrix} \hat{Z}_n \\ \hat{z}(x_0) \end{bmatrix} - m \cdot u_{n+1} = \begin{bmatrix} V_{Z,xn} \\ V_{Z,x0}^T \end{bmatrix} V_{Z,xn}^{-1} (Z_n - m \cdot u_n) \quad (\text{B-14})$$

Eq. (B-14) can be rewritten in the form of ‘simple kriging’ for estimation of $\hat{z}(x_0)$ (Hoshiya & Yoshida 1996), as follows:

$$\hat{z}(x_0) - m = V_{Z,x_0}^T V_{Z,xn}^{-1} (Z_n - m \cdot u_n) = w_{sm} (Z_n - m \cdot u_n) \quad (\text{B-15})$$

where w_{sm} = the weight factor for simple kriging (see APPENDIX C); and

In this thesis, the extension for the derivation of ‘ordinary kriging’ will be presented. It is clear from the above equations that the simple kriging actually gives the estimation of the residual, i.e. $\hat{z}(x_0) - m$, assuming that the constant mean, m , is known. On the other hand, ordinary kriging directly estimates the unknown value, i.e. $\hat{z}(x_0)$, implying that determination of m is already included in the ordinary kriging formulation. Therefore, prior to deriving ordinary kriging, a formulation to estimate m has to be derived. The term ‘kriging the mean’ is used to represent this calculation in some literatures (e.g. Wackernagel 1998).

In this case, $\hat{z}(x_0)$ will be excluded from the calculation, while m will be considered as an unknown parameter to be estimated. Thus, Eq. (B-2) becomes

$$Y = MZ = I_{n,n} Z_n = Z_n \quad (\text{B-16})$$

It is clear that, in this case, $V_Z = V_{Z,xn}$. By substitute this and Eq. (B-9), Eq. (B-16) into Eq. (B-7) we have

$$J = (Z_n - m \cdot u_n)^T V_{Z,xn}^{-1} (Z_n - m \cdot u_n) \quad (\text{B-17})$$

The best estimator of m , i.e. \hat{m} , can be determined by minimizing J as follows:

$$\frac{\partial J}{\partial m} = -2u_n^T V_{Z,xn}^{-1} (Z_n - \hat{m} \cdot u_n) = 0 \quad (\text{B-18})$$

$$\hat{m} = \left\{ \left[u_n^T V_{Z,xn}^{-1} u_n \right]^{-1} u_n^T V_{Z,xn}^{-1} \right\} Z_n = w_m Z_n \quad (\text{B-19})$$

where w_m is the weight factor for ‘kriging the mean’ (see APPENDIX C).

Substitute \hat{m} to m in Eq. (B-15), i.e. simple kriging equation, we have

$$\hat{z}(x_0) = (w_m + w_{sm} - w_{sm}u_n w_m)Z_n = w_{or}Z_n \quad (\text{B-20})$$

where w_{or} actually equivalent to the weight factor for ‘ordinary kriging’. As a result, the relationship between the weights for simple, mean, and ordinary kriging is obtained. In order to prove this, the common form of ordinary kriging equation will be shown and rearranged in the corresponding terms. Using the previously defined parameters, the common form of the weight factor of ordinary kriging (Wackernagel 1998) can be expressed as follows:

$$\begin{bmatrix} w_{or}^T \\ \mu \end{bmatrix} = \begin{bmatrix} V_{Z,xn} & -u_n \\ u_n^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} V_{Z,x0} \\ 1 \end{bmatrix} \quad (\text{B-21})$$

where μ denotes Lagrange multiplier. Based on technique of the matrix inversion by partitioning, it can be proved that

$$\begin{bmatrix} A_{11} & \vdots & A_{12} \\ \dots & \vdots & \dots \\ A_{21} & \vdots & A_{22} \end{bmatrix}^{-1} = \begin{bmatrix} A_{11}^{-1} + A_{11}^{-1}A_{12}[A_{22} - A_{21}A_{11}^{-1}A_{12}]^{-1}A_{21}A_{11}^{-1} & \vdots & -A_{11}^{-1}A_{12}[A_{22} - A_{21}A_{11}^{-1}A_{12}]^{-1} \\ \dots & \vdots & \dots \\ -[A_{22} - A_{21}A_{11}^{-1}A_{12}]^{-1}A_{21}A_{11}^{-1} & \vdots & [A_{22} - A_{21}A_{11}^{-1}A_{12}]^{-1} \end{bmatrix}^{-1} \quad (\text{B-22})$$

Considering the right-hand side of Eq. (B-21), let

$$\begin{bmatrix} V_{Z,xn} & \vdots & -u_n \\ \dots & \vdots & \dots \\ u_n^T & \vdots & 0 \end{bmatrix} = \begin{bmatrix} A_{11} & \vdots & A_{12} \\ \dots & \vdots & \dots \\ A_{21} & \vdots & A_{22} \end{bmatrix} \quad (\text{B-23})$$

From Eq. (B-21), (B-22), and (B-23), we have

$$w_{or} = \left[u_n^T V_{Z,xn}^{-1} u_n \right]^{-1} u_n^T V_{Z,xn}^{-1} + V_{Z,x0}^T V_{Z,xn}^{-1} - V_{Z,x0}^T V_{Z,xn}^{-1} u_n \left[u_n^T V_{Z,xn}^{-1} u_n \right]^{-1} u_n^T V_{Z,xn}^{-1} \quad (\text{B-24})$$

which is equivalent to

$$w_{or} = w_m + w_{sm} - w_{sm}u_n w_m \quad (\text{B-25})$$

The above equation is identical to the one shown in Eq. (B-20) which has been derived based on Bayesian formulation. Therefore, the derivation of ordinary kriging using Bayesian approach is now proved to be completed.

APPENDIX C

Kriging techniques: assumptions and equations

The summary of the basic assumptions and equations of kriging method (Krige 1966, Matheron 1973, Wackernagel 1998) is presented in this section. Three different techniques of this method, namely Simple kriging, kriging the mean, and Ordinary kriging, are chosen and the comparisons among them are illustrated in the following table:

		Simple kriging	Kriging the mean	Ordinary kriging
Estimated value		estimate of residual of random function at x_0 , $\hat{z}(x_0) - m$	estimate of mean of random function, m	estimate of random function at x_0 , $\hat{z}(x_0)$
Assumption of random function	Mean	constant and known; $E[Z(x+h)] = E[Z(x)]$	constant, but unknown; $E[Z(x+h)] = E[Z(x)]$	
	Covariance*	known & depends only on vector ‘ h ’ which separates the points: $\text{cov}[Z(x+h), Z(x)] = C(h)$		
	Distribution	spatially identical		
Estimator conditions	Linear estimator	satisfy		
	Unbiased estimator	satisfy		
	Best estimator	satisfy		
Constraint on kriging weight		none	$\sum_{\alpha=1}^n w_{\alpha} = 1$	$\sum_{\alpha=1}^n w_{\alpha} = 1$
Kriging estimation		Kriging weight, $\begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix} = \begin{bmatrix} C(\underline{x}_1 - \underline{x}_1) & \cdots & C(\underline{x}_1 - \underline{x}_n) \\ \vdots & \ddots & \vdots \\ C(\underline{x}_n - \underline{x}_1) & \cdots & C(\underline{x}_n - \underline{x}_n) \end{bmatrix}^{-1} \begin{bmatrix} C(\underline{x}_1 - \underline{x}_0) \\ \vdots \\ C(\underline{x}_n - \underline{x}_0) \end{bmatrix}$ Kriging estimation, $\hat{z}(x_0) - m = \begin{bmatrix} \hat{z}(x_1) - m \\ \vdots \\ \hat{z}(x_n) - m \end{bmatrix}^T \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$	Kriging weight, $\begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \mu \end{bmatrix} = \begin{bmatrix} C(\underline{x}_1 - \underline{x}_1) & \cdots & C(\underline{x}_1 - \underline{x}_n) & -1 \\ \vdots & \ddots & \vdots & \vdots \\ C(\underline{x}_n - \underline{x}_1) & \cdots & C(\underline{x}_n - \underline{x}_n) & -1 \\ 1 & \cdots & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$ Kriging estimation, $m = \begin{bmatrix} \hat{z}(x_1) \\ \vdots \\ \hat{z}(x_n) \\ 0 \end{bmatrix}^T \begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \mu \end{bmatrix}$	Kriging weight, $\begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \mu \end{bmatrix} = \begin{bmatrix} C(\underline{x}_1 - \underline{x}_1) & \cdots & C(\underline{x}_1 - \underline{x}_n) & -1 \\ \vdots & \ddots & \vdots & \vdots \\ C(\underline{x}_n - \underline{x}_1) & \cdots & C(\underline{x}_n - \underline{x}_n) & -1 \\ 1 & \cdots & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} C(\underline{x}_1 - \underline{x}_0) \\ \vdots \\ C(\underline{x}_n - \underline{x}_0) \\ 1 \end{bmatrix}$ Kriging estimation, $\hat{z}(x_0) = \begin{bmatrix} \hat{z}(x_1) \\ \vdots \\ \hat{z}(x_n) \\ 0 \end{bmatrix}^T \begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \mu \end{bmatrix}$
Kriging error		$\sigma_{sm}^2 = C(\underline{x}_0 - \underline{x}_0) - \begin{bmatrix} C(\underline{x}_1 - \underline{x}_0) \\ \vdots \\ C(\underline{x}_n - \underline{x}_0) \end{bmatrix}^T \begin{bmatrix} w_1 \\ \vdots \\ w_n \end{bmatrix}$	$\sigma_m^2 = \mu$	$\sigma_{or}^2 = C(\underline{x}_0 - \underline{x}_0) - \begin{bmatrix} C(\underline{x}_1 - \underline{x}_0) \\ \vdots \\ C(\underline{x}_n - \underline{x}_0) \\ -1 \end{bmatrix}^T \begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \mu \end{bmatrix}$

* $C(h)$ denotes covariance function of spatial vector h

APPENDIX D

Derivation of spectral density function

This section shows the derivation of the spectral density functions for one- and two-dimensional random process as presented in Section 4.1.2. This derivation is, in fact, based on the formation presented by Shinozuka (1971). The theoretical detail of this frequency-domain technique can be found in Shinozuka (1971), and Shinozuka and Jan (1972).

Spectral density function for one-dimensional random process

Consider a one-dimensional Gaussian random process with mean zero and the two-sided mean-square spectral density function $S(\omega)$. By Wiener-Khintchine relationship, the auto-correlation function $C(\tau)$ can be described as a Fourier transform of $S(\omega)$ as follows:

$$C(\tau) = \int_{-\infty}^{\infty} S(\omega) e^{i\omega\tau} d\omega \quad (\text{D-1})$$

and, by the inverse form of Fourier transform, we have

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} C(\tau) e^{-i\omega\tau} d\tau \quad (\text{D-2})$$

where ω represents frequency domain and τ denotes spatial or temporal lag.

By assuming the exponential type auto-correlation function (See Figure D-1), we have

$$C(\tau) = e^{-|\tau|/\eta} \quad (\text{D-3})$$

where η denotes auto-correlation distance. Substituting Eq. (D-3) to (D-2) gives

$$S(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-|\tau|/\eta} e^{-i\omega\tau} d\tau \quad (D-4)$$

Due to the fact that $e^{-|\tau|/\eta}$ is an even function, Eq. (D-4) becomes

$$\begin{aligned} S(\omega) &= \frac{1}{\pi} \int_0^{\infty} e^{-\tau/\eta} \cos(\omega\tau) d\tau \\ &= \frac{1}{\pi} \frac{1/\eta}{\frac{1}{\eta^2} + \omega^2} \end{aligned} \quad (D-5)$$

By rearranging the above equation, we have the spectral density function for one-dimensional random process as (See Figure D-2)

$$S(\omega) = \frac{1}{\pi\eta \left(\frac{1}{\eta^2} + \omega^2 \right)} \quad (D-6)$$

Assuming that the power of the employed SDF is negligible outside the interval $[-\omega_0, \omega_0]$, the simulated stationary Gaussian random field at any coordinate, x , can be expressed as the following series of cosine functions

$$X(x) = \sum_{j=1}^M \sqrt{2S(\omega_j)\Delta\omega} \cdot \cos(\omega_j x + \phi_j) \quad (D-7)$$

where $\Delta\omega = 2\omega_0/M$, $\omega_j = -\omega_0 + (j-1/2)\Delta\omega$, and ϕ_j = random phase angles which uniformly and independently distribute in the interval $(0, 2\pi)$. M is the number of equally divided intervals of the range $[-\omega_0, \omega_0]$.

Spectral density function for two-dimensional random process

Consider a two-dimensional Gaussian random process with mean zero and the mean-square spectral density function $S(\omega_1, \omega_2)$. Similar to Eq. (D-1) and (D-2), the spectral density function can be described by the inverse form of Fourier transform of auto-correlation function $C(\tau, \xi)$ as follows:

$$S(\omega_1, \omega_2) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C(\lambda, \xi) e^{-i(\omega_1 \lambda + \omega_2 \xi)} d\lambda d\xi \quad (D-8)$$

where ω_1 and ω_2 represent frequency domain and λ and ξ denotes spatial or temporal lag.

By introducing the polar coordinate r and θ ,

$$\lambda = r \cos \theta, \quad \xi = r \sin \theta \quad (D-9)$$

Eq. (D-8) becomes

$$\begin{aligned} S(\omega_1, \omega_2) &= \frac{1}{(2\pi)^2} \int_0^{\infty} \int_0^{2\pi} C^*(r) e^{-ir(\omega_1 \cos \theta + \omega_2 \sin \theta)} r dr d\theta \\ &= \frac{1}{2\pi} \int_0^{\infty} J_0(r\omega) C^*(r) r dr \end{aligned} \quad (D-10)$$

where

$$\omega = (\omega_1^2 + \omega_2^2)^{1/2} \quad (D-11)$$

and $J_0(r\omega)$ denotes the integral representation of Bessel function of order zero as follows:

$$J_0(r\omega) = \frac{1}{2\pi} \int_0^{2\pi} e^{-ir\omega \cos \theta} d\theta \quad (D-12)$$

By assuming the exponential type auto-correlation function, we have (See Figure D-3)

$$C^*(r) = e^{-|r|/\eta} \quad (D-13)$$

where η denotes auto-correlation distance. Substituting above equation to Eq. (D-10) gives

$$\begin{aligned}
S(\omega_1, \omega_2) &= \frac{1}{2\pi} \int_0^\infty J_0(r\omega) e^{-|r|/\eta} r dr \\
&= \frac{1}{2\pi} \frac{2\omega^0 (1/\eta) \Gamma(3/2)}{\sqrt{\pi} ((1/\eta)^2 + \omega^2)^{3/2}} \\
&= \frac{1}{2\pi} \frac{2(1/\eta) (\sqrt{\pi}/2)}{\sqrt{\pi} ((1/\eta)^2 + \omega^2)^{3/2}} \\
&= \frac{1}{2\pi} \frac{(1/\eta)}{((1/\eta)^2 + \omega^2)^{3/2}} \tag{D-14}
\end{aligned}$$

By rearranging the above equation, we have the spectral density function for two-dimensional random process as (See Figure D-4)

$$S(\omega_1, \omega_2) = \frac{1}{2\pi\eta \left(\frac{1}{\eta^2} + (\omega_1^2 + \omega_2^2) \right)^{3/2}} \tag{D-15}$$

Assuming that the power of the employed SDF is negligible outside the interval $[-\omega_{1,0}, \omega_{1,0}]$ and $[-\omega_{2,0}, \omega_{2,0}]$, the simulated stationary Gaussian random field at any coordinate (x, y) can be expressed as the following series of cosine functions

$$X(x, y) = \sum_{k=1}^{M_2} \sum_{j=1}^{M_1} \sqrt{2S(\omega_{1j}, \omega_{2k}) \Delta\omega_1 \Delta\omega_2} \cdot \cos(\omega_{1j}x + \omega_{2k}y + \phi_{jk}) \tag{D-16}$$

where $\Delta\omega_1 = 2\omega_{1,0}/M_1$, $\Delta\omega_2 = 2\omega_{2,0}/M_2$, $\omega_{1j} = -\omega_{1,0} + (j-1/2)\Delta\omega_1$, $\omega_{2k} = -\omega_{2,0} + (k-1/2)\Delta\omega_2$, and ϕ_{jk} = random phase angles which uniformly and independently distribute in the interval $(0, 2\pi)$. M_1 and M_2 are the number of equally divided intervals of the range $[-\omega_{1,0}, \omega_{1,0}]$ and $[-\omega_{2,0}, \omega_{2,0}]$, respectively.

Spectral density function for three-dimensional random process

Consider a three-dimensional Gaussian random process with mean zero and the mean-square spectral density function $S(\omega)$. It is assumed, in this case, that the random field is heterogeneous but isotropic. By Wiener-Khintchine relationship, the auto-correlation function $C(r)$ can be described as a Fourier transform of $S(\omega)$ as follows:

$$C(r) = 4\pi \int_0^{\infty} \frac{\sin(\omega r)}{r} S(\omega) \omega d\omega \quad (\text{D-17})$$

and, by the inverse form of Fourier transform, we have

$$S(\omega) = \frac{1}{(2\pi)^2} \int_0^{\infty} \frac{\sin(\omega r)}{\omega} C(r) r dr \quad (\text{D-18})$$

where $S(\omega)$ is a nonnegative bounded function; ω represents frequency domain; and r denotes spatial or temporal lag.

By assuming the exponential typed auto-correlation function, we have

$$C(r) = e^{-|r|/\eta} \quad (\text{D-19})$$

where η denotes auto-correlation distance. Substituting above equation to Eq. (D-18) gives

$$S(\omega) = \frac{1}{(2\pi)^2} \int_0^{\infty} \frac{\sin(\omega r)}{\omega} e^{-|r|/\eta} r dr \quad (\text{D-20})$$

Based on Laplace – Euler transform:

$$\int_0^{\infty} e^{-cx} x^{\alpha-1} (\sin bx) dx = \frac{\Gamma(\alpha)}{(c^2 + b^2)^{\alpha/2}} \sin(\alpha \arctan \frac{b}{c}) \quad (\text{D-21})$$

where α , b , and c are real number, and Γ is Gamma function and it is defined as:

$$\Gamma(\alpha) = (\alpha - 1)! \quad (\text{D-22})$$

In case $\alpha = 2$, Eq. (D-21) becomes

$$\int_0^{\infty} e^{-cx} x(\sin bx) dx = \frac{1}{c^2 + b^2} \sin(2 \arctan \frac{b}{c}) \quad (D-23)$$

Let $c = 1/\eta$, $b = \omega$, and $x = r$, we have from Eq. (D-20) and Eq. (D-23)

$$S(\omega) = \frac{1}{(2\pi)^2 \omega} \cdot \frac{1}{\frac{1}{\eta^2} + \omega^2} \sin(2 \arctan(\eta\omega)) \quad (D-24)$$

where ω represent three-dimensional frequency domain consisting of ω_1 , ω_2 , and ω_3 as follows:

$$\omega = (\omega_1^2 + \omega_2^2 + \omega_3^2)^{1/2} \quad (D-25)$$

Assuming that the power of the employed SDF is negligible outside the interval $[-\omega_{1,0}, \omega_{1,0}]$, $[-\omega_{2,0}, \omega_{2,0}]$, and $[-\omega_{3,0}, \omega_{3,0}]$, the simulated stationary Gaussian random field at any coordinate (x, y, z) can be expressed as the following series of cosine functions

$$X(x, y, z) = \sum_{l=1}^{M_3} \sum_{k=1}^{M_2} \sum_{j=1}^{M_1} \sqrt{2S(\omega_{1j}, \omega_{2k}, \omega_{3l}) \Delta\omega_1 \Delta\omega_2 \Delta\omega_3} \cdot \cos(\omega_{1j}x + \omega_{2k}y + \omega_{3l}z + \phi_{jkl}) \quad (D-26)$$

where $\Delta\omega_1 = 2\omega_{1,0}/M_1$, $\Delta\omega_2 = 2\omega_{2,0}/M_2$, $\Delta\omega_3 = 2\omega_{3,0}/M_3$, $\omega_{1j} = -\omega_{1,0} + (j-1/2)\Delta\omega_1$, $\omega_{2k} = -\omega_{2,0} + (k-1/2)\Delta\omega_2$, $\omega_{3l} = -\omega_{3,0} + (l-1/2)\Delta\omega_3$, and ϕ_{jkl} = random phase angles which uniformly and independently distribute in the interval $(0, 2\pi)$. M_1 , M_2 , and M_3 are the number of equally divided intervals of the range $[-\omega_{1,0}, \omega_{1,0}]$, $[-\omega_{2,0}, \omega_{2,0}]$, and $[-\omega_{3,0}, \omega_{3,0}]$, respectively.

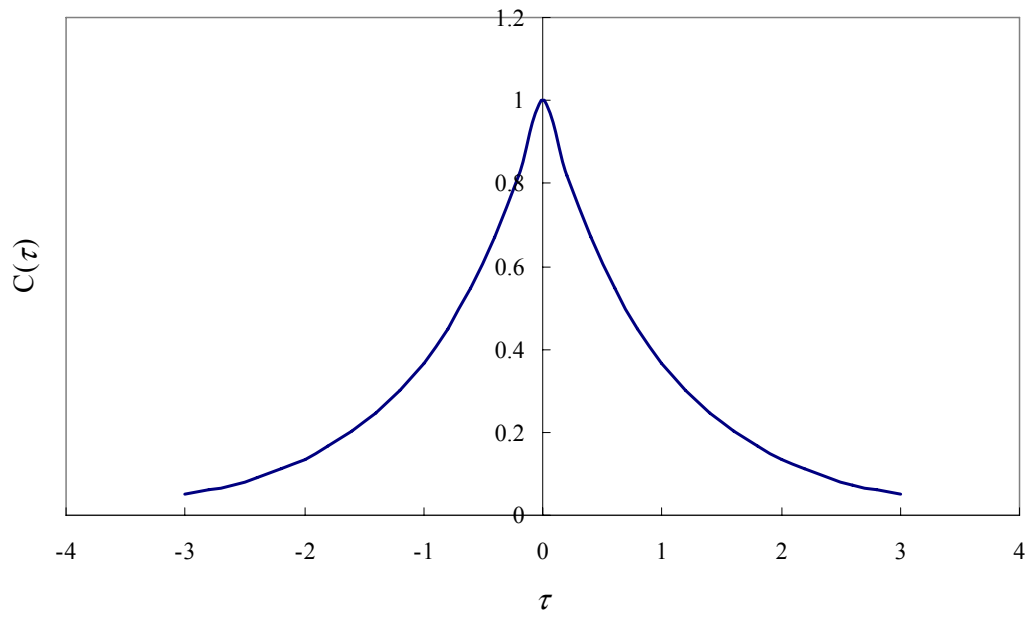


Figure D-1: Auto-correlation function for one-dimensional random process.

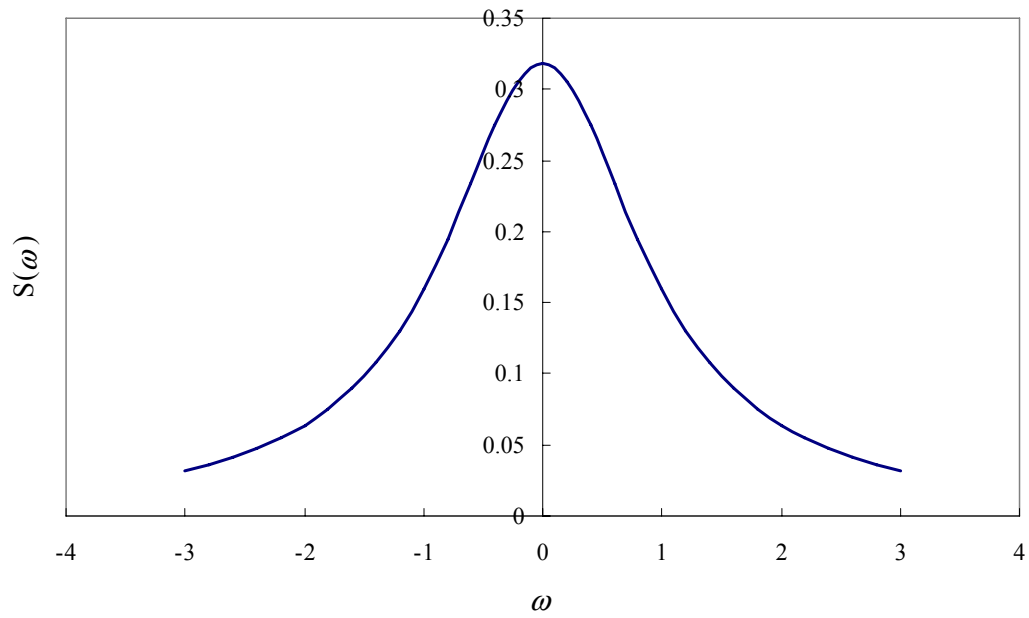


Figure D-2: Spectral density function for one-dimensional random process.

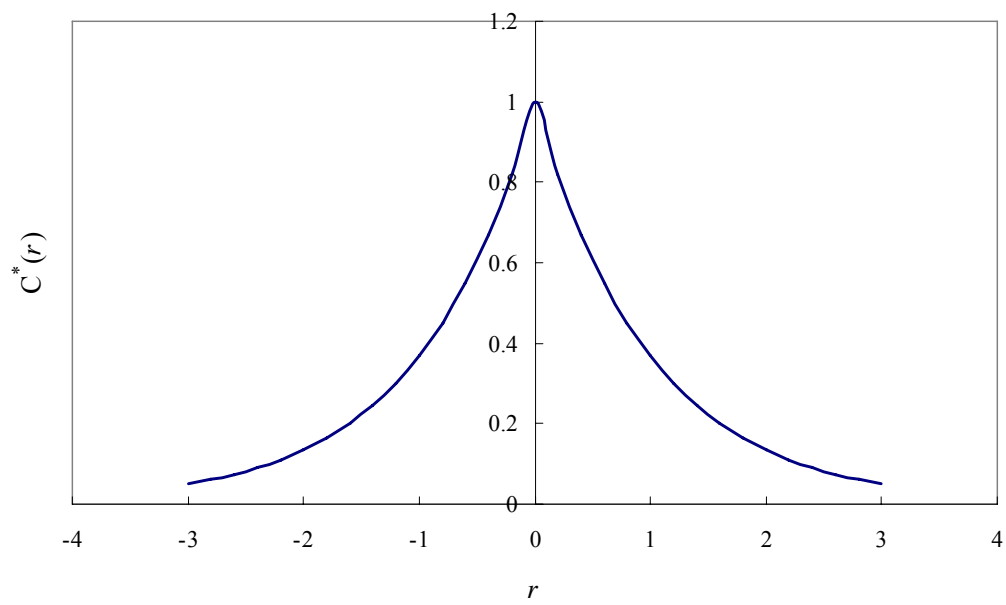
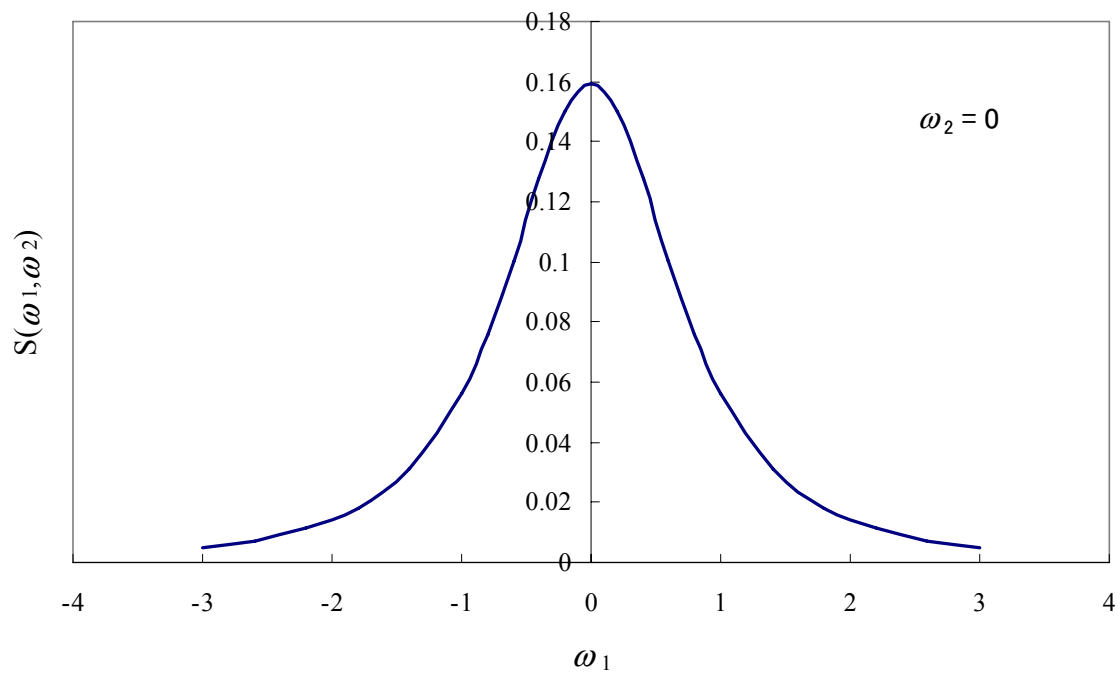
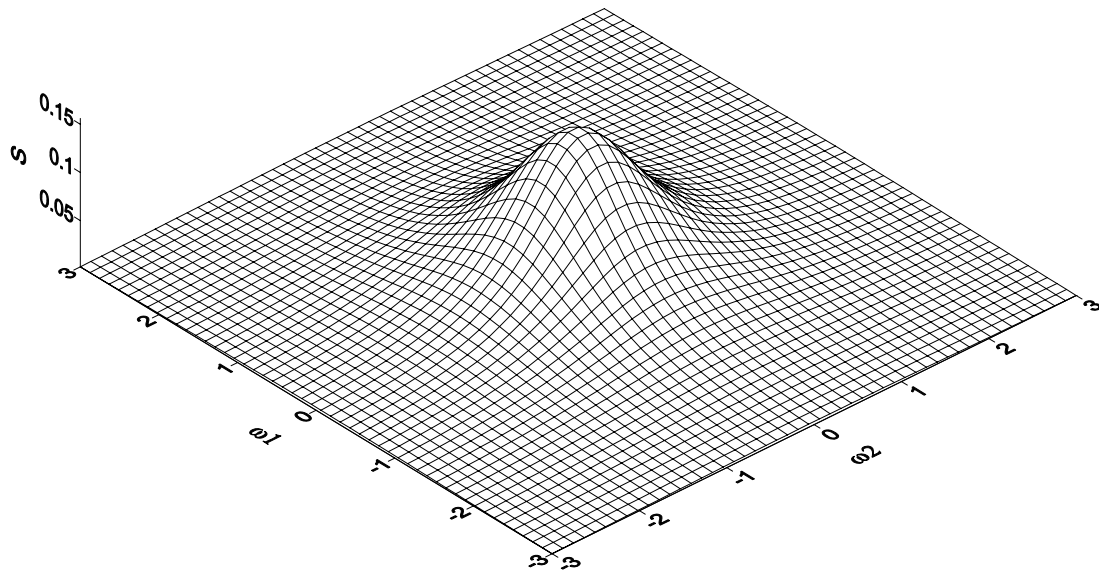


Figure D-3: Auto-correlation function for two-dimensional random process.



(a) a section in 1D space, for $\omega_2 = 0$



(b) a surface in 2D space

Figure D-4: Spectral density function for two-dimensional random process.

APPENDIX E

Field observation data of secondary compression in peat

The section presents the raw data of the observed settlement from a residential land development project in suburb area of Tokyo (Ueda *et al.* 1986) used for the case study in Section 4.2.3. The original data contains 55 observation points, but only 42 points are considered acceptable in quality and selected to use for the analysis. Note that the data before the day 103rd are also excluded from the calculation due to the rebound effect as described in Section 4.2.3.1.

Observation point no. (original data)	Observation point no. (after selection)	Coordinates (m)		Observation time (day)																												= the data removed from the analysis due to its irregularity									
		x	y	0	9	22	38	49	60	71	87	103	117	130	145	165	188	202	214	247	278	325	351	390	430	520	599	696	776	820	1017										
1	1	297.0	143.0	0	4	5	3	0	3	2	1	9	14	12	26	29	31		29	30	32	40	41	62	78	98	105	109	108	124	134										
2		252.0	166.0	0	3	3	2	4	0	4		3	10	7	20	16	15		18	14	15	11	7	19	23	30	30	32		37	42										
3		222.0	192.0	0	-2	3	-1	7	9	9	10	14	8	5	16	19	20		18	17	18	11	10	17	20	26	28	26		31	37										
4	2	212.0	233.0	0	8	18	10	19	23	21	23	29	29	30	28	37	36	90	37	35	35	39	42	46	49	51	51	39	215	47	68										
5	3	215.0	272.0	0	2	7	0	17	18	18	24	27	28	42	43	50	54		50	63	63	63	65	69	85	81	81	72		83	101										
6	4	316.0	121.0	0	6	14	0	24	28	36	48	49	61	62	79	88	96		92	111	118	123	124	145	157		192	201		221	253										
7	5	268.0	143.0	0	-3	4	8	4	10	10		13	23	21	31	26	26		28	35	38	51	52	62	65		89	95		107	131										
8	6	223.0	165.0	0	2	6	6	9	13	12		18	26	21	27	33	35		41	49	53	64	65	77	85	104	116	125		141	167										
9	7	200.0	208.0	0	10	10	3	10	13	13	14	17	19	12	19	29	36		40	40	44	41	40	46	51	65	65	65		74	191										
10	8	201.0	260.0	0	8	12	3	13	12	13	16	21	20	28	29	36	57		59	62	64	66	67	67	81	82	82	75		90	108										
11		285.0	110.0	0	3	4	6	5	12	11	12	12	-12	-16	1	1	6		2	-4	-1	7	8	17	19	25	31		43												
12		239.0	136.0	0	4	2	1	0	1	2			9	5	19	17	17		17	12	11	16	14	21	20	23	28		36												
13	9	196.0	166.0	0	-1	5	5	8	11	12	16	21	31	26	47	53	51	79	55	64	71	84	88	101	110	125	140	150		168	200										
14	10	182.0	197.0	0	11	12	7	11	14	14	17	20	22	26	25	29	39		36	41	46	45	47	51	58	79	79		101	119											
15	11	180.0	234.0	0	11	-6	-16	5	5	7	4	0	1	2	3	3	8		11	12	20	25	27	40	50	56	56														
16	12	183.0	274.0	0	13	19	18	30	34	36	43	56	53	60	67	81	89		105	112	114																				
17	13	288.0	89.0	0	4	6	11	14	20	24	36	39	49	57	56	66	74		84	92	98	128	132	195	156		160		197	231											
18	14	242.0	113.0	0	6	6	6	7	8	13	16	17	21	23	26	30	35		37	45	50	58	58	70	75		82	96	106	119	140										
19	15	202.0	184.0	0	4	3	5	9	8	10	14	16	18	21	25	31	37		40	47	52	64	65	76	82	93	108	119	131	155											
20	16	183.0	154.0	0	-8	2	0	5	7	11	15	20	28	29	45	52	42		52	62	69	82	87	95	104	125	139	151	166	194											
21	17	165.0	190.0	0	11	15	11	18	25	24	32	39	41	46	45	52	59		52	63	68	62	58	63	71	84	93	92	101	116											
22	18	161.0	227.0	0	9	11	8	15	20	19	23	26	29	32	36	39	44		51	56	61	63	75	85	93	94	112	121	139	161											
23	19	162.0	276.0	0	17	36	52	59	70	77	92	135	142	153	165	183	192																								
24	20	259.0	91.0	0	5	2	12	16	0	25	28	36	42	48	54	63	71		79	90	100	119	122	140	149	174	187		124	152											
25	21	216.0	114.0	0	2	3	5	3	8	10	15	15	16	19	27	29	33	39	47	50	63	67	72	80	94	101	116	112	139												
26	22	184.0	123.0	0	3	2	5	9	11	19	16	18	17	20	25	29	33	37	44	48	60	62	69	75	84	93	106														
27		162.0	163.0	0	3	6	3	3	2	2		6	6	9	8	11	15		21	17	18	13	16	12	17	21	24	19	26	31											
28		148.0	200.0	0	2	6	4	3	3	3	14		18	15	18	18	27	30	25	23	27	20	22	28	38	38	31	40	49												
29	23	149.0	252.0	0	8	13	6	15	21	18	19	33	30	37	40	43	53		57	65	71	80	88	94	107	110	131	142		165	193										
30	24	263.0	64.0	0	3	7	14	20	26	35	41	50	59	63	70	79	87	96	105	111	128	132	146	155	168	191		221	223												
31	25	224.0	82.0	0	3	2	3	6	9	10	10	14	16	16	19	25	27	33	40	43	52	55	58	64	76	80	88	93	116												
32	26	180.0	106.0	0	4	2	4	10	14	14	21	20	29	30	33	37	42	47	59	67	78	84	9	99	113	125	140	150	182												
33	27	151.0	131.0	0	3	1	6	4	7	12	8	8	9	9	17	20	24	26	29	34	42	42	48	49	50	57		66	82												
34	28	136.0	163.0	0	0	3	11	11	14	19	20	17	21	20	29	35	41	47	53	58	71	74	84	93	105	118	127	143	161												
35	29	126.0	195.0	0	-1	9	8	9	12	16	13	14	19	17	22	26	30	36	39	44	50	53	60	59	66	74	78	91	103												
36	30	134.0	228.0	0	7	11	7	19	22	20	25	31	34	35	37	47	49	56	61	67	68	74	92	94		119															
37	31	137.0	276.0	0		5	-3	4	23	24	21	15	12	18	26	28	29	37	44	45	55	62	69	76	90	100	104	119													
38	32	232.0	65.0	0	3	2	2	3	3	7	5	7	9	8	13	14	19	27	30	33	42	43	48	51	66	66															
39	33	180.0	88.0	0	2	2	4	4	8	7	13	14	20	23	28	32	39	47	56	63	79	83	92	98	115	128															
40	34	155.0	108.0	0	4	-2	0	2	6	5	11	10	17	16	19	23	27	28	31	43	51	50	56	57	62	66		73	61												
41		135.0	133.0	0	1	0	3	1	2	8	-2	-1	-5	-5	-5	1	4		3	8	5	10	11	11	10	12	13	16		13	22										
42	35	118.0	169.0	0	-1	1	6	5	4	4	5		5	5	6	6	13		16	21	22	33	43	38	40	42	50														
43	36	112.0	194.0	0	0	6	11	10	12	14	15	18	19	21	28	35	36	40	47	45	63	65	75	79	83		92	109													
44	37	249.0	46.0	0	3	3	3	30	32	38	36	39	44	46	48	55	63	64	64	62	66	66	68	71	74	75															
45		200.0	67.0	0	1	1	3	1	4	4	2	2	4	3	-6	-6	1	4	4	0	2	-1	4	2	6	6	4														

APPENDIX F

The FORTRAN program's source code

This section presents the source codes of the computer programs used for implementing the application examples described in Section 4. The programs are written by FORTRAN 90 and consist of two main programs. These two programs are basically similar, but the 1st one is built especially for primary consolidation settlement prediction by Asaoka's method (Section 4.1) while the 2nd one is aimed for secondary compression prediction by $S \sim \log(t)$ method (Section 4.2). The source codes of them are attached here as a reference of the analysis performed in this thesis.

1st PROGRAM: *the program for primary consolidation settlement prediction by Asaoka's method*

Input

Input file name: 'IN.TXT'

Input data:

```
N,KT
NDIM,SSRAT,N1,NRAN,GEN_SETT,EST_ARBIT
BETA1_SM,SDBETA1_SM,BETA0_SM,SDBETA0_SM
BETA1_0,SDBETA1_0,BETA0_0,SDBETA0_0,QP,SDE,SDE_ASSUM,ZA,ZA_ASSUM
EST_ZA

[ IF (EST_ZA=1) THEN]
    SDE_L,SDE_H,NDIV_SDE
    ZA_L,ZA_H,NDIV_ZA
[END IF]

EST_ZETA0
KRIG,NKRIG

[ IF (KRIG=1) THEN]
    COR_KRIG(1,:)
    COR_KRIG (2,:)
```

```

      .
      .
      COR_KRIG(NKRIG,:)
[END IF]

KALMAN_INC
KALMAN_IGN

CORX(1,:)
CORX(2,:)
      .
      .
CORX(N,:)

[IF (GEN_SETT=1) THEN]
  Y(1,1) Y(1,2) ... Y(1,N)
[ELSE IF (GEN_SETT=0) THEN]
  T(1) T(2) ... T(KT) T_EST

  Y(1,1) Y(2,1) ... Y(KT,1) YT_EST(1)
  Y(1,2) Y(2,2) ... Y(KT,2) YT_EST(2)
      .
      .
  Y(1,N) Y(2,N) ... Y(KT,N) YT_EST(N)
[END IF]

```

(Note: for the meaning of each variable, please refer to ‘VARIABLE DESCRIPTION’ section at the beginning of the source code

Output

1. Output file name: ‘OUT.TXT’

Output data: main output data, including generated data, estimated model parameters and settlement, and estimation error.

2. Output file name: ‘OUT_MISC.TXT’

Output data: miscellaneous output data, including estimated auto-correlation distance & standard deviation of observation error, estimated prior mean & variance of unknown parameters, estimated values at arbitrary points, and estimation errors.

Source code

PROGRAM STSETT1_5_2

```
! *****
! SPATIAL-TEMPORAL PREDICTION OF CONSOLIDATION SETTLEMENT BASED ON ASAOKA'S METHOD
!
! CREATED BY P. RUNGBANAPHAN - FEB 14, 2010
! *****
USE MATHLIB ; IMPLICIT NONE

! ***** VARIABLE DECLARATION *****
INTEGER I,J,K,N,KT,LMAT,NRANK,IER,NRAN,IRAN,IRD,NDIM,N1,IR,EST_ZA,NDIV_ZA,IDIV,NDIV_SDE,JDIV, &
KRIG,IKRIG,NKRIG,KALMAN_INC,KALMAN_IGN,IC_BEGIN,IRT,IP,IC,TIMEI,GEN_SETT,NT_EST,NDAT, &
EST_ZETA0,EST_ARBIT,N_ARBIT,NA,N_CURR,N_FUT,NT

INTEGER, ALLOCATABLE :: LW_EI(:)
REAL(8) BETA1_0,BETA0_0,SDBETA1_0,SDBETA0_0,SDE,ZA,DX,AINV(3,3),EPS,RHO,MEAN,SD,OM1,SSRAT, &
ZA_L,ZA_H,DET_VE_ZA,SDE_L,SDE_H,ZA_ASSUM,SDE_ASSUM,SETT_FN,MINABIC,TIMER, &
YF_EST,ERB1_PT,ERBO_PT,SDERB1_PT,SDERBO_PT,ERIB1_PT,ERIBO_PT,SDERIB1_PT,SDERIBO_PT, &
ERYF_PT,SDERYF_PT,ERIYF_PT,SDERIYF_PT,MERB1_PT,MERBO_PT,MERIB1_PT,MERIBO_PT,BERB1_PT, &
BERBO_PT,BERIB1_PT,BERIBO_PT,BERYF_PT,BERIYF_PT,MERYF_PT,MERIYF_PT,T_EST,QP,IM,IM_IG, &
SDE_MISS,MEAN_X,MEAN_Y,XXYY,XX2,YY2,S2_YX,YT_EST_ARBIT,SETT_CURR,SETT_FUT,ERR_CURR, &
ERR_FUT,ERR_CURR_BIAS,ERR_FUT_BIAS,ERR_CURR_ABS,ERR_FUT_ABS,BETA1_SM,BETA0_SM, &
SDBETA1_SM,SDBETA0_SM,BETA1_ARBIT,BETA0_ARBIT,ERR_M1,ERR_MO,ERR_BIAS_M1,ERR_ABS_M1, &
ERR_BIAS_MO,ERR_ABS_MO,N_M1MO,YT_CURR_ARBIT,YF_ARBIT,ELN_B1_T,ELN_BO_T,VLN_B1_T, &
VLN_BO_T,ELN_YF_T,VLN_YF_T,ELNI_B1_T,ELNI_BO_T,VLNI_B1_T,VLNI_BO_T,ELNI_YF_T, &
VLNI_YF_T
REAL(16) DET_VZETA_ZA,DET_MVM
REAL(8), ALLOCATABLE :: ZETA(:),VZETA(:,:),MK(:,:),YK(:),QK(:,:),VE(:,:),SK(:,:),SKINV(:,:), &
YK_IG(:),MK_IG(:,:),ZETA_IG(:),KK_IG(:,:),SK_IG(:,:),VZETA_IG(:,:), &
VE_IG(:,:),SKINV_IG(:,:),QK_IG(:,:),KK(:,:),CORX(:,:),X1(:),X0(:), &
BETA1(:),BETA0(:),Y(:,:),XY(:),ZETA_I(:,:),VZETA_I(:,:),ZA_TRIAL(:), &
ABIC(:,:),VZETA_ZA(:,:),VZETA_ZA_INV(:,:),VE_ZA(:,:),VE_ZA_INV(:,:), &
MVM(:,:),MVM_INV(:,:),MVY(:,:),MK_ZA(:,:),YK_ZA(:,:),ZETA_ZA(:,:), &
ZETA_MLM(:,:),UNIT(:,:),W_EI(:,:),E_EI(:),V_EI(:,:),A_EI(:,:), &
D_N(:),V_N(:,:),E_N(:),D_2N(:),V_2N(:),E_2N(:),D_2(:), &
V_2(:,:),E_2(:),D_1(:),V_1(:,:),E_1(:),SDE_TRIAL(:),LH1(:,:), &
COR_KRIG(:,:),V_KRIG(:,:),V_KRIG_INV(:,:),VXO_KRIG(:,:),WGHT(:,:), &
Z_KRIG(:,:),ZXO_KRIG(:,:),D_N1(:),V_N1(:,:),E_N1(:),ERB1_P(:), &
ERBO_P(:),ERIB1_P(:),ERIBO_P(:),SDERB1_P(:),SDERBO_P(:), &
SDERIB1_P(:),SDERIBO_P(:),YF(:),ERYF_P(:),ERIYF_P(:),SDERYF_P(:), &
SDERIYF_P(:),BERB1_P(:),BERBO_P(:),BERIB1_P(:),BERIBO_P(:), &
BERYF_P(:),BERIYF_P(:),T(:),YT_EST(:),IMI(:),IMI_IG(:), &
CORX_ARBIT(:),CORX_T(:,:),Y_T(:,:),YT_EST_T(:),Y_ARBIT(:), &
CORX_NT(:,:),BETA1_KR(:),BETA0_KR(:),YF_KR(:),ERB1_KR_P(:), &
ERBO_KR_P(:),BERB1_KR_P(:),BERBO_KR_P(:),ERYF_KR_P(:),BERYF_KR_P(:), &
ELN_B1(:),ELN_BO(:),VLN_B1(:),VLN_BO(:),ELN_YF(:),VLN_YF(:), &
ELNI_B1(:),ELNI_BO(:),VLNI_B1(:),VLNI_BO(:),ELNI_YF(:),VLNI_YF(:), &
ELN_B1_KR(:),ELN_BO_KR(:),VLN_B1_KR(:),VLN_BO_KR(:),ELN_YF_KR(:), &
VLN_YF_KR(:)

!===== VARIABLE DESCRIPTION =====
! N : TOTAL NO. OF OBSERVATION POINTS
! KT : TOTAL TIME STEP OF OBSERVED SETTLEMENT
! NDIM : NO. OF DIMENSIONS OF THE PROBLEM (1 OR 2 FOR 1D OR 2D)
```

```

!   SSRAT      : RATIO OF SPECTRAL DENSITY VALUES AT MAXIMUM POINT TO THAT AT THE POINT CONSIDERED
!               NEGLIGIBLE
!   N1         : NUMBER OF DEVIDED INTERVALS IN FREQUENCY DOMAIN (BOTH X & Y AXIS, FOR 2D CASE)
!   N1         : NO. OF RANDOM SAMPLINGS OF MODEL PARAMETERS
!   GEN_SETT   : = 1 = PERFORM SETTLEMENT GENERATION BASED ON THE ASSIGNED RANDOM FIELD PARAMETERS
!               USING PREDICATION MODEL
!               0 = READ THE OBSERVED SETTLEMENT DATA FROM THE INPUT FILE
!   EST_ARBIT  : = 1 OR 0 = DO OR DON'T PERFORM ESTIMATION OF SETTLEMENT AT ARBITRARY POINTS
!   BETA1_SM   : EXPECTED VALUE OF MODEL PARAMETER, BETA1, USED FOR DATA SIMULATION
!   SDBETA1_SM : STANDARD DEVIATION OF MODEL PARAMETER, BETA1, USED FOR DATA SIMULATION
!   BETA0_SM   : EXPECTED VALUE OF MODEL PARAMETER, BETA0, USED FOR DATA SIMULATION
!   SDBETA0_SM : STANDARD DEVIATION OF MODEL PARAMETER, BETA0, USED FOR DATA SIMULATION
!   BETA1_0    : INITIAL EXPECTED VALUE OF MODEL PARAMETER, BETA1
!   BETA0_0    : INITIAL EXPECTED VALUE OF MODEL PARAMETER, BETA0
!   SDBETA1_0  : INITIAL STANDARD DEVIATION OF MODEL PARAMETER, BETA1
!   SDBETA0_0  : INITIAL STANDARD DEVIATION OF MODEL PARAMETER, BETA0
!   QP         : CONSTANT PARAMETER, REPRESENTING PROCESS NOISE (REFERED AS 'a' IN THESIS BOOK)
!   SDE        : ASSUMED STANDARD DEVIATION OF OBSERVATION MODEL
!   SDE_ASSUM  : STANDARD DEVIATION OF OBSERVATION MODEL, USED FOR DATA SIMULATION
!   ZA         : ASSUMED AUTO-CORRELATION DISTANCE
!   ZA_ASSUM   : AUTO-CORRELATION DISTANCE, USED FOR DATA SIMULATION
!   EST_ZA     : = 1 OR 0 = DO OR DON'T PERFORM ESTIMATION OF AUTO-CORRELATION DISTANCE AND SD OF
!               OBSERVATION ERROR
!   SDE_L      : LOWER BOUNDARY FOR TRIAL AND ERROR VALUES OF SD OF OBSERVATION ERROR
!   SDE_H      : HIGHER BOUNDARY FOR TRIAL AND ERROR VALUES OF SD OF OBSERVATION ERROR
!   NDIV_SDE   : NUMBER OF DIVISIONS FOR TRIAL AND ERROR RANGE OF SD OF OBSERVATION ERROR
!   ZA_L       : LOWER BOUNDARY FOR TRIAL AND ERROR VALUES OF AUTO-CORRELATION DISTANCE
!   ZA_H       : HIGHER BOUNDARY FOR TRIAL AND ERROR VALUES OF AUTO-CORRELATION DISTANCE
!   NDIV_ZA    : NUMBER OF DIVISIONS FOR TRIAL AND ERROR RANGE OF AUTO-CORRELATION DISTANCE
!   EST_ZETA0  : = 1 OR 0 = DO OR DON'T PERFORM ESTIMATION OF PRIOR MEAN OF UNKNOWN PARAMETERS
!   KRIG       : = 1 OR 0 = DO OR DON'T KRIG FOR MODEL PARAMETERS AT AN ARBITRARY POINT
!   NKRIG      : NUMBER OF ARBITRARY POINTS TO BE ESTIMATED BY KRIGING
!   COR_KRIG   : COORDINATES OF ARBITRARY POINTS TO BE ESTIMATED BY KRIGING
!   KALMAN_INC : = 1 OR 0 = DO OR DON'T PERFORM KALMAN FILTER UPDATE, INCLUDING SPATIAL CORRELATION
!   KALMAN_IGN : = 1 OR 0 = DO OR DON'T PERFORM KALMAN FILTER UPDATE, IGNORING SPATIAL CORRELATION
!   CORX(I,:)  : COORDINATES (X,Y) OF OBSERVATION POINTS AT POINT 'I'
!   Y(K,I)     : SETTLEMENT AT TIME STEP 'K' AND AT OBSERVATION POINT 'I'
!   T(K)       : TIME AT TIME STEP K
!   T_EST      : TIME AT WHICH THE SETTLEMENT IS TO BE ESTIMATED
!   YT_EST(I)  : SETTLEMENT AT TIME 'T_EST' AND AT OBSERVATION POINT 'I'
!   NT         : TOTAL NO. OF OBSERVATION POINTS INCLUDING ARBITRARY POINTS
!   ZETA       : STATE VECTOR [BETA1(X1) ... BETA1(XN):BETA0(X0) ... BETA0(XN)]
!   VZETA      : COVARIANCE MATRIX OF ZETA
!   QK         : COVARIANCE MATRIX OF PROCESS NOISE
!   MK         : OBSERVATION-PARAMETER MODEL[DIAG[Z_K-1(X1) ... Z_K-1(XN)]:I]
!   YK         : OBSERVED SETTLEMENTS FOR CURRENT TIMESTEP [Z_K(X1) ... Z_K(XN)]T
!   VE         : COVARIANCE MATRIX OF OBSERVATION MODEL
!   KK         : KALMAN GAIN
!   OM1        : CONSIDERED REGION IN FREQUENCY DOMAIN (BOTH X & Y AXIS, FOR 2D CASE)
!   DET_VE_ZA  : DETERMINANT OF COVARIANCE MATRIX OF THE OBSERVATION MODEL ERROR
!   DET_VZETA_ZA : DETERMINANT OF COVARIANCE MATRIX OF THE STATE VECTOR
!=====
! ***** INITIALIZATION AND READ CONTROL VARIABLES *****

```

```

OPEN(UNIT=5, FILE='IN.TXT')
OPEN(UNIT=6, FILE='OUT.TXT')
OPEN(UNIT=7, FILE='OUT_MISC.TXT')

```

```

!===== INPUT/OUTPUT FILE DESCRIPTION =====
!   File No.5 : [INPUT] MAIN INPUT FILE
!   File No.6 : [OUTPUT] MAIN OUTPUT FILE
!   File No.7 : [OUTPUT] MISCELLANEOUS OUTPUT FILE
!=====
WRITE(*,*) 'START PROGRAM'
READ(5,*) N,KT
READ(5,*) NDIM,SSRAT,N1,NRAN,GEN_SETT,EST_ARBIT
IF (EST_ARBIT==1) THEN
    N=N-1
    N_ARBIT=N+1
ELSEIF (EST_ARBIT==0) THEN
    N_ARBIT=1
ELSE
    WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [EST_ARBIT] !!!!!!!!!!!!!!!'
    READ(*,*)
    STOP
END IF
READ(5,*) BETA1_SM,SDBETA1_SM,BETA0_SM,SDBETA0_SM
READ(5,*) BETA1_0,SDBETA1_0,BETA0_0,SDBETA0_0,QP,SDE,SDE_ASSUM,ZA,ZA_ASSUM

READ(5,*) EST_ZA
IF (EST_ZA==1) THEN
    READ(5,*) SDE_L,SDE_H,NDIV_SDE
    READ(5,*) ZA_L,ZA_H,NDIV_ZA
    ALLOCATE (ZA_TRIAL(NDIV_ZA+1),SDE_TRIAL(NDIV_SDE+1),ABIC(NDIV_ZA+1,NDIV_SDE+1))
ELSEIF (EST_ZA/=0) THEN
    WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [EST_ZA] !!!!!!!!!!!!!!!'
    READ(*,*)
    STOP
END IF

READ(5,*) EST_ZETA0

READ(5,*) KRIG,NKRIG
IF (KRIG==1) THEN
    ALLOCATE (COR_KRIG(NKRIG,2))
    DO I=1,NKRIG
        READ(5,*) COR_KRIG(I,:)
    END DO
    IF (EST_ARBIT==1) THEN
        NT=N_ARBIT
    ELSEIF (EST_ARBIT==0) THEN
        NT=N+NKRIG
    END IF
ELSEIF (KRIG==0) THEN
    IF (EST_ARBIT==1) THEN
        NT=N_ARBIT
    ELSEIF (EST_ARBIT==0) THEN
        NT=N
    END IF

```

```

ELSE
  WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [KRIG] !!!!!!!!!!!!!!!'
  READ(*,*)
  STOP
END IF

READ(5,*) KALMAN_INC
READ(5,*) KALMAN_IGN

WRITE(6,*) '===== GENERAL INPUT PARAMETERS ====='
WRITE(6,'(10I6)') N,NT,KT,NRAN
WRITE(6,'(20F12.6)') BETA1_SM,SDBETA1_SM,BETA0_SM,SDBETA0_SM
WRITE(6,'(20F12.6)') BETA1_O,SDBETA1_O,BETA0_O,SDBETA0_O,QP,SDE,SDE_ASSUM,ZA,ZA_ASSUM
ALLOCATE (ZETA(2*NT),VZETA(2*NT,2*NT),MK(N,2*NT),YK(N),QK(2*NT,2*NT),VE(N,N),KK(2*NT,N), &
  SK(N,N),SKINV(N,N),YK_IG(1),MK_IG(1,2),ZETA_IG(2),KK_IG(2,1),SK_IG(1,1), &
  VZETA_IG(2,2),VE_IG(1,1),SKINV_IG(1,1),QK_IG(2,2),CORX(N,2),X1(NT),X0(NT),BETA1(N), &
  BETA0(N),Y(KT,N),XY((KT)*N),ZETA_I(KT-1,2*N),VZETA_I(2*N,2*N),VZETA_ZA(2*NT,2*NT), &
  VZETA_ZA_INV(2*NT,2*NT),VE_ZA(N,N),VE_ZA_INV(N,N),MVM(2*NT,2*NT),MVM_INV(2*NT,2*NT), &
  MVY(2*NT,1),MK_ZA(N,2*NT),YK_ZA(N,1),ZETA_ZA(2*NT,1),ZETA_MLM(2*NT,1),UNIT(1,1), &
  W_EI(2*NT,7),E_EI(2*NT),V_EI(2*NT,2*NT),LW_EI(2*NT),A_EI(2*NT,2*NT),D_N(N),V_N(N,N), &
  E_N(N),D_2N(2*NT),V_2N(2*NT,2*NT),E_2N(2*NT),D_2(2),V_2(2,2),E_2(2),D_1(1),V_1(1,1), &
  E_1(1),V_KRIG(N+1,N+1),V_KRIG_INV(N+1,N+1),VXO_KRIG(N+1,1),WGHT(N+1,1),Z_KRIG(N+1,1), &
  ZXO_KRIG(1,2),D_N1(N+1),V_N1(N+1,N+1),E_N1(N+1),ERB1_P(N),ERBO_P(N),ERIB1_P(N), &
  ERIBO_P(N),SDERB1_P(N),SDERBO_P(N),SDERIB1_P(N),SDERIBO_P(N),YF(N),ERYF_P(N), &
  ERIYF_P(N),SDERYF_P(N),SDERIYF_P(N),BERB1_P(N),BERBO_P(N),BERIB1_P(N),BERIBO_P(N), &
  BERYF_P(N),BERIYF_P(N),T(KT),YT_EST(N),IMI(2*NT),IMI_IG(2*NT),CORX_ARBIT(2), &
  CORX_T(N+1,2),Y_T(KT,N+1),YT_EST_T(N+1),Y_ARBIT(KT),CORX_NT(NT,2),BETA1_KR(NKRIG), &
  BETA0_KR(NKRIG),YF_KR(NKRIG),ERB1_KR_P(NKRIG),ERBO_KR_P(NKRIG),BERB1_KR_P(NKRIG), &
  BERBO_KR_P(NKRIG),ERYF_KR_P(NKRIG),BERYF_KR_P(NKRIG),ELN_B1(N),ELN_BO(N),VLN_B1(N), &
  VLN_BO(N),ELN_YF(N),VLN_YF(N),ELNI_B1(N),ELNI_BO(N),VLNI_B1(N),VLNI_BO(N),ELNI_YF(N), &
  VLNI_YF(N),ELN_B1_KR(NKRIG),ELN_BO_KR(NKRIG),VLN_B1_KR(NKRIG),VLN_BO_KR(NKRIG), &
  ELN_YF_KR(NKRIG),VLN_YF_KR(NKRIG))

N_CURR=0
N_FUT=0
N_M1M0=0

ERR_ABS_M1=0.0D0
ERR_BIAS_M1=0.0D0
ERR_ABS_M0=0.0D0
ERR_BIAS_M0=0.0D0

ERR_CURR_ABS=0.0D0
ERR_FUT_ABS=0.0D0
ERR_CURR_BIAS=0.0D0
ERR_FUT_BIAS=0.0D0

DO NA=1,N_ARBIT
  IF (EST_ARBIT==1) THEN
    WRITE(*,'(A,I6,A,I6)') 'NA = ', NA, ' / ', N_ARBIT
    IF (NA==1) THEN
      DO I=1,N+1
        READ(5,*) CORX_T(I,:)
      END DO
    END IF
  END IF

```



```

J=0
DO I=1,N+1
  IF (I==NA) THEN
    CORX_ARBIT(:)=CORX_T(I,:)
  ELSE
    J=J+1
    CORX(J,:)=CORX_T(I,:)
  END IF
END DO
CORX_NT(1:N,:)=CORX
CORX_NT(NT,:)=CORX_ARBIT(:)
ELSEIF (EST_ARBIT==0) THEN
  DO I=1,N
    READ(5,*) CORX(I,:)
  END DO
  CORX_NT(1:N,:)=CORX
  IF (KRIG==1) THEN
    CORX_NT(N+1:NT,:)=COR_KRIG(1:NKRIG,:)
  END IF
END IF

```

! ***** GENERATE SETTLEMENT DATA, ASSUMING CORRELATED RANDOM FIELD *****

```

ELN_B1=0.0D0
ELN_B0=0.0D0
VLN_B1=0.0D0
VLN_B0=0.0D0
ELN_YF=0.0D0
VLN_YF=0.0D0
ELNI_B1=0.0D0
ELNI_B0=0.0D0
VLNI_B1=0.0D0
VLNI_B0=0.0D0
ELNI_YF=0.0D0
VLNI_YF=0.0D0
SDE_MISS=10000.0D0

```

```

IF (KRIG==1) THEN
  ELN_B1_KR=0.0D0
  ELN_B0_KR=0.0D0
  VLN_B1_KR=0.0D0
  VLN_B0_KR=0.0D0
  ELN_YF_KR=0.0D0
  VLN_YF_KR=0.0D0
END IF

```

```

CALL SYSTEM_CLOCK (COUNT= IC_BEGIN, COUNT_RATE= IRT,COUNT_MAX= IP)

```

```

DO IRAN=1,NRAN

```

```

  IF (GEN_SETT==1) THEN ! FOR THE CASE USING GENERATED SETTLEMENT
    IF (IRAN<=3) THEN
      WRITE(6,*) ' '
      WRITE(6,'(A,I6)') 'RANDOM SAMPLING# ', IRAN
    END IF
    IF (IRAN<=3) WRITE(6,*) '===== GENERATED MODEL PARAMETERS

```

```

=====
IF (EST_ARBIT==1) THEN ! FOR THE CASE WITH REMOVED OBSERVATION POINTS
  DEALLOCATE(X1,X0)
  ALLOCATE (X1(N_ARBIT),X0(N_ARBIT))

  IF (NDIM==1) THEN
    IR=23455+(IRAN-1)*50
    CALL RFGEN1D(SSRAT,N1,CORX_T,N_ARBIT,ZA,IR,X1)
    IR=34567+(IRAN-1)*50
    CALL RFGEN1D(SSRAT,N1,CORX_T,N_ARBIT,ZA,IR,X0)
  ELSEIF (NDIM==2) THEN
    IR=23455+(IRAN-1)*50
    CALL RFGEN2D(SSRAT,N1,CORX_T,N_ARBIT,ZA,IR,X1)
    IR=34567+(IRAN-1)*50
    CALL RFGEN2D(SSRAT,N1,CORX_T,N_ARBIT,ZA,IR,X0)
  ELSE
    WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT NO. OF DIMENSIONS [NDIM] !!!!!!!!!!!!!!!'
    READ(*,*)
    STOP
  END IF

  !===== GENERATION OF MODEL PARAMETERS =====
  J=0
  DO I=1,N+1
    IF (I==NA) THEN
      BETA1_ARBIT=X1(I)*SDBETA1_SM+BETA1_SM
      BETA0_ARBIT=X0(I)*SDBETA0_SM+BETA0_SM
      YF_ARBIT=BETA0_ARBIT/(1.0D0-BETA1_ARBIT)
    ELSE
      J=J+1
      BETA1(J)=X1(I)*SDBETA1_SM+BETA1_SM
      BETA0(J)=X0(I)*SDBETA0_SM+BETA0_SM
      YF(I)=BETA0(I)/(1.0D0-BETA1(I))
    END IF
  END DO
  IF (IRAN<=3) WRITE(6,'(100F12.6)') BETA1(:),BETA0(:)
  IF (IRAN<=3) WRITE(6,*) '----- MODEL PARAMETERS AT ARBITRARY POINT -----'

  IF (IRAN<=3) WRITE(6,'(100F12.6)') BETA1_ARBIT,BETA0_ARBIT

  !===== GENERATION OF SETTLEMENT DATA =====
  DEALLOCATE(XY)
  ALLOCATE (XY((KT)*N_ARBIT))
  IF (IRAN==1.AND.NA==1) THEN
    J=0
    DO I=1,N+1 ! READ OBSERVED SETTLEMENT AT FIRST TIME STEP
      IF (I==NA) THEN
        READ(5,*) Y_ARBIT(1)
      ELSE
        J=J+1
        READ(5,*) Y(1,J)
      END IF
    END DO
  END IF
  IF (IRAN<=3) WRITE(6,*) '===== GENERATED SETTLEMENT DATA

```

```

=====
      IR=12345+(IRAN-1)*50
      CALL NRRAND((KT)*N_ARBIT, XY, IR)

      IRD=0
      J=0
      DO I=1,N+1
        IF (I==NA) THEN
          DO K=2,KT
            IRD=IRD+1
            Y_ARBIT(K)=BETA0_ARBIT+BETA1_ARBIT*Y_ARBIT(K-1)+XY(IRD)*SDE
          END DO
        ELSE
          J=J+1
          DO K=2,KT
            IRD=IRD+1
            Y(K,J)=BETA0(J)+BETA1(J)*Y(K-1,J)+XY(IRD)*SDE
          END DO
          IF (IRAN<=3) WRITE(6,'(100F12.6)') Y(:,J),YF(J)
        END IF
      END DO
      IF (IRAN<=3) WRITE(6,*) '----- SETTLEMENT AT ARBITRARY POINT -----'
      IF (IRAN<=3) WRITE(6,'(100F12.6)') Y_ARBIT(:),YF_ARBIT

    ELSEIF (EST_ARBIT==0) THEN ! FOR THE CASE WITHOUT REMOVED OBSERVATION POINTS
      IF (NDIM==1) THEN
        IR=23455+(IRAN-1)*50
        CALL RFGEN1D(SSRAT,N1,CORX_NT,NT,ZA,IR,X1)
        IR=34567+(IRAN-1)*50
        CALL RFGEN1D(SSRAT,N1,CORX_NT,NT,ZA,IR,X0)
      ELSEIF (NDIM==2) THEN
        IR=23455+(IRAN-1)*50 ! FOR ALL THE OTHER CASES
        ! IR=23459+(IRAN-1)*50 ! FOR FINAL SETTLEMENT PREDICTION AT ARBITRARY POINTS [S/ZA = 1]
        ! IR=23465+(IRAN-1)*50 ! FOR FINAL SETTLEMENT PREDICTION AT ARBITRARY POINTS [S/ZA = 5]
        CALL RFGEN2D(SSRAT,N1,CORX_NT,NT,ZA,IR,X1)
        IR=34567+(IRAN-1)*50 ! FOR ALL THE OTHER CASES
        ! IR=34571+(IRAN-1)*50 ! FOR FINAL SETTLEMENT PREDICTION AT ARBITRARY POINTS [S/ZA = 1]
        ! IR=34579+(IRAN-1)*50 ! FOR FINAL SETTLEMENT PREDICTION AT ARBITRARY POINTS [S/ZA = 5]

        CALL RFGEN2D(SSRAT,N1,CORX_NT,NT,ZA,IR,X0)
      ELSE
        WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT NO. OF DIMENSIONS [NDIM] !!!!!!!!!!!!!!!'
        READ(*,*)
        STOP
      END IF
      !===== GENERATION OF MODEL PARAMETERS =====
      DO I=1,N
        BETA1(I)=X1(I)*SDBETA1_SM+BETA1_SM
        BETA0(I)=X0(I)*SDBETA0_SM+BETA0_SM
        YF(I)=BETA0(I)/(1.0D0-BETA1(I))
      END DO
      IF (KRIG==1) THEN
        J=0
        DO I=1,NKRIG
          BETA1_KR(I)=X1(N+I)*SDBETA1_SM+BETA1_SM

```

```

        BETAO_KR(I)=X0(N+I)*SDBETAO_SM+BETAO_SM
        YF_KR(I)=BETAO_KR(I)/(1.0D0-BETA1_KR(I))
    END DO
END IF
IF (IRAN<=3) WRITE(6,'(100F24.16)') BETA1(:),BETA1_KR(:),BETAO(:),BETAO_KR(:)

!===== GENERATION OF SETTLEMENT DATA =====
IF (IRAN==1) READ(5,*) (Y(1,I),I=1,N)
IF (IRAN<=3) WRITE(6,*) '===== GENERATED SETTLEMENT DATA
===== '
IF (IRAN<=3) WRITE(6,'(100F12.6)') Y(1,:)
IR=12345+(IRAN-1)*50 ! FOR ALL THE OTHER CASES
! IR=12355+(IRAN-1)*50 ! FOR FINAL SETTLEMENT PREDICTION AT ARBITRARY POINTS [S/ZA = 1]
! IR=12365+(IRAN-1)*50 ! FOR FINAL SETTLEMENT PREDICTION AT ARBITRARY POINTS [S/ZA = 5]

CALL NRAND((KT)*N, XY, IR)
IRD=0
DO K=2,KT
    DO I=1,N
        IRD=IRD+1
        Y(K,I)=BETAO(I)+BETA1(I)*Y(K-1,I)+XY(IRD)*SDE
    END DO
    IF (IRAN<=3) WRITE(6,'(100F12.6)') Y(K,:)
END DO
END IF

ELSEIF (GEN_SETT==0) THEN ! FOR THE CASE USING DIRECTLY INPUT DATA
    IF (EST_ARBIT==1) THEN ! FOR THE CASE WITH REMOVED OBSERVATION POINTS
        IF (NA==1) THEN
            READ(5,*) (T(K),K=1,KT),T_EST
            DO I=1,N+1
                READ(5,*) (Y_T(K,I),K=1,KT),YT_EST_T(I)
            END DO
        END IF
        J=0
        DO I=1,N+1
            IF (I==NA) THEN
                Y_ARBIT(:)=Y_T(:,I)
                YT_CURR_ARBIT=Y_ARBIT(KT)
                YT_EST_ARBIT=YT_EST_T(I)
            ELSE
                J=J+1
                Y(:,J)=Y_T(:,I)
                YT_EST(J)=YT_EST_T(I)
            END IF
        END DO
    ELSEIF (EST_ARBIT==0) THEN ! FOR THE CASE WITHOUT REMOVED OBSERVATION POINTS
        READ(5,*) (T(K),K=1,KT),T_EST
        DO I=1,N
            READ(5,*) (Y(K,I),K=1,KT),YT_EST(I)
        END DO
    END IF
END IF

```

! ***** ESTIMATE PRIOR MEANS & VARIANCE OF UNKNOWN PARAMETERS *****

```

IF (EST_ZETA0==1) THEN
  MEAN_X=0.0D0
  MEAN_Y=0.0D0
  NDAT=0
  DO I=1,N
    DO K=1,KT
      IF (Y(K,I)/=-999) THEN
        NDAT=NDAT+1
        MEAN_X=MEAN_X+DLOG10(T(K))
        MEAN_Y=MEAN_Y+Y(K,I)
      END IF
    END DO
  END DO
  MEAN_X=MEAN_X/NDAT
  MEAN_Y=MEAN_Y/NDAT
  DO I=1,N
    DO K=1,KT
      IF (Y(K,I)/=-999) THEN
        XXYY=XXYY+(DLOG10(T(K))-MEAN_X)*(Y(K,I)-MEAN_Y)
        XX2=XX2+(DLOG10(T(K))-MEAN_X)**2
        YY2=YY2+(Y(K,I)-MEAN_Y)**2
      END IF
    END DO
  END DO
  BETA1_0=XXYY/XX2
  BETA0_0=MEAN_Y-BETA1_0*MEAN_X
  S2_YX=(YY2-(BETA1_0**2)*XX2)/(NDAT-2)
  SDBETA1_0=DSQRT(S2_YX*(1.0D0/XX2))
  SDBETA0_0=DSQRT(S2_YX*(1.0D0/NDAT+(MEAN_X**2)/XX2))

  SDBETA1_0=0.40D0*DABS(BETA1_0)
  SDBETA0_0=0.40D0*DABS(BETA0_0)

  WRITE(7,*) '===== ESTIMATION OF PRIOR MEAN/SD OF UNKNOWN PARAMETERS ====='
  WRITE(7,'(A20,1F12.6)') 'EST. BETA1_0: ',BETA1_0
  WRITE(7,'(A20,1F12.6)') 'EST. BETA0_0: ',BETA0_0
  WRITE(7,'(A20,1F12.6)') 'EST. S2_YX: ',S2_YX
  WRITE(7,'(A20,1F12.6)') 'EST. SDBETA1_0: ',SDBETA1_0
  WRITE(7,'(A20,1F12.6)') 'EST. SDBETA0_0: ',SDBETA0_0

ELSEIF (EST_ZETA0/=0) THEN
  WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [EST_ZETA0] !!!!!!!!!!!!!!!'
  READ(*,*)
  STOP
END IF

```

! ***** ESTIMATION OF AUTO-CORRELATION DISTANCE & OBSERVATION-MODEL ERROR

```

IF (EST_ZA==1) THEN
  DO JDIV=1,NDIV_SDE+1
    IF (NDIV_SDE==0) THEN
      SDE_TRIAL(JDIV)=SDE_L
    ELSE
      SDE_TRIAL(JDIV)=SDE_L+((SDE_H-SDE_L)/NDIV_SDE)*(JDIV-1)
    END IF
  END DO

```

```

END IF
VE_ZA=0.0D0
VE_ZA_INV=0.0D0
DO I=1,N
    VE_ZA(I,I)=SDE_TRIAL(JDIV)**2
    VE_ZA_INV(I,I)=1.0D0/VE_ZA(I,I)
END DO
DET_VE_ZA=1.0D0
DO I=1,N
    DET_VE_ZA=DET_VE_ZA*VE_ZA(I,I)
END DO
MK_ZA=0.0D0
DO I=1,N
    MK_ZA(I,I+NT)=1.0D0
END DO
ZETA_ZA(1:NT,1)=BETA1_0
ZETA_ZA(NT+1:2*NT,1)=BETA0_0
DO IDIV=1,NDIV_ZA+1
    ! CALCULATE THE STATE VECTOR WHICH MINIMIZES ABIC
    IF (NDIV_ZA==0) THEN
        ZA_TRIAL(IDIV)=ZA_L
    ELSE
        ZA_TRIAL(IDIV)=ZA_L+((ZA_H-ZA_L)/NDIV_ZA)*(IDIV-1)
    END IF
    VZETA_ZA=0.0D0
    DO I=1,NT
        DO J=1,NT
            DX=DSQRT((CORX_NT(I,1)-CORX_NT(J,1))**2+(CORX_NT(I,2)-CORX_NT(J,2))**2) ; !
DISTANCE BETWEEN OBSERVED POINTS
            RHO=DEXP(-DX/ZA_TRIAL(IDIV))
            VZETA_ZA(I,J)=(SDBETA1_0**2)*RHO
            VZETA_ZA(I+NT,J+NT)=(SDBETA0_0**2)*RHO
        END DO
    END DO
    A_EI=VZETA_ZA
    EPS=1.0E-16
    CALL EIGRS(A_EI, 2*NT, 2*NT, 2*NT, 2*NT, EPS, W_EI, LW_EI, E_EI, V_EI, IER)
    DET_VZETA_ZA=1.0D0
    DO I=1,2*N
        DET_VZETA_ZA=DET_VZETA_ZA*E_EI(I)
    END DO
    EPS=0.0D0
    LMAT=1
    D_2N=0.0D0
    V_2N=0.0D0
    E_2N=0.0D0
    A_EI=VZETA_ZA
    CALL SVDEC(A_EI, 2*NT, 2*NT, 2*NT, EPS, LMAT, D_2N, V_2N, 2*NT, NRANK, E_2N, IER)
    CALL GINV(A_EI, 2*NT, 2*NT, 2*NT, D_2N, V_2N, 2*NT, NRANK, VZETA_ZA_INV, 2*NT, IER)

    MVM=0.0D0
    MVY=0.0D0
    DO K=1,KT-1
        YK_ZA(:,1)=Y(K+1,:)
    DO I=1,N

```

```

        MK_ZA(I,I)=Y(K,I)
        IF (YK_ZA(I,1)==-999) THEN
            VE_ZA(I,I)=SDE_MISS**2
        ELSE
            VE_ZA(I,I)=SDE_TRIAL(JDIV)**2
        END IF
        VE_ZA_INV(I,I)=1.0D0/VE_ZA(I,I)
    END DO
    MVM=MVM+MATMUL(MATMUL(TRANSPose(MK_ZA),VE_ZA_INV),MK_ZA)
    MVY=MVY+MATMUL(MATMUL(TRANSPose(MK_ZA),VE_ZA_INV),YK_ZA)
END DO
MVM=MVM+VZETA_ZA_INV
A_EI=MVM
EPS=1.0E-16
CALL EIGRS(A_EI, 2*NT, 2*NT, 2*NT, 2*NT, EPS, W_EI, LW_EI, E_EI, V_EI, IER)
DET_MVM=1.0D0
DO I=1,2*N
    DET_MVM=DET_MVM*E_EI(I)
END DO

EPS=0.0D0
LMAT=1
D_2N=0.0D0
V_2N=0.0D0
E_2N=0.0D0
CALL SVDEC(MVM, 2*NT, 2*NT, 2*NT, EPS, LMAT, D_2N, V_2N, 2*NT, NRANK, E_2N, IER)
CALL GINV(MVM, 2*NT, 2*NT, 2*NT, D_2N, V_2N, 2*NT, NRANK, MVM_INV, 2*NT, IER)
ZETA_MLM=MATMUL(MVM_INV,(MATMUL(VZETA_ZA_INV,ZETA_ZA)+MVY))

! CALCULATE THE ABIC VALUE
UNIT=0.0D0
DO K=1,KT-1
    YK_ZA(:,1)=Y(K+1,:)
    DO I=1,N
        MK_ZA(I,I)=Y(K,I)
        IF (YK_ZA(I,1)==-999) THEN
            VE_ZA(I,I)=SDE_MISS**2
        ELSE
            VE_ZA(I,I)=SDE_TRIAL(JDIV)**2
        END IF
        VE_ZA_INV(I,I)=1.0D0/VE_ZA(I,I)
    END DO
    UNIT=UNIT+MATMUL(MATMUL(TRANSPose(YK_ZA-MATMUL(MK_ZA,ZETA_MLM)),VE_ZA_INV),YK_ZA-
MATMUL(MK_ZA,ZETA_MLM))
    END DO
    UNIT=UNIT+MATMUL(MATMUL(TRANSPose(ZETA_MLM-ZETA_ZA),VZETA_ZA_INV),ZETA_MLM-ZETA_ZA)
    ABIC(IDIV,JDIV)=QLOG(QABS(DET_VZETA_ZA))+(KT-
1)*DLOG(DABS(DET_VE_ZA))+QLOG(QABS(DET_MVM))+UNIT(1,1)
    END DO
END DO
WRITE(7,*) ' '
WRITE(7,*) '===== BAYESIAN INFORMATION CRITERION, ABIC ====='
WRITE(7, '(A,14)') 'IRAN =', IRAN
WRITE(7, '(3A25)') 'SD OF OBS. ERROR', 'AUTO-COR. DIST.', 'ABIC'
DO JDIV=1,NDIV_SDE+1

```

```

DO IDIV=1,NDIV_ZA+1
  WRITE(7,'(100F25.10)') SDE_TRIAL(JDIV),ZA_TRIAL(IDIV),ABIC(IDIV,JDIV)
END DO
END DO
MINABIC=MINVAL(ABIC)
DO JDIV=1,NDIV_SDE+1
  DO IDIV=1,NDIV_ZA+1
    IF(ABIC(IDIV,JDIV)==MINABIC) THEN
      ZA_ASSUM=ZA_TRIAL(IDIV)
      SDE_ASSUM=SDE_TRIAL(JDIV)
      WRITE(7,*) '----- MINIMUM VALUE OF ABIC -----'
      WRITE(7,'(A1,F24.10,100F25.10)') 'A' ,
SDE_TRIAL(JDIV),ZA_TRIAL(IDIV),ABIC(IDIV,JDIV)
    END IF
  END DO
END DO
END IF
! ***** SPATIAL-TEMPORAL UPDATING BASED ON KALMAN FILTER *****
IF (KALMAN_INC==1) THEN
  QK=0.0D0
  VE=0.0D0
  DO I=1,N
    VE(I,I)=SDE_ASSUM**2
  END DO
  MK=0.0D0
  DO I=1,N
    MK(I,I+NT)=1.0D0
  END DO
  ZETA(1:NT)=BETA1_0
  ZETA(NT+1:2*NT)=BETA0_0
  VZETA=0.0D0
  DO I=1,NT
    DO J=1,NT
      DX=DSQRT((CORX_NT(I,1)-CORX_NT(J,1))**2+(CORX_NT(I,2)-CORX_NT(J,2))**2) ; ! DISTANCE
BETWEEN OBSERVED POINTS
      RHO=DEXP(-DX/ZA_ASSUM)
      VZETA(I,J)=(SDBETA1_0**2)*RHO
      VZETA(I+NT,J+NT)=(SDBETA0_0**2)*RHO
    END DO
  END DO
  IF (IRAN==1) THEN
    WRITE(6,*) '===== INITIAL STATE VECTOR, ZETA ====='
    WRITE(6,'(100F12.6)') ZETA(:)
    WRITE(6,*) '===== INITIAL COVARIANCE MATRIX OF ZETA ====='
    DO I=1,2*NT
      WRITE(6,'(100F12.6)') VZETA(I,:)
    END DO
    WRITE(6,*) '===== INITIAL COVARIANCE MATRIX OF OBSERVATION ERROR ====='
    DO I=1,N
      WRITE(6,'(100F12.6)') VE(I,:)
    END DO
    WRITE(6,*) '===== INITIAL COVARIANCE MATRIX OF PROCESS NOISE ====='
    DO I=1,2*NT
      WRITE(6,'(100F12.6)') QK(I,:)
    END DO
  END IF

```



```

END IF

IF (IRAN<=3) WRITE(6,*) '===== SPATIAL-TEMPORAL UPDATED STATE VECTOR, ZETA
=====
DO K=1,KT-1
  YK(:)=Y(K+1,:)
  DO I=1,N
    MK(I,I)=Y(K,I)
    IF (YK(I)==-999) THEN
      VE(I,I)=SDE_MISS**2
    ELSE
      VE(I,I)=SDE_ASSUM**2
    END IF
  END DO

  !===== TIME UPDATING =====
  IF (K>=2) THEN
    VZETA=(1.0D0+QP)*VZETA
  END IF

  !===== OBSERVATION UPDATING =====
  SK=MATMUL(MATMUL(MK,VZETA),TRANPOSE(MK))+VE
  EPS=0.0D0
  LMAT=1
  CALL SVDEC(SK, N, N, N, EPS, LMAT, D_N, V_N, N, NRANK, E_N, IER)
  CALL GINV(SK, N, N, N, D_N, V_N, N, NRANK, SKINV, N, IER)
  KK=MATMUL(MATMUL(VZETA,TRANPOSE(MK)),SKINV) ! KALMAN GAIN
  ZETA=ZETA+MATMUL(KK,YK-MATMUL(MK,ZETA))
  VZETA=VZETA-MATMUL(MATMUL(KK,MK),VZETA)
  IF (IRAN<=3) WRITE(6,'(100F18.6)') ZETA(:)

  IF (IRAN<=3) THEN
    WRITE(6,'(100F24.16)') ZETA(:)
  END IF
END DO

! ERROR DETERMINATION
DO I=1,N
  ELN_B1(I)=ELN_B1(I)+DLOG(ZETA(I)/BETA1(I))
  ELN_B0(I)=ELN_B0(I)+DLOG(ZETA(I+NT)/BETA0(I))
  VLN_B1(I)=VLN_B1(I)+(DLOG(ZETA(I)/BETA1(I)))**2
  VLN_B0(I)=VLN_B0(I)+(DLOG(ZETA(I+NT)/BETA0(I)))**2

  YF_EST=ZETA(I+NT)/(1.0D0-ZETA(I))
  ELN_YF(I)=ELN_YF(I)+DLOG(YF_EST/YF(I))
  VLN_YF(I)=VLN_YF(I)+(DLOG(YF_EST/YF(I)))**2

  IMI(I)=VZETA(I,I)/SDBETA1_0
  IMI(I+NT)=VZETA(I+NT,I+NT)/SDBETA0_0
END DO

!=====
IF (EST_ARBIT==1) THEN
  IF (GEN_SETT==1) THEN
    ERR_M1=(ZETA(NT)-BETA1_ARBIT)*100/BETA1_ARBIT
    ERR_ABS_M1=ERR_ABS_M1+DABS(ERR_M1)
    ERR_BIAS_M1=ERR_BIAS_M1+ERR_M1
  END IF
END IF

```

```

ERR_MO=(ZETA(2*NT)-BETAO_ARBIT)*100/BETAO_ARBIT
ERR_ABS_MO=ERR_ABS_MO+DABS(ERR_MO)
ERR_BIAS_MO=ERR_BIAS_MO+ERR_MO

N_M1MO=N_M1MO+1
END IF
YF_EST=ZETA(2*NT)/(1.0D0-ZETA(NT))
IF (YF_ARBIT== -999) THEN
    ERR_FUT= -999
ELSE
    ERR_FUT=(YF_EST-YF_ARBIT)*100/YF_ARBIT
    ERR_FUT_ABS=ERR_FUT_ABS+DABS(ERR_FUT)
    ERR_FUT_BIAS=ERR_FUT_BIAS+ERR_FUT
    N_FUT=N_FUT+1
END IF
WRITE(7, '(A,100F30.18)')
'B', CORX_ARBIT(1), CORX_ARBIT(2), ZETA(NT), ZETA(2*NT), YF_EST, ERR_M1, ERR_MO, ERR_FUT, ZA_ASSUM
END IF
!=====
IF (KRIG==1) THEN
    WRITE(7,*) '----- ESTIMATION AT ARBIT POINTS BY KRIGING -----'
    WRITE(7, '(A11,3A30)') 'POINT#', 'BETA1', 'BETAO', 'ESTIMATED AUTO-COR. DIST.'
    DO I=1, NKRIG
        WRITE(7, '(A,110,4F30.18)') 'B', I, ZETA(N+I), ZETA(N+NT+I), ZA_ASSUM
        ELN_B1_KR(I)=ELN_B1_KR(I)+DLOG(ZETA(N+I)/BETA1_KR(I))
        ELN_B0_KR(I)=ELN_B0_KR(I)+DLOG(ZETA(N+NT+I)/BETAO_KR(I))
        VLN_B1_KR(I)=VLN_B1_KR(I)+(DLOG(ZETA(N+I)/BETA1_KR(I)))**2
        VLN_B0_KR(I)=VLN_B0_KR(I)+(DLOG(ZETA(N+NT+I)/BETAO_KR(I)))**2

        YF_EST=ZETA(N+NT+I)/(1.0D0-ZETA(N+I))
        ELN_YF_KR(I)=ELN_YF_KR(I)+DLOG(YF_EST/YF_KR(I))
        VLN_YF_KR(I)=VLN_YF_KR(I)+(DLOG(YF_EST/YF_KR(I)))**2
    END DO
END IF

ELSEIF (KALMAN_INC/=0) THEN
    WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [KALMAN_INC] !!!!!!!!!!!!!!!'
    READ(*,*)
    STOP
END IF

!***** KALMAN FILTER UPDATE, IGNORING SPATIAL CORRELATION *****
IF (KALMAN_IGN==1) THEN
    MK_IG(1,2)=1.0D0
    QK_IG=0.0D0
    ZETA_I=0.0D0
    VZETA_I=0.0D0
    IF (IRAN<=3) WRITE(6,*) '===== KALMAN FILTER UPDATE, IGNORING SPATIAL CORRELATION
=====
DO I=1,N
    ZETA_IG(1)=BETA1_0
    ZETA_IG(2)=BETAO_0
    VZETA_IG=0.0D0
    VZETA_IG(1,1)=SDBETA1_0**2

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```

VZETA_IG(2,2)=SDBETA0_0**2
DO K=1,KT-1
    MK_IG(1,1)=Y(K,I)
    YK_IG(1)=Y(K+1,I)
    IF (YK_IG(1)=-999) THEN
        VE_IG(1,1)=SDE_MISS**2
    ELSE
        VE_IG(1,1)=SDE_ASSUM**2
    END IF
!===== TIME UPDATING =====
    IF (K>=2) THEN
        VZETA_IG=(1.0D0+QP)*VZETA_IG
    END IF
!===== OBSERVATION UPDATING =====
    SK_IG=MATMUL(MATMUL(MK_IG,VZETA_IG),TRANSPPOSE(MK_IG))+VE_IG
    EPS=0.0D0
    LMAT=1
    CALL SVDEC(SK_IG, 1, 1, 1, EPS, LMAT, D_1, V_1, 1, NRANK, E_1, IER)
    CALL GINV(SK_IG, 1, 1, 1, D_1, V_1, 1, NRANK, SKINV_IG, 1, IER)
    KK_IG=MATMUL(MATMUL(VZETA_IG,TRANSPPOSE(MK_IG)),SKINV_IG) ! KALMAN GAIN
    ZETA_IG=ZETA_IG+MATMUL(KK_IG,YK_IG-MATMUL(MK_IG,ZETA_IG))
    VZETA_IG=VZETA_IG-MATMUL(MATMUL(KK_IG,MK_IG),VZETA_IG)
    ZETA_I(K,I)=ZETA_IG(1)
    ZETA_I(K,I+N)=ZETA_IG(2)
    VZETA_I(I,I)=VZETA_IG(1,1)
    VZETA_I(I+N,I+N)=VZETA_IG(2,2)
END DO
! ERROR DETERMINATION
ELNI_B1(I)=ELNI_B1(I)+DLOG(ZETA_IG(1)/BETA1(I))
ELNI_B0(I)=ELNI_B0(I)+DLOG(ZETA_IG(2)/BETA0(I))
VLNI_B1(I)=VLNI_B1(I)+(DLOG(ZETA_IG(1)/BETA1(I)))**2
VLNI_B0(I)=VLNI_B0(I)+(DLOG(ZETA_IG(2)/BETA0(I)))**2

YF_EST=ZETA_IG(2)/(1.0D0-ZETA_IG(1))
ELNI_YF(I)=ELNI_YF(I)+DLOG(YF_EST/YF(I))
VLNI_YF(I)=VLNI_YF(I)+(DLOG(YF_EST/YF(I)))**2

IMI_IG(I)=VZETA_IG(1,1)/SDBETA1_0
IMI_IG(I+N)=VZETA_IG(2,2)/SDBETA0_0
END DO
IF (IRAN<=3) THEN
    DO K=1,KT-1
        WRITE(6,'(100F24.16)') ZETA_I(K,:)
    END DO
END IF
ELSEIF (KALMAN_IGN/=0) THEN
    WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [KALMAN_IGN] !!!!!!!!!!!!!!!'
    READ(*,*)
    STOP
END IF
!----- TOTAL RUNNING TIME APPROXIMATION -----
IF (IRAN==1) THEN
    CALL SYSTEM_CLOCK (COUNT= IC)
    TIMEI=(IC-IC_BEGIN)/(60*IRT)
    TIMER=(REAL(IC)-REAL(IC_BEGIN))/(60*IRT)

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        WRITE(*,'(A,I6,A,F5.1,A)') 'RUNNING TIME FOR 1ST SAMPLING:  ', TIMEI,' MIN',(TIMER-TIMEI)*60,'
SEC'
        TIMEI=(IC-IC_BEGIN)*NRAN/(60*IRT)
        TIMER=(REAL(IC)-REAL(IC_BEGIN))*NRAN/(60*IRT)
        WRITE(*,'(A,I6,A,F5.1,A)') 'TOTAL RUNNING TIME (APPROX.):  ', TIMEI,' MIN',(TIMER-TIMEI)*60,'
SEC'
        END IF
        WRITE(*,'(A,I6,A,I6)') 'RUNNING PROGRESS:  ', IRAN , ' / ' , NRAN
    END DO

!===== OUTPUT CALCULATION =====
ELN_B1_T=0.0D0
ELN_B0_T=0.0D0
VLN_B1_T=0.0D0
VLN_B0_T=0.0D0
ELN_YF_T=0.0D0
VLN_YF_T=0.0D0

ELNI_B1_T=0.0D0
ELNI_B0_T=0.0D0
VLNI_B1_T=0.0D0
VLNI_B0_T=0.0D0
ELNI_YF_T=0.0D0
VLNI_YF_T=0.0D0

NT_EST=0

IM=0.0D0
IM_IG=0.0D0

DO I=1,N
    ELN_B1_T=ELN_B1_T+ELN_B1(I)
    ELN_B0_T=ELN_B0_T+ELN_B0(I)
    VLN_B1_T=VLN_B1_T+VLN_B1(I)
    VLN_B0_T=VLN_B0_T+VLN_B0(I)

    ELNI_B1_T=ELNI_B1_T+ELNI_B1(I)
    ELNI_B0_T=ELNI_B0_T+ELNI_B0(I)
    VLNI_B1_T=VLNI_B1_T+VLNI_B1(I)
    VLNI_B0_T=VLNI_B0_T+VLNI_B0(I)

    IF (YT_EST(I)/=-999) THEN
        NT_EST=NT_EST+1
        ELN_YF_T=ELN_YF_T+ELN_YF(I)
        VLN_YF_T=VLN_YF_T+VLN_YF(I)
        ELNI_YF_T=ELNI_YF_T+ELNI_YF(I)
        VLNI_YF_T=VLNI_YF_T+VLNI_YF(I)
    END IF
    IM=IM+DLOG(IMI(I))+DLOG(IMI(I+N))
    IM_IG=IM_IG+DLOG(IMI_IG(I))+DLOG(IMI_IG(I+N))
END DO

IF (KRIG==1) THEN
    DO I=1,NKRIG
        ELN_B1_KR(I)=ELN_B1_KR(I)/NRAN

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```

        ELN_BO_KR(I)=ELN_BO_KR(I)/NRAN
        VLN_B1_KR(I)=(VLN_B1_KR(I)-NRAN*ELN_B1_KR(I)**2)/(NRAN-1)
        VLN_BO_KR(I)=(VLN_BO_KR(I)-NRAN*ELN_BO_KR(I)**2)/(NRAN-1)
        ELN_YF_KR(I)=ELN_YF_KR(I)/NRAN
        VLN_YF_KR(I)=(VLN_YF_KR(I)-NRAN*ELN_YF_KR(I)**2)/(NRAN-1)
    END DO
END IF
ELN_B1_T=ELN_B1_T/(NRAN*N)
ELN_BO_T=ELN_BO_T/(NRAN*N)
VLN_B1_T=(VLN_B1_T-(NRAN*N)*ELN_B1_T**2)/(NRAN*N-1)
VLN_BO_T=(VLN_BO_T-(NRAN*N)*ELN_BO_T**2)/(NRAN*N-1)
ELNI_B1_T=ELNI_B1_T/(NRAN*N)
ELNI_BO_T=ELNI_BO_T/(NRAN*N)
VLNI_B1_T=(VLNI_B1_T-(NRAN*N)*ELNI_B1_T**2)/(NRAN*N-1)
VLNI_BO_T=(VLNI_BO_T-(NRAN*N)*ELNI_BO_T**2)/(NRAN*N-1)
ELN_YF_T=ELN_YF_T/(NRAN*NT_EST)
VLN_YF_T=(VLN_YF_T-(NRAN*NT_EST)*ELN_YF_T**2)/(NRAN*NT_EST-1)
ELNI_YF_T=ELNI_YF_T/(NRAN*NT_EST)
VLNI_YF_T=(VLNI_YF_T-(NRAN*NT_EST)*ELNI_YF_T**2)/(NRAN*NT_EST-1)

IM=DEXP(IM/(2*N))
IM_IG=DEXP(IM_IG/(2*N))

WRITE(6,*) ' '
WRITE(6,*) '===== ESTIMATION ERROR AT THE LAST TIME STEP ====='
! (CONSID) REPRESENTS THE CASE OF CONSIDERING AUTO-CORRELATION DISTANCE
! (IGNOR) REPRESENTS THE CASE OF IGNORING AUTO-CORRELATION DISTANCE
WRITE(6,*) '----- MEAN & VARIANCE OF ERROR RATIO (ER) -----'
WRITE(6, '(A20,4A36)') 'PARAMETERS', 'MEAN(CONSID)', 'MEAN(IGNOR)', 'VARIANCE(CONSID)',
'VARIANCE(IGNOR)'
WRITE(6, '(A20,100F36.18)') 'BETA1', ELN_B1_T, ELNI_B1_T, VLN_B1_T, VLNI_B1_T
WRITE(6, '(A20,100F36.18)') 'BETA0', ELN_BO_T, ELNI_BO_T, VLN_BO_T, VLNI_BO_T
WRITE(6, '(A20,200F36.18)') 'FINAL SETTLEMENT', ELN_YF_T, ELNI_YF_T, VLN_YF_T, VLNI_YF_T

END DO
IF (EST_ARBIT==1) THEN

    ERR_ABS_M1=ERR_ABS_M1/N_M1MO
    ERR_BIAS_M1=ERR_BIAS_M1/N_M1MO
    ERR_ABS_M0=ERR_ABS_M0/N_M1MO
    ERR_BIAS_M0=ERR_BIAS_M0/N_M1MO

    ERR_CURR_BIAS=ERR_CURR_BIAS/N_CURR
    ERR_FUT_BIAS=ERR_FUT_BIAS/N_FUT
    ERR_CURR_ABS=ERR_CURR_ABS/N_CURR
    ERR_FUT_ABS=ERR_FUT_ABS/N_FUT
    WRITE(7,*) '----- MEAN ABSOLUTE ERROR -----'
    WRITE(7, '(100F30.18)') ERR_ABS_M1, ERR_ABS_M0, ERR_CURR_ABS, ERR_FUT_ABS
    WRITE(7,*) '----- MEAN BIAS -----'
    WRITE(7, '(100F30.18)') ERR_BIAS_M1, ERR_BIAS_M0, ERR_CURR_BIAS, ERR_FUT_BIAS
END IF

IF (KRIG==1) THEN
    WRITE(7,*) ' '
    WRITE(7,*) '***** ERROR OF ESTIMATION AT ARBITRARY POINTS (KRIGING) *****'

```

```

WRITE(7,*) '----- ERROR RATIO (ER) OF M1 -----'
WRITE(7, '(A15,100F30.18)') 'MEAN',ELN_B1_KR
WRITE(7, '(A15,100F30.18)') 'VARIANCE',VLN_B1_KR
WRITE(7,*) '----- ERROR RATIO (ER) OF M0 -----'
WRITE(7, '(A15,100F30.18)') 'MEAN',ELN_B0_KR
WRITE(7, '(A15,100F30.18)') 'VARIANCE',VLN_B0_KR
WRITE(7,*) '----- ERROR RATIO (ER) OF FINAL SETTLEMENT -----'
WRITE(7, '(A15,100F30.18)') 'MEAN',ELN_YF_KR
WRITE(7, '(A15,100F30.18)') 'VARIANCE',VLN_YF_KR
END IF

CALL SYSTEM_CLOCK (COUNT= IC)
TIMEI=(IC-IC_BEGIN)/(60*IRT)
TIMER=(REAL(IC)-REAL(IC_BEGIN))/(60*IRT)
WRITE(*, '(A,I6,A,F5.1,A)') 'TOTAL RUNNING TIME: ', TIMEI, ' MIN', (TIMER-TIMEI)*60, ' SEC'
WRITE(*,*) 'END PROGRAM'
READ(*,*)
! *****
CLOSE(5)
CLOSE(6)
CLOSE(7)

END PROGRAM STSETT1_5_2

```

2nd PROGRAM: the program for secondary compression prediction by $S \sim \log(t)$ method

Input

Input file name: 'IN.TXT'

Input data:

```

N,KT
NDIM,SSRAT,N1,NRAN,GEN_SETT,EST_ARBIT
BETA1_SM,SDBETA1_SM,BETA0_SM,SDBETA0_SM
BETA1_0,SDBETA1_0,BETA0_0,SDBETA0_0,QP,SDE,SDE_ASSUM,ZA,ZA_ASSUM
EST_ZA

```

```

[ IF (EST_ZA=1) THEN]
  SDE_L,SDE_H,NDIV_SDE
  ZA_L,ZA_H,NDIV_ZA
[END IF]

```

```

EST_ZETA0
KRIG,NKRIG

```

```

[ IF (KRIG=1) THEN]
  COR_KRIG(1,:)
  COR_KRIG (2,:)
.

```

```

COR_KRIG(NKRIG,:)
[END IF]

KALMAN_INC
KALMAN_IGN

CORX(1,:)
CORX(2,:)
.
.
CORX(N,:)

[IF (GEN_SETT=1) THEN]
  T(1) T(2) ... T(KT) T_EST
[ELSEIF (GEN_SETT=0) THEN]
  T(1) T(2) ... T(KT) T_EST

  Y(1,1) Y(2,1) ... Y(KT,1) YT_EST(1)
  Y(1,2) Y(2,2) ... Y(KT,2) YT_EST(2)
  .
  .
  Y(1,N) Y(2,N) ... Y(KT,N) YT_EST(N)
[END IF]

```

(Note: for the meaning of each variable, please refer to ‘VARIABLE DESCRIPTION’ section at the beginning of the source code

Output

1. Output file name: ‘OUT.TXT’

Output data: main output data, including generated data, estimated model parameters and settlement, and estimation error.

2. Output file name: ‘OUT_MISC.TXT’

Output data: miscellaneous output data, including estimated auto-correlation distance & standard deviation of observation error, estimated prior mean & variance of unknown parameters, estimated values at arbitrary points, and estimation errors.

Source code

PROGRAM STSETT1_6_2

```
! *****
! SPATIAL-TEMPORAL PREDICTION OF SECONDARY COMPRESSION BASED ON S~LOG(T) METHOD
!
! CREATED BY P. RUNGBANAPHAN - FEB 14, 2010
! *****
USE MATHLIB ; IMPLICIT NONE

! ***** VARIABLE DECLARATION *****
INTEGER I,J,K,N,KT,LMAT,NRANK,IER,NRAN,IRAN,IRD,NDIM,N1,IR,EST_ZA,NDIV_ZA,IDIV,NDIV_SDE,JDIV, &
KRIG,IKRIG,NKRIG,KALMAN_INC,KALMAN_IGN,IC_BEGIN,IRT,IP,IC,TIMEI,GEN_SETT,NT_EST,NDAT, &
EST_ZETA0,EST_ARBIT,N_ARBIT,NA,N_CURR,N_FUT,NT
INTEGER, ALLOCATABLE :: LW_EI(:)
REAL(8) BETA1_0,BETA0_0,SDBETA1_0,SDBETA0_0,SDE,ZA,DX,AINV(3,3),EPS,RHO,MEAN,SD,OM1,SSRAT, &
ZA_L,ZA_H,DET_VE_ZA,SDE_L,SDE_H,ZA_ASSUM,SDE_ASSUM,SETT_FN,MINABIC,TIMER, &
YF_EST,ERB1_PT,ERBO_PT,SDERB1_PT,SDERBO_PT,ERIB1_PT,ERIBO_PT,SDERIB1_PT,SDERIBO_PT, &
ERYF_PT,SDERYF_PT,ERIYF_PT,SDERIYF_PT,MERB1_PT,MERBO_PT,MERIB1_PT,MERIBO_PT,BERB1_PT, &
BERBO_PT,BERIB1_PT,BERIBO_PT,BERYF_PT,BERIYF_PT,MERYF_PT,MERIYF_PT,T_EST,QP,IM,IM_IG, &
SDE_MISS,MEAN_X,MEAN_Y,XXYY,XX2,YY2,S2_YX,YT_EST_ARBIT,SETT_CURR,SETT_FUT,ERR_CURR, &
ERR_FUT,ERR_VLN_CURR,ERR_VLN_FUT,ERR_ELN_CURR,ERR_ELN_FUT,BETA1_SM,BETA0_SM, &
SDBETA1_SM,SDBETA0_SM,BETA1_ARBIT,BETA0_ARBIT,ERR_M1,ERR_MO,ERR_VLN_M1,ERR_ELN_M1, &
ERR_VLN_MO,ERR_ELN_MO,N_M1MO,YT_CURR_ARBIT,DET_MVM,ELN_B1_T,ELN_BO_T,VLN_B1_T, &
VLN_BO_T,ELN_YF_T,VLN_YF_T,ELNI_B1_T,ELNI_BO_T,VLNI_B1_T,VLNI_BO_T,ELNI_YF_T, &
VLNI_YF_T
REAL(16) DET_VZETA_ZA
REAL(8), ALLOCATABLE :: ZETA(:),VZETA(:,:),MK(:,:),YK(:),QK(:,:),VE(:,:),SK(:,:),SKINV(:,:), &
YK_IG(:),MK_IG(:,:),ZETA_IG(:),KK_IG(:,:),SK_IG(:,:),VZETA_IG(:,:), &
VE_IG(:,:),SKINV_IG(:,:),QK_IG(:,:),KK(:,:),CORX(:,:),X1(:),X0(:), &
BETA1(:),BETA0(:),Y(:,:),XY(:),ZETA_I(:,:),VZETA_I(:,:),ZA_TRIAL(:), &
ABIC(:,:),VZETA_ZA(:,:),VZETA_ZA_INV(:,:),VE_ZA(:,:),VE_ZA_INV(:,:), &
MVM(:,:),MVM_INV(:,:),MVY(:,:),MK_ZA(:,:),YK_ZA(:,:),ZETA_ZA(:,:), &
ZETA_MLM(:,:),UNIT(:,:),W_EI(:,:),E_EI(:),V_EI(:,:),A_EI(:,:), &
D_N(:),V_N(:,:),E_N(:),D_2N(:),V_2N(:,:),E_2N(:),D_2(:), &
V_2(:,:),E_2(:),D_1(:),V_1(:,:),E_1(:),SDE_TRIAL(:),LH1(:,:), &
COR_KRIG(:,:),V_KRIG(:,:),V_KRIG_INV(:,:),VXO_KRIG(:,:),WGHT(:,:), &
Z_KRIG(:,:),ZXO_KRIG(:,:),D_N1(:),V_N1(:,:),E_N1(:),ERB1_P(:), &
ERBO_P(:),ERIB1_P(:),ERIBO_P(:),SDERB1_P(:),SDERBO_P(:), &
SDERIB1_P(:),SDERIBO_P(:),ERYF_P(:),ERIYF_P(:),SDERYF_P(:), &
SDERIYF_P(:),BERB1_P(:),BERBO_P(:),BERIB1_P(:),BERIBO_P(:), &
BERYF_P(:),BERIYF_P(:),T(:),YT_EST(:),IMI(:),IMI_IG(:), &
CORX_ARBIT(:),CORX_T(:,:),Y_T(:,:),YT_EST_T(:),Y_ARBIT(:), &
CORX_NT(:,:),ELN_B1(:),ELN_BO(:),VLN_B1(:),VLN_BO(:),ELN_YF(:), &
VLN_YF(:),ELNI_B1(:),ELNI_BO(:),VLNI_B1(:),VLNI_BO(:),ELNI_YF(:), &
VLNI_YF(:),ELN_B1_KR(:),ELN_BO_KR(:),VLN_B1_KR(:),VLN_BO_KR(:), &
ELN_YF_KR(:),VLN_YF_KR(:)

!===== VARIABLE DESCRIPTION =====
! N : TOTAL NO. OF OBSERVATION POINTS
! KT : TOTAL TIME STEP OF OBSERVED SETTLEMENT
! NDIM : NO. OF DIMENSIONS OF THE PROBLEM (1 OR 2 FOR 1D OR 2D)
! SSRAT : RATIO OF SPECTRAL DENSITY VALUES AT MAXIMUM POINT TO THAT AT THE POINT CONSIDERED
! NEGLIGIBLE
```



```

!  N1          : NUMBER OF DEVIDED INTERVALS IN FREQUENCY DOMAIN (BOTH X & Y AXIS, FOR 2D CASE)
!  N1         : NO. OF RANDOM SAMPLINGS OF MODEL PARAMETERS
!  GEN_SETT    : = 1 = PERFORM SETTLEMENT GENERATION BASED ON THE ASSIGNED RANDOM FIELD PARAMETERS
!               USING PREDICATION MODEL
!               0 = READ THE OBSERVED SETTLEMENT DATA FROM THE INPUT FILE
!  EST_ARBIT   : = 1 OR 0 = DO OR DON'T PERFORM ESTIMATION OF SETTLEMENT AT ARBITRARY POINTS
!  BETA1_SM    : EXPECTED VALUE OF MODEL PARAMETER, M1, USED FOR DATA SIMULATION
!  SDBETA1_SM  : STANDARD DEVIATION OF MODEL PARAMETER, M1, USED FOR DATA SIMULATION
!  BETA0_SM    : EXPECTED VALUE OF MODEL PARAMETER, M0, USED FOR DATA SIMULATION
!  SDBETA0_SM  : STANDARD DEVIATION OF MODEL PARAMETER, M0, USED FOR DATA SIMULATION
!  BETA1_0     : INITIAL EXPECTED VALUE OF MODEL PARAMETER, M1
!  BETA0_0     : INITIAL EXPECTED VALUE OF MODEL PARAMETER, M0
!  SDBETA1_0   : INITIAL STANDARD DEVIATION OF MODEL PARAMETER, M1
!  SDBETA0_0   : INITIAL STANDARD DEVIATION OF MODEL PARAMETER, M0
!  QP          : CONSTANT PARAMETER, REPRESENTING PROCESS NOISE (REFERED AS 'a' IN THESIS BOOK)
!  SDE         : ASSUMED STANDARD DEVIATION OF OBSERVATION MODEL
!  SDE_ASSUM   : STANDARD DEVIATION OF OBSERVATION MODEL, USED FOR DATA SIMULATION
!  ZA          : ASSUMED AUTO-CORRELATION DISTANCE
!  ZA_ASSUM    : AUTO-CORRELATION DISTANCE, USED FOR DATA SIMULATION
!  EST_ZA      : = 1 OR 0 = DO OR DON'T PERFORM ESTIMATION OF AUTO-CORRELATION DISTANCE AND SD OF
!               OBSERVATION ERROR
!  SDE_L       : LOWER BOUNDARY FOR TRIAL AND ERROR VALUES OF SD OF OBSERVATION ERROR
!  SDE_H       : HIGHER BOUNDARY FOR TRIAL AND ERROR VALUES OF SD OF OBSERVATION ERROR
!  NDIV_SDE    : NUMBER OF DIVISIONS FOR TRIAL AND ERROR RANGE OF SD OF OBSERVATION ERROR
!  ZA_L        : LOWER BOUNDARY FOR TRIAL AND ERROR VALUES OF AUTO-CORRELATION DISTANCE
!  ZA_H        : HIGHER BOUNDARY FOR TRIAL AND ERROR VALUES OF AUTO-CORRELATION DISTANCE
!  NDIV_ZA     : NUMBER OF DIVISIONS FOR TRIAL AND ERROR RANGE OF AUTO-CORRELATION DISTANCE
!  EST_ZETA0   : = 1 OR 0 = DO OR DON'T PERFORM ESTIMATION OF PRIOR MEAN OF UNKNOWN PARAMETERS
!  KRIG        : = 1 OR 0 = DO OR DON'T KRIG FOR MODEL PARAMETERS AT AN ARBITRARY POINT
!  NKRIG       : NUMBER OF ARBITRARY POINTS TO BE ESTIMATED BY KRIGING
!  COR_KRIG    : COORDINATES OF ARBITRARY POINTS TO BE ESTIMATED BY KRIGING
!  KALMAN_INC  : = 1 OR 0 = DO OR DON'T PERFORM KALMAN FILTER UPDATE, INCLUDING SPATIAL CORRELATION
!  KALMAN_IGN  : = 1 OR 0 = DO OR DON'T PERFORM KALMAN FILTER UPDATE, IGNORING SPATIAL CORRELATION
!  CORX(I,:)   : COORDINATES (X,Y) OF OBSERVATION POINTS AT POINT 'I'
!  Y(K,I)      : SETTLEMENT AT TIME STEP 'K' AND AT OBSERVATION POINT 'I'
!  T(K)        : TIME AT TIME STEP K
!  T_EST       : TIME AT WHICH THE SETTLEMENT IS TO BE ESTIMATED
!  YT_EST(I)   : SETTLEMENT AT TIME 'T_EST' AND AT OBSERVATION POINT 'I'
!  NT         : TOTAL NO. OF OBSERVATION POINTS INCLUDING ARBITRARY POINTS
!  ZETA        : STATE VECTOR [BETA1(X1) ... BETA1(XN):BETA0(X0) ... BETA0(XN)]
!  VZETA       : COVARIANCE MATRIX OF ZETA
!  QK          : COVARIANCE MATRIX OF PROCESS NOISE
!  MK          : OBSERVATION-PARAMETER MODEL[DIAG[Z_K-1(X1) ... Z_K-1(XN)]:I]
!  YK          : OBSERVED SETTLEMENTS FOR CURRENT TIMESTEP [Z_K(X1) ... Z_K(XN)]T
!  VE          : COVARIANCE MATRIX OF OBSERVATION MODEL
!  KK          : KALMAN GAIN
!  OM1         : CONSIDERED REGION IN FREQUENCY DOMAIN (BOTH X & Y AXIS, FOR 2D CASE)
!  DET_VE_ZA   : DETERMINANT OF COVARIANCE MATRIX OF THE OBSERVATION MODEL ERROR
!  DET_VZETA_ZA : DETERMINANT OF COVARIANCE MATRIX OF THE STATE VECTOR
!=====

```

```

!***** INITIALIZATION AND READ CONTROL VARIABLES *****

```

```

OPEN(UNIT=5, FILE='IN.TXT')

```

```

OPEN(UNIT=6, FILE='OUT.TXT')
OPEN(UNIT=7, FILE='OUT_MISC.TXT')

```

```

===== INPUT/OUTPUT FILE DESCRIPTION =====
!      File No.5  : [INPUT] MAIN INPUT FILE
!      File No.6  : [OUTPUT] MAIN OUTPUT FILE
!      File No.7  : [OUTPUT] MISCELLANEOUS OUTPUT FILE
=====
WRITE(*,*) 'START PROGRAM'
READ(5,*) N,KT
READ(5,*) NDIM,SSRAT,N1,NRAN,GEN_SETT,EST_ARBIT
IF (EST_ARBIT==1) THEN
    N=N-1
    N_ARBIT=N+1
ELSEIF (EST_ARBIT==0) THEN
    N_ARBIT=1
ELSE
    WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [EST_ARBIT] !!!!!!!!!!!!!!!'
    READ(*,*)
    STOP
END IF
READ(5,*) BETA1_SM,SDBETA1_SM,BETA0_SM,SDBETA0_SM
READ(5,*) BETA1_0,SDBETA1_0,BETA0_0,SDBETA0_0,QP,SDE,SDE_ASSUM,ZA,ZA_ASSUM

READ(5,*) EST_ZA
IF (EST_ZA==1) THEN
    READ(5,*) SDE_L,SDE_H,NDIV_SDE
    READ(5,*) ZA_L,ZA_H,NDIV_ZA
    ALLOCATE (ZA_TRIAL(NDIV_ZA+1),SDE_TRIAL(NDIV_SDE+1),ABIC(NDIV_ZA+1,NDIV_SDE+1))
ELSEIF (EST_ZA/=0) THEN
    WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [EST_ZA] !!!!!!!!!!!!!!!'
    READ(*,*)
    STOP
END IF

READ(5,*) EST_ZETA0

READ(5,*) KRIG,NKRIG
IF (KRIG==1) THEN
    ALLOCATE (COR_KRIG(NKRIG,2))
    DO I=1,NKRIG
        READ(5,*) COR_KRIG(I,:)
    END DO
    IF (EST_ARBIT==1) THEN
        NT=N_ARBIT
    ELSEIF (EST_ARBIT==0) THEN
        NT=N+NKRIG
    END IF
ELSEIF (KRIG==0) THEN
    IF (EST_ARBIT==1) THEN
        NT=N_ARBIT
    ELSEIF (EST_ARBIT==0) THEN
        NT=N
    END IF
ELSE

```

```

WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [KRIG] !!!!!!!!!!!!!!!'
READ(*,*)
STOP
END IF

READ(5,*) KALMAN_INC
READ(5,*) KALMAN_IGN

WRITE(6,*) '===== GENERAL INPUT PARAMETERS ====='
WRITE(6, '(10I6)') N,NT,KT,NRAN
WRITE(6, '(20F12.6)') BETA1_SM,SDBETA1_SM,BETA0_SM,SDBETA0_SM
WRITE(6, '(20F12.6)') BETA1_0,SDBETA1_0,BETA0_0,SDBETA0_0,QP,SDE,SDE_ASSUM,ZA,ZA_ASSUM
ALLOCATE (ZETA(2*NT),VZETA(2*NT,2*NT),MK(N,2*NT),YK(N),QK(2*NT,2*NT),VE(N,N),KK(2*NT,N), &
          SK(N,N),SKINV(N,N),YK_IG(1),MK_IG(1,2),ZETA_IG(2),KK_IG(2,1),SK_IG(1,1), &
          VZETA_IG(2,2),VE_IG(1,1),SKINV_IG(1,1),QK_IG(2,2),CORX(N,2),X1(N),X0(N),BETA1(N), &
          BETA0(N),Y(KT,N),XY((KT+1)*N),ZETA_I(KT,2*N),VZETA_I(2*N,2*N),VZETA_ZA(2*NT,2*NT), &
          VZETA_ZA_INV(2*NT,2*NT),VE_ZA(N,N),VE_ZA_INV(N,N),MVM(2*NT,2*NT),MVM_INV(2*NT,2*NT), &
          MVY(2*NT,1),MK_ZA(N,2*NT),YK_ZA(N,1),ZETA_ZA(2*NT,1),ZETA_MLM(2*NT,1),UNIT(1,1), &
          W_EI(2*NT,7),E_EI(2*NT),V_EI(2*NT,2*NT),LW_EI(2*NT),A_EI(2*NT,2*NT),D_N(N),V_N(N,N), &
          E_N(N),D_2N(2*NT),V_2N(2*NT,2*NT),E_2N(2*NT),D_2(2),V_2(2,2),E_2(2),D_1(1),V_1(1,1), &
          E_1(1),V_KRIG(N+1,N+1),V_KRIG_INV(N+1,N+1),VX0_KRIG(N+1,1),WGHT(N+1,1),Z_KRIG(N+1,1), &
          ZX0_KRIG(1,2),D_N1(N+1),V_N1(N+1,N+1),E_N1(N+1),ERB1_P(N),ERBO_P(N),ERIB1_P(N), &
          ERIBO_P(N),SDERB1_P(N),SDERBO_P(N),SDERIB1_P(N),SDERIBO_P(N),ERYF_P(N),ERIFYF_P(N), &
          SDERYF_P(N),SDERIFYF_P(N),BERB1_P(N),BERBO_P(N),BERIB1_P(N),BERIBO_P(N),BERYF_P(N), &
          BERIFYF_P(N),T(KT),YT_EST(N),IMI(2*NT),IMI_IG(2*NT),CORX_ARBIT(2),CORX_T(N+1,2), &
          Y_T(KT,N+1),YT_EST_T(N+1),Y_ARBIT(KT),CORX_NT(NT,2),ELN_B1(N),ELN_BO(N),VLN_B1(N), &
          VLN_BO(N),ELN_YF(N),VLN_YF(N),ELNI_B1(N),ELNI_BO(N),VLNI_B1(N),VLNI_BO(N),ELNI_YF(N), &
          VLNI_YF(N),ELN_B1_KR(NKRIG),ELN_BO_KR(NKRIG),VLN_B1_KR(NKRIG),VLN_BO_KR(NKRIG), &
          ELN_YF_KR(NKRIG),VLN_YF_KR(NKRIG))

N_CURR=0
N_FUT=0
N_M1M0=0

ERR_ELN_M1=0.0D0
ERR_VLN_M1=0.0D0
ERR_ELN_M0=0.0D0
ERR_VLN_M0=0.0D0

ERR_ELN_CURR=0.0D0
ERR_VLN_CURR=0.0D0
ERR_ELN_FUT=0.0D0
ERR_VLN_FUT=0.0D0

DO NA=1,N_ARBIT
  IF (EST_ARBIT==1) THEN
    WRITE(*, '(A,I6,A,I6)') 'NA = ', NA, ' / ', N_ARBIT
    IF (NA==1) THEN
      DO I=1,N+1
        READ(5,*) CORX_T(I,:)
      END DO
    END IF
    J=0
    DO I=1,N+1

```

```

        IF (I==NA) THEN
            CORX_ARBIT(:)=CORX_T(I,:)
        ELSE
            J=J+1
            CORX(J,:)=CORX_T(I,:)
        END IF
    END DO
    CORX_NT(1:N,:)=CORX
    CORX_NT(NT,:)=CORX_ARBIT(:)
ELSEIF (EST_ARBIT==0) THEN
    DO I=1,N
        READ(5,*) CORX(I,:)
    END DO
    CORX_NT(1:N,:)=CORX
    IF (KRIG==1) THEN
        CORX_NT(N+1:NT,:)=COR_KRIG(1:NKRIG,:)
    END IF
END IF

```

! ***** GENERATE SETTLEMENT DATA, ASSUMING CORRELATED RANDOM FIELD *****

```

ELN_B1=0.0D0
ELN_B0=0.0D0
VLN_B1=0.0D0
VLN_B0=0.0D0
ELN_YF=0.0D0
VLN_YF=0.0D0
ELNI_B1=0.0D0
ELNI_B0=0.0D0
VLNI_B1=0.0D0
VLNI_B0=0.0D0
ELNI_YF=0.0D0
VLNI_YF=0.0D0

```

```

ERB1_P=0.0D0
ERB0_P=0.0D0
BERB1_P=0.0D0
BERB0_P=0.0D0
SDERB1_P=0.0D0
SDERB0_P=0.0D0
ERIB1_P=0.0D0
ERIB0_P=0.0D0
BERIB1_P=0.0D0
BERIB0_P=0.0D0
SDERIB1_P=0.0D0
SDERIB0_P=0.0D0

```

```

ERYF_P=0.0D0
BERYF_P=0.0D0
SDERYF_P=0.0D0
ERIYF_P=0.0D0
BERIYF_P=0.0D0
SDERIYF_P=0.0D0

```

```

SDE_MISS=10000.0D0

```

```

CALL SYSTEM_CLOCK (COUNT= IC_BEGIN, COUNT_RATE= IRT,COUNT_MAX= IP)

DO IRAN=1,NRAN

  IF (GEN_SETT==1) THEN ! FOR THE CASE USING GENERATED SETTLEMENT
    IF (IRAN<=3) THEN
      WRITE(6,*) ' '
      WRITE(6, '(A,I6)') 'RANDOM SAMPLING# ', IRAN
    END IF
    IF (IRAN<=3) WRITE(6,*) '===== GENERATED MODEL PARAMETERS
===== '
    IF (EST_ARBIT==1) THEN ! FOR THE CASE WITH REMOVED OBSERVATION POINTS
      DEALLOCATE(X1,X0)
      ALLOCATE (X1(N_ARBIT),X0(N_ARBIT))

      IF (NDIM==1) THEN
        IR=23455+(IRAN-1)*50
        CALL RFGEN1D(SSRAT,N1,CORX_T,N_ARBIT,ZA,IR,X1)
        IR=34567+(IRAN-1)*50
        CALL RFGEN1D(SSRAT,N1,CORX_T,N_ARBIT,ZA,IR,X0)
      ELSEIF (NDIM==2) THEN
        IR=23455+(IRAN-1)*50
        CALL RFGEN2D(SSRAT,N1,CORX_T,N_ARBIT,ZA,IR,X1)
        IR=34567+(IRAN-1)*50
        CALL RFGEN2D(SSRAT,N1,CORX_T,N_ARBIT,ZA,IR,X0)
      ELSE
        WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT NO. OF DIMENSIONS [NDIM] !!!!!!!!!!!!!!!'
        READ(*,*)
        STOP
      END IF

      !===== GENERATION OF S-LOG(T) MODEL PARAMETERS
      =====
      J=0
      DO I=1,N+1
        IF (I==NA) THEN
          BETA1_ARBIT=X1(I)*SDBETA1_SM+BETA1_SM
          BETA0_ARBIT=X0(I)*SDBETA0_SM+BETA0_SM
        ELSE
          J=J+1
          BETA1(J)=X1(I)*SDBETA1_SM+BETA1_SM
          BETA0(J)=X0(I)*SDBETA0_SM+BETA0_SM
        END IF
      END DO
      IF (IRAN<=3) WRITE(6, '(100F12.6)') BETA1(:),BETA0(:)
      IF (IRAN<=3) WRITE(6,*) '----- MODEL PARAMETERS AT ARBITRARY POINT -----
-- '

      IF (IRAN<=3) WRITE(6, '(100F12.6)') BETA1_ARBIT,BETA0_ARBIT

      !===== GENERATION OF SETTLEMENT DATA =====
      DEALLOCATE(XY)
      ALLOCATE (XY((KT+1)*N_ARBIT))
      IF (IRAN==1.AND.NA==1) READ(5,*) (T(K),K=1,KT),T_EST
      IF (IRAN<=3) WRITE(6,*) '===== GENERATED SETTLEMENT DATA
===== '

```

```

IR=12345+(IRAN-1)*50
CALL NRAND((KT+1)*N_ARBIT, XY, IR)
IRD=0

J=0
DO I=1,N+1
  IF (I==NA) THEN
    DO K=1,KT
      IRD=IRD+1
      Y_ARBIT(K)=BETA0_ARBIT+BETA1_ARBIT*DLOG10(T(K))+XY(IRD)*SDE
    END DO
    IRD=IRD+1
    YT_CURR_ARBIT=BETA0_ARBIT+BETA1_ARBIT*DLOG10(T(KT))
    YT_EST_ARBIT=BETA0_ARBIT+BETA1_ARBIT*DLOG10(T_EST)
  ELSE
    J=J+1
    DO K=1,KT
      IRD=IRD+1
      Y(K,J)=BETA0(J)+BETA1(J)*DLOG10(T(K))+XY(IRD)*SDE
    END DO
    IRD=IRD+1
    YT_EST(J)=BETA0(J)+BETA1(J)*DLOG10(T_EST)
    IF (IRAN<=3) WRITE(6,'(100F12.6)') Y(:,J),YT_EST(J)
  END IF
END DO
IF (IRAN<=3) WRITE(6,*) '----- SETTLEMENT AT ARBITRARY POINT -----'
IF (IRAN<=3) WRITE(6,'(100F12.6)') Y_ARBIT(:),YT_EST_ARBIT

ELSEIF (EST_ARBIT==0) THEN ! FOR THE CASE WITHOUT REMOVED OBSERVATION POINTS
  IF (NDIM==1) THEN
    IR=23455+(IRAN-1)*50
    CALL RFGEN1D(SSRAT,N1,CORX,N,ZA,IR,X1)
    IR=34567+(IRAN-1)*50
    CALL RFGEN1D(SSRAT,N1,CORX,N,ZA,IR,X0)
  ELSEIF (NDIM==2) THEN
    IR=23455+(IRAN-1)*50
    CALL RFGEN2D(SSRAT,N1,CORX,N,ZA,IR,X1)
    IR=34567+(IRAN-1)*50
    CALL RFGEN2D(SSRAT,N1,CORX,N,ZA,IR,X0)
  ELSE
    WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT NO. OF DIMENSIONS [NDIM] !!!!!!!!!!!!!!!'
    READ(*,*)
    STOP
  END IF
  !===== GENERATION OF S-LOG(T) MODEL PARAMETERS
  =====
  DO I=1,N
    BETA1(I)=X1(I)*SDBETA1_SM+BETA1_SM
    BETA0(I)=X0(I)*SDBETA0_SM+BETA0_SM
  END DO
  IF (IRAN<=3) WRITE(6,'(100F12.6)') BETA1(:),BETA0(:)

  !===== GENERATION OF SETTLEMENT DATA =====
  IF (IRAN==1) READ(5,*) (T(K),K=1,KT),T_EST
  IF (IRAN<=3) WRITE(6,*) '===== GENERATED SETTLEMENT DATA

```

```

=====
      IR=12345+(IRAN-1)*50
      CALL NRRAND((KT+1)*N, XY, IR)
      IRD=0
      DO I=1,N
        DO K=1,KT
          IRD=IRD+1
          Y(K,I)=BETA0(I)+BETA1(I)*DLOG10(T(K))+XY(IRD)*SDE
        END DO
        IRD=IRD+1
        YT_EST(I)=BETA0(I)+BETA1(I)*DLOG10(T_EST)
        IF (IRAN<=3) WRITE(6,'(100F12.6)') Y(:,I),YT_EST(I)
      END DO
    END IF

    ELSEIF (GEN_SETT==0) THEN ! FOR THE CASE USING DIRECTLY INPUT DATA
      IF (EST_ARBIT==1) THEN ! FOR THE CASE WITH REMOVED OBSERVATION POINTS
        IF (NA==1) THEN
          READ(5,*) (T(K),K=1,KT),T_EST
          DO I=1,N+1
            READ(5,*) (Y_T(K,I),K=1,KT),YT_EST_T(I)
          END DO
        END IF
        J=0
        DO I=1,N+1
          IF (I==NA) THEN
            Y_ARBIT(:)=Y_T(:,I)
            YT_CURR_ARBIT=Y_ARBIT(KT)
            YT_EST_ARBIT=YT_EST_T(I)
          ELSE
            J=J+1
            Y(:,J)=Y_T(:,I)
            YT_EST(J)=YT_EST_T(I)
          END IF
        END DO
      ELSEIF (EST_ARBIT==0) THEN ! FOR THE CASE WITHOUT REMOVED OBSERVATION POINTS
        READ(5,*) (T(K),K=1,KT),T_EST
        DO I=1,N
          READ(5,*) (Y(K,I),K=1,KT),YT_EST(I)
        END DO
      END IF
    END IF

    ! ***** ESTIMATE PRIOR MEANS & VARIANCE OF UNKNOWN PARAMETERS *****
    IF (EST_ZETA0==1) THEN
      MEAN_X=0.0D0
      MEAN_Y=0.0D0
      NDAT=0
      DO I=1,N
        DO K=1,KT
          IF (Y(K,I)/=-999) THEN
            NDAT=NDAT+1
            MEAN_X=MEAN_X+DLOG10(T(K))
            MEAN_Y=MEAN_Y+Y(K,I)
          END IF
        END DO
      END DO
    END IF
  END IF

```

```

        END IF
    END DO
END DO
MEAN_X=MEAN_X/NDAT
MEAN_Y=MEAN_Y/NDAT
DO I=1,N
    DO K=1,KT
        IF (Y(K,I)/=-999) THEN
            XXYY=XXYY+(DLOG10(T(K))-MEAN_X)*(Y(K,I)-MEAN_Y)
            XX2=XX2+(DLOG10(T(K))-MEAN_X)**2
            YY2=YY2+(Y(K,I)-MEAN_Y)**2
        END IF
    END DO
END DO
BETA1_0=XXYY/XX2
BETA0_0=MEAN_Y-BETA1_0*MEAN_X
S2_YX=(YY2-(BETA1_0**2)*XX2)/(NDAT-2)
SDBETA1_0=DSQRT(S2_YX*(1.0D0/XX2))
SDBETA0_0=DSQRT(S2_YX*(1.0D0/NDAT+(MEAN_X**2)/XX2))

SDBETA1_0=0.40D0*DABS(BETA1_0)
SDBETA0_0=0.40D0*DABS(BETA0_0)

WRITE(7,*) '===== ESTIMATION OF PRIOR MEAN/SD OF UNKNOWN PARAMETERS ====='
WRITE(7, '(A20,1F12.6)') 'EST. BETA1_0: ', BETA1_0
WRITE(7, '(A20,1F12.6)') 'EST. BETA0_0: ', BETA0_0
WRITE(7, '(A20,1F12.6)') 'EST. S2_YX: ', S2_YX
WRITE(7, '(A20,1F12.6)') 'EST. SDBETA1_0: ', SDBETA1_0
WRITE(7, '(A20,1F12.6)') 'EST. SDBETA0_0: ', SDBETA0_0

ELSEIF (EST_ZETA0/=0) THEN
    WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [EST_ZETA0] !!!!!!!!!!!!!!!'
    READ(*,*)
    STOP
END IF

! ***** ESTIMATION OF AUTO-CORRELATION DISTANCE & OBSERVATION-MODEL ERROR
*****

IF (EST_ZA==1) THEN
    DO JDIV=1,NDIV_SDE+1
        IF (NDIV_SDE==0) THEN
            SDE_TRIAL(JDIV)=SDE_L
        ELSE
            SDE_TRIAL(JDIV)=SDE_L+((SDE_H-SDE_L)/NDIV_SDE)*(JDIV-1)
        END IF
        VE_ZA=0.0D0
        VE_ZA_INV=0.0D0
        DO I=1,N
            VE_ZA(I,I)=SDE_TRIAL(JDIV)**2
            VE_ZA_INV(I,I)=1.0D0/VE_ZA(I,I)
        END DO
        DET_VE_ZA=1.0D0
        DO I=1,N
            DET_VE_ZA=DET_VE_ZA*VE_ZA(I,I)
        END DO
    END DO

```



```

MK_ZA=0.0D0
DO I=1,N
    MK_ZA(I,I+NT)=1.0D0
END DO
ZETA_ZA(1:NT,1)=BETA1_0
ZETA_ZA(NT+1:2*NT,1)=BETA0_0
DO IDIV=1,NDIV_ZA+1
    ! CALCULATE THE STATE VECTOR WHICH MINIMIZES ABIC
    IF (NDIV_ZA==0) THEN
        ZA_TRIAL(IDIV)=ZA_L
    ELSE
        ZA_TRIAL(IDIV)=ZA_L+((ZA_H-ZA_L)/NDIV_ZA)*(IDIV-1)
    END IF
    VZETA_ZA=0.0D0
    DO I=1,NT
        DO J=1,NT
            DX=DSQRT((CORX_NT(I,1)-CORX_NT(J,1))**2+(CORX_NT(I,2)-CORX_NT(J,2))**2) ; !
DISTANCE BETWEEN OBSERVED POINTS
            RHO=DEXP(-DX/ZA_TRIAL(IDIV))
            VZETA_ZA(I,J)=(SDBETA1_0**2)*RHO
            VZETA_ZA(I+NT,J+NT)=(SDBETA0_0**2)*RHO
        END DO
    END DO
    A_EI=VZETA_ZA
    EPS=1.0E-16
    CALL EIGRS(A_EI, 2*NT, 2*NT, 2*NT, 2*NT, EPS, W_EI, LW_EI, E_EI, V_EI, IER)
    DET_VZETA_ZA=1.0D0
    DO I=1,2*N
        DET_VZETA_ZA=DET_VZETA_ZA*E_EI(I)
    END DO
    EPS=0.0D0
    LMAT=1
    D_2N=0.0D0
    V_2N=0.0D0
    E_2N=0.0D0
    A_EI=VZETA_ZA
    CALL SVDEC(A_EI, 2*NT, 2*NT, 2*NT, EPS, LMAT, D_2N, V_2N, 2*NT, NRANK, E_2N, IER)
    CALL GINV(A_EI, 2*NT, 2*NT, 2*NT, D_2N, V_2N, 2*NT, NRANK, VZETA_ZA_INV, 2*NT, IER)

MVM=0.0D0
MVY=0.0D0
DO K=1,KT
    YK_ZA(:,1)=Y(K,:)
    DO I=1,N
        MK_ZA(I,I)=DLOG10(T(K))
        IF (YK_ZA(I,1)==-999) THEN
            VE_ZA(I,I)=SDE_MISS**2
        ELSE
            VE_ZA(I,I)=SDE_TRIAL(JDIV)**2
        END IF
        VE_ZA_INV(I,I)=1.0D0/VE_ZA(I,I)
    END DO
    MVM=MVM+MATMUL(MATMUL(TRANSPose(MK_ZA),VE_ZA_INV),MK_ZA)
    MVY=MVY+MATMUL(MATMUL(TRANSPose(MK_ZA),VE_ZA_INV),YK_ZA)
END DO

```

```

MVM=MVM+VZETA_ZA_INV
A_EI=MVM
EPS=1.0E-16
CALL EIGRS(A_EI, 2*NT, 2*NT, 2*NT, 2*NT, EPS, W_EI, LW_EI, E_EI, V_EI, IER)
DET_MVM=1.0D0
DO I=1,2*N
    DET_MVM=DET_MVM*E_EI(I)
END DO

EPS=0.0D0
LMAT=1
D_2N=0.0D0
V_2N=0.0D0
E_2N=0.0D0
CALL SVDEC(MVM, 2*NT, 2*NT, 2*NT, EPS, LMAT, D_2N, V_2N, 2*NT, NRANK, E_2N, IER)
CALL GINV(MVM, 2*NT, 2*NT, 2*NT, D_2N, V_2N, 2*NT, NRANK, MVM_INV, 2*NT, IER)
ZETA_MLM=MATMUL(MVM_INV, (MATMUL(VZETA_ZA_INV, ZETA_ZA)+MVY))

! CALCULATE THE ABIC VALUE
UNIT=0.0D0
DO K=1,KT
    YK_ZA(:,1)=Y(K,:)
    DO I=1,N
        MK_ZA(I,1)=DLOG10(T(K))
        IF (YK_ZA(I,1)==-999) THEN
            VE_ZA(I,1)=SDE_MISS**2
        ELSE
            VE_ZA(I,1)=SDE_TRIAL(JDIV)**2
        END IF
        VE_ZA_INV(I,1)=1.0D0/VE_ZA(I,1)
    END DO
    UNIT=UNIT+MATMUL(MATMUL(TRANPOSE(YK_ZA-MATMUL(MK_ZA, ZETA_MLM)), VE_ZA_INV), YK_ZA-
MATMUL(MK_ZA, ZETA_MLM))
END DO
UNIT=UNIT+MATMUL(MATMUL(TRANPOSE(ZETA_MLM-ZETA_ZA), VZETA_ZA_INV), ZETA_MLM-ZETA_ZA)

ABIC(IDIV,JDIV)=QLOG(QABS(DET_VZETA_ZA))+(KT)*DLOG(DABS(DET_VE_ZA))+DLOG(DABS(DET_MVM))+UNIT(1,1)
END DO
END DO
WRITE(7,*) ' '
WRITE(7,*) '===== BAYESIAN INFORMATION CRITERION, ABIC ====='
WRITE(7, '(A, I4)') 'IRAN =', IRAN
WRITE(7, '(3A25)') 'SD OF OBS. ERROR', 'AUTO-COR. DIST.', 'ABIC'
DO JDIV=1,NDIV_SDE+1
    DO IDIV=1,NDIV_ZA+1
        WRITE(7, '(10F25.10)') SDE_TRIAL(JDIV), ZA_TRIAL(IDIV), ABIC(IDIV,JDIV)
    END DO
END DO
MINABIC=MINVAL(ABIC)
DO JDIV=1,NDIV_SDE+1
    DO IDIV=1,NDIV_ZA+1
        IF(ABIC(IDIV,JDIV)==MINABIC) THEN
            ZA_ASSUM=ZA_TRIAL(IDIV)
            SDE_ASSUM=SDE_TRIAL(JDIV)
            WRITE(7,*) '----- MINIMUM VALUE OF ABIC -----'
        END IF
    END DO
END DO

```

```

WRITE(7, '(A1,F24.10,100F25.10)') 'A' ,
SDE_TRIAL(JDIV),ZA_TRIAL(IDIV),ABIC(IDIV,JDIV)
END IF
END DO
END DO
END IF
! END DO
! ***** SPATIAL-TEMPORAL UPDATING BASED ON KALMAN FILTER *****
IF (KALMAN_INC==1) THEN
  QK=0.0D0
  VE=0.0D0
  DO I=1,N
    VE(I,I)=SDE_ASSUM**2
  END DO
  MK=0.0D0
  DO I=1,N
    MK(I,I+NT)=1.0D0
  END DO
  ZETA(1:NT)=BETA1_0
  ZETA(NT+1:2*NT)=BETA0_0
  VZETA=0.0D0
  DO I=1,NT
    DO J=1,NT
      DX=DSQRT((CORX_NT(I,1)-CORX_NT(J,1))**2+(CORX_NT(I,2)-CORX_NT(J,2))**2) ; ! DISTANCE
      RHO=DEXP(-DX/ZA_ASSUM)
      VZETA(I,J)=(SDBETA1_0**2)*RHO
      VZETA(I+NT,J+NT)=(SDBETA0_0**2)*RHO
    END DO
  END DO
  IF (IRAN==1) THEN
    WRITE(6,*) '===== INITIAL STATE VECTOR, ZETA ====='
    WRITE(6, '(100F12.6)') ZETA(:)
    WRITE(6,*) '===== INITIAL COVARIANCE MATRIX OF ZETA ====='
    DO I=1,2*NT
      WRITE(6, '(100F12.6)') VZETA(I,:)
    END DO
    WRITE(6,*) '===== INITIAL COVARIANCE MATRIX OF OBSERVATION ERROR ====='
    DO I=1,N
      WRITE(6, '(100F12.6)') VE(I,:)
    END DO
    WRITE(6,*) '===== INITIAL COVARIANCE MATRIX OF PROCESS NOISE ====='
    DO I=1,2*NT
      WRITE(6, '(100F12.6)') QK(I,:)
    END DO
  END IF

  IF (IRAN<=3) WRITE(6,*) '===== SPATIAL-TEMPORAL UPDATED STATE VECTOR, ZETA
=====
DO K=1,KT
  YK(:)=Y(K,:)
  DO I=1,N
    MK(I,I)=DLOG10(T(K))
    IF (YK(I)==-999) THEN
      VE(I,I)=SDE_MISS**2

```

```

        ELSE
            VE(I,I)=SDE_ASSUM**2
        END IF
    END DO

!===== TIME UPDATING =====
    IF (K>=2) THEN
        VZETA=(1.0D0+QP)*VZETA
    END IF

!===== OBSERVATION UPDATING =====
    SK=MATMUL(MATMUL(MK,VZETA),TRANPOSE(MK))+VE
    EPS=0.0D0
    LMAT=1
    CALL SVDEC(SK, N, N, N, EPS, LMAT, D_N, V_N, N, NRANK, E_N, IER)
    CALL GINV(SK, N, N, N, D_N, V_N, N, NRANK, SKINV, N, IER)
    KK=MATMUL(MATMUL(VZETA,TRANPOSE(MK)),SKINV) ! KALMAN GAIN
    ZETA=ZETA+MATMUL(KK,YK-MATMUL(MK,ZETA))
    VZETA=VZETA-MATMUL(MATMUL(KK,MK),VZETA)
!
    IF (IRAN<=3) WRITE(6,'(100F18.6)') ZETA(:)

    IF (IRAN<=3) THEN
        WRITE(6,'(A,2I5)') 'OBSERVATION TIME STEP #' , K
        WRITE(6,*) '===== UPDATED STATE VECTOR, ZETA ====='
        WRITE(6,'(100F18.6)') ZETA(:)
        WRITE(6,*) '===== UPDATED COVARIANCE MATRIX OF ZETA ====='
        DO I=1,2*NT
            WRITE(6,'(100F12.6)') VZETA(I,:)
        END DO
    END IF
END DO

! ERROR DETERMINATION
DO I=1,N

    ELN_B1(I)=ELN_B1(I)+DLOG(ZETA(I)/BETA1(I))
    ELN_B0(I)=ELN_B0(I)+DLOG(ZETA(I+NT)/BETA0(I))
    VLN_B1(I)=VLN_B1(I)+(DLOG(ZETA(I)/BETA1(I)))**2
    VLN_B0(I)=VLN_B0(I)+(DLOG(ZETA(I+NT)/BETA0(I)))**2

    ERB1_P(I)=ERB1_P(I)+DABS((ZETA(I)-BETA1(I))*100/BETA1(I))
    ERB0_P(I)=ERB0_P(I)+DABS((ZETA(I+NT)-BETA0(I))*100/BETA0(I))
    BERB1_P(I)=BERB1_P(I)+(ZETA(I)-BETA1(I))*100/BETA1(I)
    BERB0_P(I)=BERB0_P(I)+(ZETA(I+NT)-BETA0(I))*100/BETA0(I)

    YF_EST=ZETA(I+NT)+ZETA(I)*DLOG10(T_EST)

    ELN_YF(I)=ELN_YF(I)+DLOG(YF_EST/YT_EST(I))
    VLN_YF(I)=VLN_YF(I)+(DLOG(YF_EST/YT_EST(I)))**2

    ERYF_P(I)=ERYF_P(I)+DABS((YF_EST-YT_EST(I))*100/YT_EST(I))
    BERYF_P(I)=BERYF_P(I)+(YF_EST-YT_EST(I))*100/YT_EST(I)
    SDERYF_P(I)=SDERYF_P(I)+((YF_EST-YT_EST(I))*100/YT_EST(I))**2

    IMI(I)=VZETA(I,I)/SDBETA1_0
    IMI(I+NT)=VZETA(I+NT,I+NT)/SDBETA0_0

```

```

END DO
!=====
IF (EST_ARBIT==1) THEN
  IF (GEN_SETT==1) THEN
    ERR_M1=DLOG(ZETA(NT)/BETA1_ARBIT)
    ERR_ELN_M1=ERR_ELN_M1+ERR_M1
    ERR_VLN_M1=ERR_VLN_M1+ERR_M1**2

    ERR_MO=DLOG(ZETA(2*NT)/BETA0_ARBIT)
    ERR_ELN_MO=ERR_ELN_MO+ERR_MO
    ERR_VLN_MO=ERR_VLN_MO+ERR_MO**2

    N_M1MO=N_M1MO+1

  END IF

  SETT_CURR=ZETA(2*NT)+ZETA(NT)*DLOG10(T(KT))
  SETT_FUT=ZETA(2*NT)+ZETA(NT)*DLOG10(T_EST)
  IF (YT_CURR_ARBIT== -999) THEN
    ERR_CURR= -999
  ELSE
    ERR_CURR=DLOG(SETT_CURR/YT_CURR_ARBIT)
    ERR_ELN_CURR=ERR_ELN_CURR+ERR_CURR
    ERR_VLN_CURR=ERR_VLN_CURR+ERR_CURR**2
    N_CURR=N_CURR+1
  END IF
  IF (YT_EST_ARBIT== -999) THEN
    ERR_FUT= -999
  ELSE
    ERR_FUT=DLOG(SETT_FUT/YT_EST_ARBIT)
    ERR_ELN_FUT=ERR_ELN_FUT+ERR_FUT
    ERR_VLN_FUT=ERR_VLN_FUT+ERR_FUT**2
    N_FUT=N_FUT+1
  END IF
  WRITE(7, '(A,100F30.18)')
  'B',CORX_ARBIT(1),CORX_ARBIT(2),ZETA(NT),ZETA(2*NT),SETT_CURR,SETT_FUT,ERR_M1,ERR_MO,ERR_CURR,ERR_FUT,ZA_ASS
UM
END IF
!=====
IF (KRIG==1) THEN
  WRITE(7,*) '----- ESTIMATION AT ARBIT POINTS BY KRIGING -----'
  WRITE(7, '(A11,3A30)') 'POINT#','BETA1','BETA0','ESTIMATED AUTO-COR. DIST.'
  DO IKRIG=1,NKRIG
    WRITE(7, '(A,110,4F30.18)') 'B',IKRIG,ZETA(N+IKRIG),ZETA(N+NT+IKRIG),ZA_ASSUM
  END DO
END IF

ELSEIF (KALMAN_INC/=0) THEN
  WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [KALMAN_INC] !!!!!!!!!!!!!!!'
  READ(*,*)
  STOP
END IF

```

! ***** KALMAN FILTER UPDATE, IGNORING SPATIAL CORRELATION *****

IF (KALMAN_IGN==1) THEN

MK_IG(1,2)=1.0D0

QK_IG=0.0D0

ZETA_I=0.0D0

VZETA_I=0.0D0

IF (IRAN<=3) WRITE(6,*) '===== KALMAN FILTER UPDATE, IGNORING SPATIAL CORRELATION

====='

DO I=1,N

ZETA_IG(1)=BETA1_0

ZETA_IG(2)=BETA0_0

VZETA_IG=0.0D0

VZETA_IG(1,1)=SDBETA1_0**2

VZETA_IG(2,2)=SDBETA0_0**2

DO K=1,KT

MK_IG(1,1)=DLOG10(T(K))

YK_IG(1)=Y(K,I)

IF (YK_IG(1)==-999) THEN

VE_IG(1,1)=SDE_MISS**2

ELSE

VE_IG(1,1)=SDE_ASSUM**2

END IF

!===== TIME UPDATING =====

IF (K>=2) THEN

VZETA_IG=(1.0D0+QP)*VZETA_IG

END IF

!===== OBSERVATION UPDATING =====

SK_IG=MATMUL(MATMUL(MK_IG,VZETA_IG),TRANSPOSE(MK_IG))+VE_IG

EPS=0.0D0

LMAT=1

CALL SVDEC(SK_IG, 1, 1, 1, EPS, LMAT, D_1, V_1, 1, NRANK, E_1, IER)

CALL GINV(SK_IG, 1, 1, 1, D_1, V_1, 1, NRANK, SKINV_IG, 1, IER)

KK_IG=MATMUL(MATMUL(VZETA_IG,TRANSPOSE(MK_IG)),SKINV_IG)

! KALMAN GAIN

ZETA_IG=ZETA_IG+MATMUL(KK_IG,YK_IG-MATMUL(MK_IG,ZETA_IG))

VZETA_IG=VZETA_IG-MATMUL(MATMUL(KK_IG,MK_IG),VZETA_IG)

ZETA_I(K,I)=ZETA_IG(1)

ZETA_I(K,I+N)=ZETA_IG(2)

VZETA_I(I,I)=VZETA_IG(1,1)

VZETA_I(I+N,I+N)=VZETA_IG(2,2)

END DO

! ERROR DETERMINATION

ELNI_B1(I)=ELNI_B1(I)+DLOG(ZETA_IG(1)/BETA1(I))

ELNI_B0(I)=ELNI_B0(I)+DLOG(ZETA_IG(2)/BETA0(I))

VLNI_B1(I)=VLNI_B1(I)+(DLOG(ZETA_IG(1)/BETA1(I)))**2

VLNI_B0(I)=VLNI_B0(I)+(DLOG(ZETA_IG(2)/BETA0(I)))**2

ERIB1_P(I)=ERIB1_P(I)+DABS((ZETA_IG(1)-BETA1(I))*100/BETA1(I))

ERIB0_P(I)=ERIB0_P(I)+DABS((ZETA_IG(2)-BETA0(I))*100/BETA0(I))

BERIB1_P(I)=BERIB1_P(I)+(ZETA_IG(1)-BETA1(I))*100/BETA1(I)

BERIB0_P(I)=BERIB0_P(I)+(ZETA_IG(2)-BETA0(I))*100/BETA0(I)

YF_EST=ZETA_IG(2)+ZETA_IG(1)*DLOG10(T_EST)

ELNI_YF(I)=ELNI_YF(I)+DLOG(YF_EST/YT_EST(I))

```

VLNI_YF(I)=VLNI_YF(I)+(DLOG(YF_EST/YT_EST(I)))**2

ERIFY_P(I)=ERIFY_P(I)+DABS((YF_EST-YT_EST(I))*100/YT_EST(I))
BERIFY_P(I)=BERIFY_P(I)+(YF_EST-YT_EST(I))*100/YT_EST(I)
SDERIFY_P(I)=SDERIFY_P(I)+((YF_EST-YT_EST(I))*100/YT_EST(I))**2

IMI_IG(I)=VZETA_IG(1,1)/SDBETA1_0
IMI_IG(I+N)=VZETA_IG(2,2)/SDBETA0_0

END DO
IF (IRAN<=3) THEN
DO K=1,KT
WRITE(6,'(100F18.6)') ZETA_I(K,:)
END DO
END IF
ELSEIF (KALMAN_IGN/=0) THEN
WRITE(*,*) '!!!!!!!!!!!!!! ERROR: INCORRECTLY INPUT VALUES OF [KALMAN_IGN] !!!!!!!!!!!!!!!'
READ(*,*)
STOP
END IF
!----- TOTAL RUNNING TIME APPROXIMATION -----
IF(IRAN==1)THEN
CALL SYSTEM_CLOCK (COUNT= IC)
TIMEI=(IC-IC_BEGIN)/(60*IRT)
TIMER=(REAL(IC)-REAL(IC_BEGIN))/(60*IRT)
WRITE(*, '(A,I6,A,F5.1,A)') 'RUNNING TIME FOR 1ST SAMPLING: ', TIMEI, ' MIN',(TIMER-TIMEI)*60,'
SEC'

TIMEI=(IC-IC_BEGIN)*NRAN/(60*IRT)
TIMER=(REAL(IC)-REAL(IC_BEGIN))*NRAN/(60*IRT)
WRITE(*, '(A,I6,A,F5.1,A)') 'TOTAL RUNNING TIME (APPROX.): ', TIMEI, ' MIN',(TIMER-TIMEI)*60,'
SEC'

END IF
WRITE(*, '(A,I6,A,I6)') 'RUNNING PROGRESS: ', IRAN , ' / ' , NRAN
END DO

!===== OUTPUT CALCULATION =====
ELN_B1_T=0.0D0
ELN_B0_T=0.0D0
VLN_B1_T=0.0D0
VLN_B0_T=0.0D0
ELN_YF_T=0.0D0
VLN_YF_T=0.0D0

ELNI_B1_T=0.0D0
ELNI_B0_T=0.0D0
VLNI_B1_T=0.0D0
VLNI_B0_T=0.0D0
ELNI_YF_T=0.0D0
VLNI_YF_T=0.0D0

ERB1_PT=0.0D0
ERB0_PT=0.0D0
MERB1_PT=0.0D0
MERB0_PT=0.0D0
BERB1_PT=0.0D0

```

```

BERB0_PT=0.0D0
SDERB1_PT=0.0D0
SDERB0_PT=0.0D0
ERIB1_PT=0.0D0
ERIB0_PT=0.0D0
MERIB1_PT=0.0D0
MERIB0_PT=0.0D0
BERIB1_PT=0.0D0
BERIB0_PT=0.0D0
SDERIB1_PT=0.0D0
SDERIB0_PT=0.0D0

```

```

ERYF_PT=0.0D0
MERYF_PT=0.0D0
BERYF_PT=0.0D0
SDERYF_PT=0.0D0
ERIYF_PT=0.0D0
MERIYF_PT=0.0D0
BERIYF_PT=0.0D0
SDERIYF_PT=0.0D0
NT_EST=0

```

```

IM=0.0D0
IM_IG=0.0D0

```

```

DO I=1,N

```

```

    ELN_B1_T=ELN_B1_T+ELN_B1(I)
    ELN_B0_T=ELN_B0_T+ELN_B0(I)
    VLN_B1_T=VLN_B1_T+VLN_B1(I)
    VLN_B0_T=VLN_B0_T+VLN_B0(I)

```

```

    ELN_B1(I)=ELN_B1(I)/NRAN
    ELN_B0(I)=ELN_B0(I)/NRAN
    VLN_B1(I)=(VLN_B1(I)-NRAN*(ELN_B1(I)**2))/(NRAN-1)
    VLN_B0(I)=(VLN_B0(I)-NRAN*(ELN_B0(I)**2))/(NRAN-1)

```

```

    ERB1_PT=ERB1_PT+ERB1_P(I)
    ERB0_PT=ERB0_PT+ERB0_P(I)
    BERB1_PT=BERB1_PT+BERB1_P(I)
    BERB0_PT=BERB0_PT+BERB0_P(I)

```

```

    ELNI_B1_T=ELNI_B1_T+ELNI_B1(I)
    ELNI_B0_T=ELNI_B0_T+ELNI_B0(I)
    VLNI_B1_T=VLNI_B1_T+VLNI_B1(I)
    VLNI_B0_T=VLNI_B0_T+VLNI_B0(I)

```

```

    ELNI_B1(I)=ELNI_B1(I)/NRAN
    ELNI_B0(I)=ELNI_B0(I)/NRAN
    VLNI_B1(I)=(VLNI_B1(I)-NRAN*(ELNI_B1(I)**2))/(NRAN-1)
    VLNI_B0(I)=(VLNI_B0(I)-NRAN*(ELNI_B0(I)**2))/(NRAN-1)

```

```

    ERIB1_PT=ERIB1_PT+ERIB1_P(I)
    ERIB0_PT=ERIB0_PT+ERIB0_P(I)
    BERIB1_PT=BERIB1_PT+BERIB1_P(I)
    BERIB0_PT=BERIB0_PT+BERIB0_P(I)

```



```

ERB1_P(I)=ERB1_P(I)/NRAN
ERB0_P(I)=ERB0_P(I)/NRAN
ERIB1_P(I)=ERIB1_P(I)/NRAN
ERIB0_P(I)=ERIB0_P(I)/NRAN
IF (YT_EST(I)/=-999) THEN
    NT_EST=NT_EST+1

    ELN_YF_T=ELN_YF_T+ELN_YF(I)
    VLN_YF_T=VLN_YF_T+VLN_YF(I)
    ELNI_YF_T=ELNI_YF_T+ELNI_YF(I)
    VLNI_YF_T=VLNI_YF_T+VLNI_YF(I)

    ELN_YF(I)=ELN_YF(I)/NRAN
    VLN_YF(I)=(VLN_YF(I)-NRAN*(ELN_YF(I)**2))/(NRAN-1)

    ELNI_YF(I)=ELNI_YF(I)/NRAN
    VLNI_YF(I)=(VLNI_YF(I)-NRAN*(ELNI_YF(I)**2))/(NRAN-1)

    ERYF_PT=ERYF_PT+ERYF_P(I)
    BERYF_PT=BERYF_PT+BERYF_P(I)
    SDERYF_PT=SDERYF_PT+SDERYF_P(I)
    ERIYF_PT=ERIYF_PT+ERIYF_P(I)
    BERIYF_PT=BERIYF_PT+BERIYF_P(I)
    SDERIYF_PT=SDERIYF_PT+SDERIYF_P(I)
END IF
ERYF_P(I)=ERYF_P(I)/NRAN
SDERYF_P(I)=DSQRT((SDERYF_P(I)-NRAN*ERYF_P(I)**2)/(NRAN)) ! SHOULD BE DIVIDED BY NRAN-1 INSTEAD
ERIYF_P(I)=ERIYF_P(I)/NRAN
SDERIYF_P(I)=DSQRT((SDERIYF_P(I)-NRAN*ERIYF_P(I)**2)/(NRAN)) ! SHOULD BE DIVIDED BY NRAN-1 INSTEAD

IM=IM+DLOG(IMI(I))+DLOG(IMI(I+N))
IM_IG=IM_IG+DLOG(IMI_IG(I))+DLOG(IMI_IG(I+N))

END DO

ELN_B1_T=ELN_B1_T/(NRAN*N)
ELN_B0_T=ELN_B0_T/(NRAN*N)

ERB1_PT=ERB1_PT/(NRAN*N)
ERB0_PT=ERB0_PT/(NRAN*N)

VLN_B1_T=(VLN_B1_T-(NRAN*N)*ELN_B1_T**2)/(NRAN*N-1)
VLN_B0_T=(VLN_B0_T-(NRAN*N)*ELN_B0_T**2)/(NRAN*N-1)

BERB1_PT=BERB1_PT/(NRAN*N)
BERB0_PT=BERB0_PT/(NRAN*N)

ELNI_B1_T=ELNI_B1_T/(NRAN*N)
ELNI_B0_T=ELNI_B0_T/(NRAN*N)

ERIB1_PT=ERIB1_PT/(NRAN*N)
ERIB0_PT=ERIB0_PT/(NRAN*N)

VLNI_B1_T=(VLNI_B1_T-(NRAN*N)*ELNI_B1_T**2)/(NRAN*N-1)

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```

VLNI_BO_T=(VLNI_BO_T-(NRAN*N)*ELNI_BO_T**2)/(NRAN*N-1)

BERIB1_PT=BERIB1_PT/(NRAN*N)
BERIBO_PT=BERIBO_PT/(NRAN*N)

ELN_YF_T=ELN_YF_T/(NRAN*NT_EST)
VLN_YF_T=(VLN_YF_T-(NRAN*NT_EST)*ELN_YF_T**2)/(NRAN*NT_EST-1)
ELNI_YF_T=ELNI_YF_T/(NRAN*NT_EST)
VLNI_YF_T=(VLNI_YF_T-(NRAN*NT_EST)*ELNI_YF_T**2)/(NRAN*NT_EST-1)

ERYF_PT=ERYF_PT/(NRAN*NT_EST)
MERYF_PT=SQRT(SDERYF_PT/(NRAN*NT_EST))
BERYF_PT=BERYF_PT/(NRAN*NT_EST)
SDERYF_PT=DSQRT((SDERYF_PT-(NRAN*NT_EST)*ERYF_PT**2)/(NRAN*NT_EST-1))
ERIYF_PT=ERIYF_PT/(NRAN*NT_EST)
MERIYF_PT=SQRT(SDERIYF_PT/(NRAN*NT_EST))
BERIYF_PT=BERIYF_PT/(NRAN*NT_EST)
SDERIYF_PT=DSQRT((SDERIYF_PT-(NRAN*NT_EST)*ERIYF_PT**2)/(NRAN*NT_EST-1))

IM=DEXP(IM/(2*N))
IM_IG=DEXP(IM_IG/(2*N))

WRITE(6,*) ' '
WRITE(6,*) '===== ESTIMATION ERROR AT THE LAST TIME STEP ====='
! (CONSID) REPRESENTS THE CASE OF CONSIDERING AUTO-CORRELATION DISTANCE
! (IGNOR) REPRESENTS THE CASE OF IGNORING AUTO-CORRELATION DISTANCE
WRITE(6,*) '----- MEAN & VARIANCE OF ERROR RATIO (ER) -----'
WRITE(6, '(A25,4A36)') 'PARAMETERS', 'MEAN(CONSID)', 'MEAN(IGNOR)', 'VARIANCE(CONSID)',
'VARIANCE(IGNOR)'
WRITE(6, '(A25,100F36.18)') 'M1', ELN_B1_T, ELNI_B1_T, VLN_B1_T, VLNI_B1_T
WRITE(6, '(A25,100F36.18)') 'M0', ELN_BO_T, ELNI_BO_T, VLN_BO_T, VLNI_BO_T
WRITE(6, '(A25,200F36.18)') 'SETTLEMENT PREDICTION', ELN_YF_T, ELNI_YF_T, VLN_YF_T, VLNI_YF_T

WRITE(6,*) '----- PERCENT ERROR -----'
WRITE(6, '(A25,4A36)') 'PARAMETERS', 'MEAN(CONSID)', 'MEAN(IGNOR)', 'BIAS(CONSID)', 'BIAS(IGNOR)'
WRITE(6, '(A25,100F36.18)') 'M1', ERB1_PT, ERIB1_PT, BERB1_PT, BERIB1_PT
WRITE(6, '(A25,100F36.18)') 'M0', ERBO_PT, ERIBO_PT, BERBO_PT, BERIBO_PT
WRITE(6, '(A25,200F36.18)') 'SETTLEMENT PREDICTION', ERYF_PT, ERIYF_PT, BERYF_PT, BERIYF_PT

! WRITE(6, '(A20,200F36.18)') 'YF-SD(D)', SDERYF_P, SDERYF_PT
! WRITE(6, '(A20,200F36.18)') 'YF-SD(I)', SDERIYF_P, SDERIYF_PT

END DO
IF (EST_ARBIT==1) THEN

ERR_ELN_M1=ERR_ELN_M1/N_M1MO
ERR_VLN_M1=(ERR_VLN_M1-N_M1MO*ERR_ELN_M1**2)/(N_M1MO-1)
ERR_ELN_MO=ERR_ELN_MO/N_M1MO
ERR_VLN_MO=(ERR_VLN_MO-N_M1MO*ERR_ELN_MO**2)/(N_M1MO-1)

ERR_ELN_CURR=ERR_ELN_CURR/N_CURR
ERR_VLN_CURR=(ERR_VLN_CURR-N_CURR*ERR_ELN_CURR**2)/(N_CURR-1)
ERR_ELN_FUT=ERR_ELN_FUT/N_FUT
ERR_VLN_FUT=(ERR_VLN_FUT-N_FUT*ERR_ELN_FUT**2)/(N_FUT-1)

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```

WRITE(7,*) '----- MEAN OF ERROR RATIO -----'
WRITE(7, '(4A30)') 'M1', 'M0', 'CURRENT SETT.', 'FUTURE SETT.'
WRITE(7, '(100F30.18)') ERR_ELN_M1, ERR_ELN_M0, ERR_ELN_CURR, ERR_ELN_FUT
WRITE(7,*) '----- VARIANCE OF ERROR RATIO -----'
WRITE(7, '(4A30)') 'M1', 'M0', 'CURRENT SETT.', 'FUTURE SETT.'
WRITE(7, '(100F30.18)') ERR_VLN_M1, ERR_VLN_M0, ERR_VLN_CURR, ERR_VLN_FUT
END IF

      CALL SYSTEM_CLOCK (COUNT= IC)
      TIMEI=(IC-IC_BEGIN)/(60*IRT)
      TIMER=(REAL(IC)-REAL(IC_BEGIN))/(60*IRT)
      WRITE(*, '(A,I6,A,F5.1,A)') 'TOTAL RUNNING TIME:  ', TIMEI, ' MIN', (TIMER-TIMEI)*60, ' SEC'
      WRITE(*,*) 'END PROGRAM'
      READ(*,*)

! *****
      CLOSE(5)
      CLOSE(6)
      CLOSE(7)

END PROGRAM STSETT1_6_2

```