Methods of S-parameter estimation for multiport circuits with indirect measurements

メタデータ	言語: eng
	出版者:
	公開日: 2015-06-26
	キーワード (Ja):
	キーワード (En):
	作成者: 前田, 登
	メールアドレス:
	所属:
URL	http://hdl.handle.net/20.500.12099/51035

DOCTORAL DISSERTATION

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March, 2015

GIFU UNIVERSITY

Electronics and Information Systems Engineering Division
Graduate School of Engineering
Gifu University
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Noboru Maeda

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by

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Submitted in partial satisfaction of the requirements for
the degree of Doctor of Philosophy
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Abstract

This thesis describes essentially two approaches to estimate the complete S-parameters of multiport reciprocal circuits with measuring some part of the ports and connecting known loads to the remaining ports:

1. An estimation method of the n-port S-parameters for reciprocal circuits with $n-1$ port measurements is presented. In the estimation procedure, all circuit equations are decomposed and combined to form a linear equation for all S-parameters, then, simultaneously solved to obtain the whole solution at once. In this method, several known loads are connected to one port in turn and reflection and transmission characteristics among the remaining ports are measured. Therefore, there is no need to connect a network analyzer to the port that is connected to the known loads. S-parameters are obtained by solving a linear least-squares equation and a quadratic equation only. Validity of this method is confirmed by applying it to estimate the S-parameters of an immunity test system. Also, as an extended variation of this method, an estimation method of the 3-port S-parameters with 1-port measurement is also presented to show its potential and limitation when applied for arbitrary configuration of the whole ports and directly measured ports.

2. Also, an estimation method of the $2r$ port S-parameters for reciprocal circuits with r port measurements is presented. To obtain the analytical solutions in the framework of linear algebra, a novel method is developed to construct equations in terms of single submatrix of the S-matrix with eliminating other submatrices and solve them in successive manner. This method can be typically applied to estimate the S-parameters of both a multiport connection and

a multiport device as follows. The multiport device is connected to one end of the multiport connection and a network analyzer is connected to the other end. Here, the two ends of the multiport connection are assumed to have different ground levels. Only measurements of reflection and transmission characteristics among the ports of the measured end of the connection are required for the estimation. Both S-parameters for the multiport connection and the multiport device are estimated by solving several linear equations and quadratic equations only. The validity of the proposed method is also confirmed by several experimental and simulated examples.

Keywords: S-parameter, multi-port circuit, immunity test, automotive components, high-voltage unit

Declaration

The work in this thesis is based on research carried out at the Toshikazu Sekine & Yasuhiro Takahashi Laboratory, Electronics and Information Systems Engineering Division, Graduate School of Engineering, Gifu University, Japan. No part of this thesis has been submitted elsewhere for any other degree or qualification and they are my own work unless referenced to the contrary in the text.

Acknowledgements

I would like to thank my supervisor Associate Prof. Toshikazu Sekine for his warm and experienced supervision and insightful discussion during my study at his lab in Gifu University.

I also would like to thank the examiners of my thesis for sparing their precious time to evaluate my study and providing valuable comments and suggestions.

Lastly, I thank Assistant Prof. Yasuhiro Takahashi and all the members of the analog/digital system LSI laboratory (also known as Sekine & Takahashi laboratory) in Gifu University.

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Chapter 1
Introduction

1.1 Background

Theoretical categories of S-parameter estimation methods

S-parameter estimation without direct measurements for all ports of the target circuit has been attracting interest of many researchers from diverse viewpoints. In such estimation, the target circuits are assumed to have some 'hard-to-access' ports which have difficulties for some reasons to be connected with measurement probes to obtain accurate directly measured characteristics. For those ports, some known loads or circuits are connected while the measurements are performed using the 'easy-to-access' ports. In this thesis, we call the measurements for such 'hard-to-access' ports as 'indirect measurements' since the whole S-parameters of the target circuits including those ports are estimated without directly connecting the measurement probes to those ports. Generally, arbitrary combination of direct measurement ports and indirect measurement ports can be exist. However, conventional theoretical view point, two categories have been of particular interest, namely, 'direct measurement for all but one ports' and 'direct measurement for half of the ports.'

For the first category, a method is proposed to estimate the 2-port S-parameter using complex-valued least-squares by connecting three known loads one at a time to one of the ports and measuring the reflection of remaining one port [1]. Another method is proposed to estimate the 3-port S-parameters with direct measurement for 2 ports using equation with rational form of equation [2]. For the second category, a method is proposed to estimate the 4-port S-parameters with direct measurement for 2 ports of the target with a large ground plane using the rational form of equation [3]. Another method is also proposed for the same configuration of ports using numerical solution of nonlinear equation [4].

In any conventional methods, the fundamental estimation equation is obtained essentially as an identical matrix quadratic equation in terms of the submatrices of the whole S-matrix. Then, the equation is decomposed and transformed into a set of linear equations or it is solved unmodified as a nonlinear equation according to the methods. In this thesis, the conventional fundamental nonlinear estimation equation is also obtained and two linearization methods are proposed and investigated corresponding to the two categories shown above. In the first method, all equation are decomposed, recombined into a linear equation, then its size is increased to solvable rank using extra measurements and simultaneously solved to obtain all S-parameters at once. This method is applied for the estimation of n-port S-parameters with 1 hard-to-access port and 3-port S-parameters with 2 hard-to-access ports. This can be considered as a generalization of [1].

In the second method which is rather unique, the submatrices are eliminated with increasing the number of equations using extra measurements to obtain a matrix quadratic equation interms of one submatrix. Then, it is transformed into a linear equation and solved. This process is repeated to solve each submatrix in successive manner. This method is applied for the estimation of 2r-port S-parameters with r hard-to-access ports. This method is developed to provide straightforward theoretical analysis in the framework of linear algebra since applying the first method to these targets requires extensive equation decomposition operation and the operation is differed according to the number of directly and indirectly measured ports which means the operation itself is required to be created using an operation construction algorithm and makes it hard to apply theoretical analysis. Also, a supplementary method is provided to improve the estimation accuracy, which can be applied to this target without extra calculation or measurement costs for the S-parameters in case of 4-port connection circuits.

Applications of indirect measurements

The needs for the indirect measurements arises in many situations typically when the target circuit is not designed to be measured with ordinary high-frequency measurement instruments, especially when it is too small or it is not for high-frequency applications which occurs often in Electromagnetic Compatibility (EMC) analysis. In such situations, to measure the target S-parameter precisely, extra connection circuit should be inserted between the target circuit and the measurement probes and the characteristics of the connection circuit must be eliminated (de-embedded) in later calculations. In another situation, we do not have common ground between the target circuit and the measurement instrument as usually expected for high-frequency circuits. Sometimes the target circuit is enclosed in a shield case which just make it physically difficult to connect measurement probes.

To obtain the S-parameters for the target circuit, network analyzers can be used. If the number of ports is not large, namely less than four, direct measurement is possible and if the number of ports is large, several methods to measure them using two or four port network analyzers have been proposed [5] [6] [7]. However, in these measurement methods the analyzer's ports are assumed to be connected to the ports of the test equipment with direct cabling.

For the target circuits with hard-to-access ports, there is a proposed method to measure the 2-port Z-parameter using 1-port measurement with the hard-to-access port being open/short assuming the application to measurement of LSI package characteristics [8].

One typical indirect measurement needs reside in the measurement for the equipments on board in vehicles. Usually many sensors are connected to them with bundles of wires to supply required information. Since these systems are now indispensable to automobile operations, electro-magnetic noise generated by on-vehicle actuators such as hybrid systems or coming from external noise sources can be a significant harm when it interferes with Electronic Control Units (ECUs) operation. To estimate the high frequency response of these vehicle-mounted devices, it is effective to analyze them using their S-parameters. However, measuring their on-board S-parameters directly by connecting measurement probes are often difficult since they are enclosed within chassis thus, physically inaccessible or even if they are accessible, connecting the measurement probes sometimes changes their high frequency environment to prevent obtaining the actual S-parameters in operating conditions. The last situation often arises from the difference of the device ground and the global system ground that is also the ground of the measurement instrument in the actual field. Because the ground potentials of all measuring probes of the instrument are usually the same, ground potentials of the ports of the device being measured then become the same forcibly to the global system ground when the probes are connected to those ports.

To avoid the difficulties described above, it is effective to measure the device characteristics through the ordinary wires on-board and de-embed the transfer characteristics of the wires afterward. To obtain the S-parameters of such targets, several estimation methods have been developed. In the field of microwave circuit design, de-embedding the effect of multiport discontinuities or local ground is often performed using high frequency CAD systems [15] [17]. As for the de-embedding using the actual measured characteristics of the connection part is shown in [16]. Also, [18] provides a detailed discussion on de-embedding several errors caused by the multiport connection between the measurement system and the DUT.

In this thesis, we present some example system taken from vehicle applications but the theory of indirect measurement applied to those examples can also be applicable to any reciprocal circuits.

1.2 Organization of the Dissertation

This thesis is organized as follows:

- *chapter 1*: Introduction.
- *chapter 2*: In this chapter, the estimation method for n port estimation with $n-1$ port measurements is presented. For this purpose we generalized and applied an S-parameter estimation method using equation decomposition and simultaneous solution.
- *chapter 3*: This chapter presents an estimation method for 3-port circuits having 2 indirectly measured ports. This was achieved by modifying the method proposed in the previous chapter to show its potential and limitation when applying for arbitrary configuration of directly and indirectly measured ports.
- *chapter 4*: $2r$ port S-parameter estimation with r port measurements is presented. Its typical application, the S-parameter estimation of both a multiport connection and a multiport device is described. To obtain a general solution in the range of linear algebra, we developed a novel S-parameter estimation method using submatrix elimination and successive solution.
- *chapter 5*: An improved method for the 4 port S-parameter estimation with 2 port measurements is presented. The estimation method in the previous chapter is modified to improve its accuracy exclusively for 4 port circuits. The result is confirmed using a typical application to the S-parameters of High Voltage (HV) units of electric vehicles.
- *chapter 6*: Conclusions and future work.

Chapter 2

Estimation of n port S-parameters with $n-1$ port measurements

An estimation method of the n-port S-parameters for reciprocal circuits with $n-1$ port measurements is presented. In the estimation procedure, all circuit equations are decomposed and combined to form a linear equation for all S-parameters, then, simultaneously solved to obtain the whole solution at once. In this method, several known loads are connected to one port in turn and reflection and transmission characteristics among the remaining ports are measured. Therefore, there is no need to connect a network analyzer to the port that is connected to the known loads. S-parameters are obtained by solving a linear least-squares equation and a quadratic equation only. Validity of this method is confirmed by applying it to estimate the S-parameters of an immunity test system.

2.1 Estimation Method

2.1.1 Derivation of the Estimating Equations

A reciprocal n port circuit shown in Fig.2.1 has an n-th port that is hard to access with a measurement probe. We measure the S-parameters of remaining n-1 ports by attaching a load to port n and estimate the whole n port S-parameters from the measured n-1 port S-parameters. The S-parameter matrix $\mathbf{S}$ can be written in block matrix form as follows where suffix a indicates the n-1 ports that are easily accessed with measurement probes and suffix u indicates the port that is hard to access with

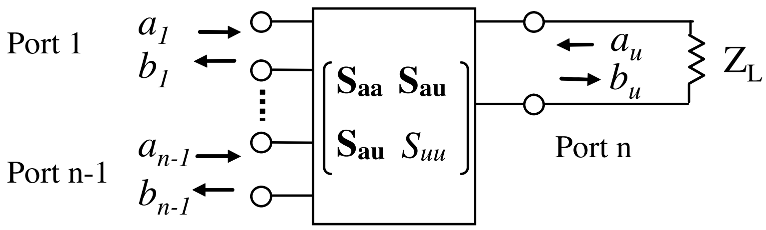


Figure 2.1: S-parameter measurement arrangement

a measurement probe,

$$\mathbf{S} = \begin{bmatrix} \mathbf{S_{aa}} & \mathbf{S_{au}} \\ \mathbf{S_{au}^T} & S_{uu} \end{bmatrix}. \tag{2.1}$$

Incident wave $\mathbf{a} = \left[\mathbf{a_a}^T \;\; a_u\right]^T$ and reflected wave $\mathbf{b} = \left[\mathbf{b_a}^T \;\; b_u\right]^T$ satisfies:

$$\begin{bmatrix} \mathbf{b_a} \\ b_u \end{bmatrix} = \begin{bmatrix} \mathbf{S_{aa}} & \mathbf{S_{au}} \\ \mathbf{S_{au}^T} & S_{uu} \end{bmatrix} \begin{bmatrix} \mathbf{a_a} \\ a_u \end{bmatrix}, \tag{2.2}$$

where

$$\mathbf{S_{aa}} \in \mathbb{C}^{(\mathrm{n}-1)\times(\mathrm{n}-1)},\; \mathbf{S_{au}} \in \mathbb{C}^{(\mathrm{n}-1)\times 1},\; S_{uu} \in \mathbb{C},$$
$$\mathbf{a_a}, \mathbf{b_a} \in \mathbb{C}^{n-1},\; a_u, b_u \in \mathbb{C}.$$

In this chapter, we only analyze those circuits with $\mathbf{S_{au}} \neq \mathbf{0}$, that is, with port n connected with other ports. When we connect a load with reflection parameter S_L to port n, the following holds:

$$a_u = S_L b_u. \tag{2.3}$$

Denoting the measured n-1 port S-parameters as $\hat{S}$, it follows that:

$$\mathbf{b_a} = \mathbf{\hat{S}} \mathbf{a_a}. \tag{2.4}$$

Rewriting (2.2),

$$\mathbf{b_a} = \mathbf{S_{aa}} \mathbf{a_a} + \mathbf{S_{au}} a_u \tag{2.5}$$
$$b_u = \mathbf{S_{au}^T} \mathbf{a_a} + S_{uu} a_u \tag{2.6}$$

holds.

From the above four equations (2.3) to (2.6), we have:

$$\mathbf{S_{aa}} + \mathbf{S_{au}} S_L \left(1 - S_L S_{uu}\right)^{-1} \mathbf{S_{au}^T} = \mathbf{\hat{S}}. \tag{2.7}$$

This is the fundamental nonlinear estimation equation for the S-parameters.

To linearize the estimation equation, multiply $(1 - S_L S_{uu})$ to the both sides and using the fact that it is a scalar, thus, commutative, we have:

$$\mathbf{S_{au}} S_L \mathbf{S_{au}^T} + (1 - S_L S_{uu})\, \mathbf{S_{aa}} + S_L S_{uu} \hat{\mathbf{S}} = \hat{\mathbf{S}}. \tag{2.8}$$

Rewriting this in block matrix form to separate the n-port S-parameters, we obtain:

$$\begin{bmatrix} \mathbf{I}_{(n-1)} & S_L \mathbf{I}_{(n-1)} & S_L \hat{\mathbf{S}} \end{bmatrix} \begin{bmatrix} \mathbf{S_{aa}} \\ \mathbf{S_{au}} \mathbf{S_{au}^T} - S_{uu} \mathbf{S_{aa}} \\ S_{uu} \mathbf{I}_{(n-1)} \end{bmatrix} = \hat{\mathbf{S}}, \tag{2.9}$$

where $\mathbf{I}_{(n-1)}$ denotes the identity matrix of dimension $n - 1$.

Furthermore, to write the equation in terms of an unknown vector consisting of the n-port S-parameters, we use vec$(\cdot)$ operator [11] [12] which denote a matrix to vector mapping by vertically concatenating the column vectors of the matrix and indicate the vectors obtained by the mapping with suffix v. The rewritten form of the equation (2.9) is:

$$\begin{bmatrix} \mathbf{I}_{(n-1)^2} & S_L \mathbf{I}_{(n-1)^2} & S_L \hat{\mathbf{S}}_\mathbf{v} \end{bmatrix} \begin{bmatrix} \mathbf{S_{aav}} \\ \mathrm{vec}\big(\mathbf{S_{au}} \mathbf{S_{au}^T}\big) - S_{uu} \mathbf{S_{aav}} \\ S_{uu} \end{bmatrix} = \hat{\mathbf{S}}_\mathbf{v} \tag{2.10}$$

where $\mathbf{I}_{(n-1)^2}$ denotes the identity matrix of dimension $(n-1)^2$. This is the basic equation to estimate the n port S-parameters with n-1 port measurement. Now, denoting

$$\mathbf{m} = \begin{bmatrix} \mathbf{S_{aav}} \\ \mathrm{vec}\big(\mathbf{S_{au}} \mathbf{S_{au}^T}\big) - S_{uu} \mathbf{S_{aav}} \\ S_{uu} \end{bmatrix}, \tag{2.11}$$

equation (2.10) becomes a linear estimating equation for $\mathbf{m}$ with the values of $\begin{bmatrix} \mathbf{I}_{(n-1)^2} & S_L \mathbf{I}_{(n-1)^2} & S_L \hat{\mathbf{S}}_\mathbf{v} \end{bmatrix}$ given where $\hat{\mathbf{S}}_\mathbf{v}$ is obtained by measurements. Moreover, denoting $\mathbf{m} = \begin{bmatrix}\mathbf{m_1}^{\mathrm{T}} \; \mathbf{m_2}^{\mathrm{T}} \; m_3\end{bmatrix}^{\mathrm{T}}$ where $\mathbf{m_1}, \mathbf{m_2} \in \mathbb{C}^{(\mathrm{n}-1)^2 \times 1}$, $m_3 \in \mathbb{C}$, $\mathbf{m}_2$ can be seen as an intermediate parameter to solve $\mathbf{S_{au}}$ with linear equations.

Once the equation for $\mathbf{m}$ is solved, as seen in (2.11), all components of $\mathbf{S_{aa}}$ and S_{uu} can be directly obtained from $\mathbf{m}_1$ and m_3 respectively. Using them and the solution for $\mathbf{m}_2$, we can obtain the value of $\mathbf{S_{au}} \mathbf{S_{au}^T}$ from

$$\mathbf{m_2} = \mathrm{vec}\left(\mathbf{S_{au}} \mathbf{S_{au}^T}\right) - S_{uu} \mathbf{S_{aav}}$$

This consists of $(n-1)^2$ quadratic equations of self-multiplication of whole $(n-1)$ elements of $\mathbf{S_{au}}$. Using the $(n-1)$ equations corresponding to the first row of $\mathbf{S_{au}}\mathbf{S_{au}^T}$ for example, we can obtain all the elements of $\mathbf{S_{au}}$. The quadratic equations provide two sets of solutions with opposite signs. This comes from the fact that even if a $1:-1$ transformer is attached to port n, it is unobservable from other ports, that is, $\mathbf{S_{aa}}$, S_{uu} are not affected by inverting the polarity of the port n. This ambiguity in signs of $\mathbf{S_{au}}$ elements is inherent one which is similar to the admittance or impedance parameter sign ambiguity stated in [13]. When calculating the noise voltage using the S-parameters, this sign ambiguity does not affect the absolute values of voltage calculation results as proved later in section 2.2.2.

2.1.2 Conditioning the estimating equation to have a unique solution

The estimating equation (2.10) consists of $(n-1)^2$ equations and $2(n-1)^2+1$ unknowns. Therefore, it does not have unique solution as it is. So, we try to increase the number of equations by changing the value of load impedance S_L and measuring $\hat{\mathbf{S}}$ several times.

Assuming the n-1 port S-parameter is measured by using three distinct values for S_L. Denoting S_L and $\hat{\mathbf{S}}$ matrix obtained in the i-th measurement with suffix (i). Then, from equation (2.10),

$$\begin{bmatrix} \mathbf{I}_{(n-1)^2} & S_L^{(1)}\mathbf{I}_{(n-1)^2} & S_L^{(1)}\hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{1})} \\ \mathbf{I}_{(n-1)^2} & S_L^{(2)}\mathbf{I}_{(n-1)^2} & S_L^{(2)}\hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{2})} \\ \mathbf{I}_{(n-1)^2} & S_L^{(3)}\mathbf{I}_{(n-1)^2} & S_L^{(3)}\hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{3})} \end{bmatrix} \begin{bmatrix} \mathbf{S_{aav}} \\ \mathrm{vec}\left(\mathbf{S_{au}}\mathbf{S_{au}^T}\right) - S_{uu}\mathbf{S_{aav}} \\ S_{uu} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{1})} \\ \hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{2})} \\ \hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{3})} \end{bmatrix}. \tag{2.12}$$

Following, it is shown that measuring all the components of $\hat{\mathbf{S}}$ three times with changing the value of S_L is sufficient to estimate the components of the $\mathbf{S}$ matrix.

The structure of a subblock:
$$\begin{bmatrix} \mathbf{I}_{(n-1)^2} & S_L^{(1)}\mathbf{I}_{(n-1)^2} \\ \mathbf{I}_{(n-1)^2} & S_L^{(2)}\mathbf{I}_{(n-1)^2} \end{bmatrix}$$
of the coefficient matrix in (2.12) shows that its all row vectors are linearly independent if $S_L^{(1)} \neq S_L^{(2)}$ and in this case, all the row vectors in the upper two row blocks:
$$\begin{bmatrix} \mathbf{I}_{(n-1)^2} & S_L^{(1)}\mathbf{I}_{(n-1)^2} & S_L^{(1)}\hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{1})} \\ \mathbf{I}_{(n-1)^2} & S_L^{(2)}\mathbf{I}_{(n-1)^2} & S_L^{(2)}\hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{2})} \end{bmatrix}$$
are guaranteed to be independent.

Now, consider to make the coefficient matrix to be square by eliminating n-2 rows from the 3rd row block:

$\begin{bmatrix} \mathbf{I}_{(n-1)^2} & S_L^{(3)}\mathbf{I}_{(n-1)^2} & S_L^{(3)}\hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{3})} \end{bmatrix}$.

If we assume the 1st row concerning $\hat{S}_{11}^{(3)}$

$\begin{bmatrix} 1 & 0\cdots0 & S_L^{(3)} & 0\cdots0 & S_L^{(3)}\hat{S}_{11}^{(3)} \end{bmatrix}$

remains, from equation (2.8),

$$\mathbf{S_{au}}S_L\mathbf{S_{au}^T} + (1 - S_L S_{uu})\,\mathbf{S_{aa}} = (1 - S_L S_{uu})\,\hat{\mathbf{S}}. \tag{2.13}$$

Here, if we assume $S_L S_{uu} = 1$, then,

$$\mathbf{S_{au}}\mathbf{S_{au}^T} = 0 \tag{2.14}$$

holds, which contradicts the assumption of $\mathbf{S_{au}} \neq \mathbf{0}$. So, $S_L S_{uu} \neq 1$. Then, from equation (2.13)

$$\hat{\mathbf{S}} = \frac{S_L}{1 - S_L S_{uu}}\mathbf{S_{au}}\mathbf{S_{au}^T} + \mathbf{S_{aa}}. \tag{2.15}$$

Therefore, $\hat{\mathbf{S}}$ matrices for different S_L values have mutually distinct corresponding elements.

So, $\hat{S}_{11}^{(3)} \neq \hat{S}_{11}^{(2)}$ and $\hat{S}_{11}^{(3)} \neq \hat{S}_{11}^{(1)}$.

Then, coefficient matrix:

$$\begin{bmatrix} \mathbf{I}_{(n-1)^2} & S_L^{(1)}\mathbf{I}_{(n-1)^2} & S_L^{(1)}\hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{1})} \\ \mathbf{I}_{(n-1)^2} & S_L^{(2)}\mathbf{I}_{(n-1)^2} & S_L^{(2)}\hat{\mathbf{S}}_{\mathbf{v}}^{(\mathbf{2})} \\ 1\,0\cdots0 & S_L^{(3)}\,0\cdots0 & S_L^{(3)}\hat{S}_{11}^{(3)} \end{bmatrix}$$

has its all row vectors linearly independent.

Therefore, the coefficient matrix in (2.12) includes the linearly independent row vectors with same number of the unknowns, that is, its rank equals to the number of the unknowns. Which means by applying least-squares estimation method, a unique solution for the unknowns can be obtained. As a result, it can be said that three load values for port n are necessary and sufficient to estimate the values of vector $\mathbf{m}$, and thus all the components in the $\mathbf{S}$ matrix. When $n = 2$ and $n = 3$, the equation (2.12) reduces to the corresponding linear least-squares estimation equation in [1] and [10], respectively.

As an overview, while the target unknown parameters might be calculated by solving n^2 nonlinear equations since they are the n^2 elements in $\mathbf{S}$ matrix, it can be considered that the number of equations increased to $3\,(n-1)^2$ in order to linearizing the main part of the problem by introducing intermediate parameters to the elements of $\mathbf{m}_2$ in the unknown vector $\mathbf{m}$ in equation (2.10).

2.2 Port Voltages

2.2.1 Calculation

In this section, we construct a formula to calculate the port voltages of the target circuit using the S-parameters estimated in the previous section. By denoting the port voltage vector and port current vector in fig.2.1 as $\mathbf{V}$ and $\mathbf{I}$ respectively, the power waves in equation (2.2) become:

$$\mathbf{a} = \mathbf{R}_0^{-\frac{1}{2}}\mathbf{V} + \mathbf{R}_0^{\frac{1}{2}}\mathbf{I} \tag{2.16}$$

$$\mathbf{b} = \mathbf{R}_0^{-\frac{1}{2}}\mathbf{V} - \mathbf{R}_0^{\frac{1}{2}}\mathbf{I}, \tag{2.17}$$

where $\mathbf{R}_0$ is a diagonal matrix consisting of the reference resistance value for each port. Then, denoting the signal source vector and internal impedance vector as $\mathbf{E}_S$ and $\mathbf{Z}_L$ respectively, we have:

$$\mathbf{E}_S = \mathbf{Z}_L\mathbf{I} + \mathbf{V}. \tag{2.18}$$

So, by defining:

$$\mathbf{v} = \mathbf{R}_0^{-\frac{1}{2}}\mathbf{V} \tag{2.19}$$

$$\mathbf{e}_s = \mathbf{R}_0^{-\frac{1}{2}}\mathbf{E}_s \tag{2.20}$$

$$\mathbf{z}_L = \mathbf{R}_0^{-\frac{1}{2}}\mathbf{Z}_L\mathbf{R}_0^{-\frac{1}{2}}, \tag{2.21}$$

an equation to calculate the normalized port voltage $\mathbf{v}$ is obtained as:

$$\left\{(\mathbf{I}_n - \mathbf{S}) + (\mathbf{I}_n + \mathbf{S})\,\mathbf{z}_L^{-1}\right\}\mathbf{v} = (\mathbf{I}_n + \mathbf{S})\mathbf{z}_L^{-1}\mathbf{e}_s, \tag{2.22}$$

where $\mathbf{I}_n$ denotes the identity matrix of dimension n.

2.2.2 Uniqueness of the absolute values

In this section, it is proved that the sign ambiguity of the S-parameters estimated as in the previous section 2.1 does not affect the absolute value calculations for the port voltages.

Let $\mathbf{S}$ defined in (2.1) denote the S-matrix estimated adopting the positive sign for the square root calculation in $\mathbf{S_{au}}$. Then, when adopting the negative sign for the square root calculation, the S-parameter can be written as:

$$\bar{\mathbf{S}} = \begin{bmatrix} \mathbf{S_{aa}} & -\mathbf{S_{au}} \\ -\mathbf{S_{au}^T} & S_{uu} \end{bmatrix}. \tag{2.23}$$

So, using the transformation matrix $\mathbf{K}$ defined as:

$$\mathbf{K} = \begin{bmatrix} \mathbf{I}_{(n-1)} & \mathbf{0} \\ \mathbf{0} & -1 \end{bmatrix}, \tag{2.24}$$

the S-matrix above can be expressed as:

$$\bar{\mathbf{S}} = \mathbf{KSK}. \tag{2.25}$$

Now, let the port voltage calculated from $\bar{\mathbf{S}}$ be $\bar{\mathbf{v}}$, then, they satisfy the following equation as in (2.22):

$$\left\{\left(\mathbf{I}_n - \bar{\mathbf{S}}\right) + \left(\mathbf{I}_n + \bar{\mathbf{S}}\right)\mathbf{z}_L^{-1}\right\}\bar{\mathbf{v}} = (\mathbf{I}_n + \bar{\mathbf{S}})\mathbf{z}_L^{-1}\mathbf{e}_s. \tag{2.26}$$

Applying (2.25) and $\mathbf{KK} = \mathbf{I_n}$ to this equation yields:

$$\left\{\mathbf{K}(\mathbf{I}_n - \mathbf{S})\mathbf{K} + \mathbf{K}(\mathbf{I}_n + \mathbf{S})\mathbf{K}\mathbf{z}_L^{-1}\right\}\bar{\mathbf{v}} = \mathbf{K}(\mathbf{I}_n + \mathbf{S})\mathbf{K}\mathbf{z}_L^{-1}\mathbf{e}_s.$$

Since both $\mathbf{K}$ and $\mathbf{z}_L^{-1}$ are diagonal matrices, and thus commutative,

$$\mathbf{K}\left\{(\mathbf{I}_n - \mathbf{S}) + (\mathbf{I}_n + \mathbf{S})\mathbf{z}_L^{-1}\right\}\mathbf{K}\bar{\mathbf{v}} = \mathbf{K}(\mathbf{I}_n + \mathbf{S})\mathbf{z}_L^{-1}\mathbf{K}\mathbf{e}_s. \tag{2.27}$$

By left multiplying $\mathbf{K}$ to the both sides:

$$\left\{(\mathbf{I}_n - \mathbf{S}) + (\mathbf{I}_n + \mathbf{S})\mathbf{z}_L^{-1}\right\}\mathbf{K}\bar{\mathbf{v}} = (\mathbf{I}_n + \mathbf{S})\mathbf{z}_L^{-1}\mathbf{K}\mathbf{e}_s. \tag{2.28}$$

Since only loads are connected to port n, $E_n = 0$, and thus the n-th component of $\mathbf{e}_s$ becomes 0, which makes:

$$\mathbf{K}\mathbf{e}_s = \mathbf{e}_s. \tag{2.29}$$

Substituting this to equation (2.28) yields:

$$\left\{(\mathbf{I}_n - \mathbf{S}) + (\mathbf{I}_n + \mathbf{S})\mathbf{z}_L^{-1}\right\}\mathbf{K}\bar{\mathbf{v}} = (\mathbf{I}_n + \mathbf{S})\mathbf{z}_L^{-1}\mathbf{e}_s. \tag{2.30}$$

Comparing this equation to equation (2.22), all terms are same excepting $\mathbf{K}\bar{\mathbf{v}}$ on the left hand side. So, we can derive:

$$\mathbf{K}\bar{\mathbf{v}} = \mathbf{v}. \tag{2.31}$$

Left multiplying $\mathbf{K}$ to the both sides of this equation and denoting $\mathbf{v} = [\mathbf{v_a}^T, v_u]^T$, we obtain:

$$\begin{aligned} \bar{\mathbf{v}} &= \mathbf{Kv} \\ &= \begin{bmatrix} \mathbf{v_a} \\ -v_u \end{bmatrix}. \end{aligned} \tag{2.32}$$

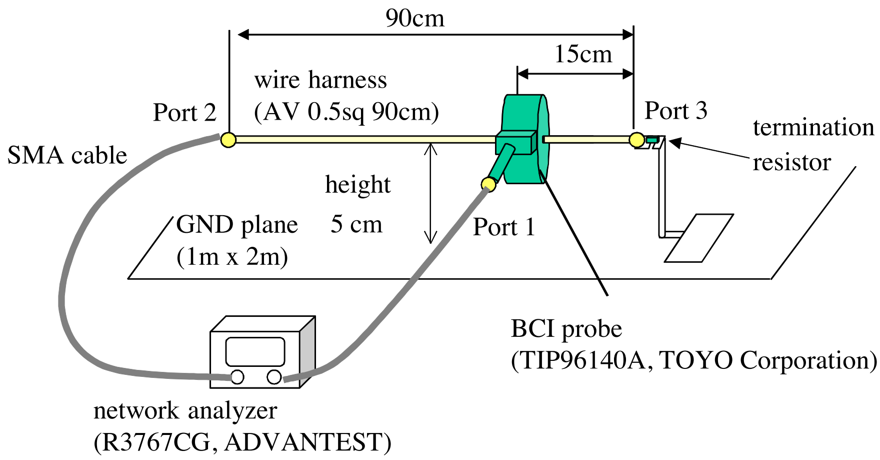


Figure 2.2: Immunity testing system using BCI probe - S-parameter measurement

Therefore, the port voltages calculated using $\bar{\mathbf{S}}$ have same absolute values as those using $\mathbf{S}$. So, the amplitudes of port voltages calculated from the estimated S-parameters are unique despite the sign ambiguities in the S-parameters. In addition, it is recognized that the above equation (2.25) expresses the operation to invert the polarity of port n.

2.2.3 Estimation for S-parameters of the connected circuit

Using the estimated S-parameters so far, we can also estimate the S-parameter of the circuit connected to the u-port. If we denote the S-parameter of the connected circuit as S_I, we can obtain the relation for it by replacing S_L with S_I in Eq.(2.8). since it occupies the same position in the circuit as S_L. This can be seen as a scalar linear equations in terms of S_I, so, we can solve for S_I using one of the equations or applying least-square method for the equations.

2.3 Evaluation of the Estimated Result

2.3.1 Measurement setting and calibration

In this section, we pick up a BCI testing system in Fig. 2.2 as a sample target 3-port network and assume that its port 3 is the port that is hard to access with a measurement probe. The measurement configuration resembles the standard test configuration [14] excepting details but the injected current to the port 3 resister replacing an ECU is not controlled nor measured as in the standard test since we are interested in the S-parameter and voltage estimation here. 3-port S-parameters of the system is estimated by measuring the 2-port S-parameters. The frequency range is from 10MHz to 400MHz and there are totally 79 frequency points at even intervals. The estimation is performed for each frequency point independently. In general, open, short and reference impedances are usually used as the calibration impedances in S-parameter measurement. However, to show the arbitrary value of resisters can be used as the references, here, 3 values of termination resisters are used to attach to port 3, namely, 75Ω, 150Ω and 300Ω. To choose the value, it was used as reference information that the characteristic impedance of the wire harness in Fig. 2.2 is approximately 150Ω. We used 9 equations in total that were obtained by collecting the 3 equations each for 75Ω, 150Ω and 300Ω load attached to port 3 in turn. Then, the least-square method was applied to estimate the 3-port S-parameters.

2.3.2 Comparison with measurement

The estimated S-parameters are shown in Fig. 2.3 with the values directly measured by a network analyzer. From this result, S_{13} and S_{23} show the sign ambiguity described in the previous section. Neglecting this sign ambiguity, the estimation and the measurement show good agreement in the frequency range. The imaginary part of S_{33} shows slight difference from the measurement in the high frequencies. This is considered to come from the extra stray capacitance at port 3 and will be analyzed in the next subsection.

Next, we calculated the port voltages from the estimated S-parameters using the calculation method shown in the previous section. As proved in the previous section, absolute values of the calculated port voltages were not affected by the sign ambiguity in the results of S_{13} and S_{23}. Also, the port 3 voltages were measured using an E/O probe to avoid the influence from the BCI probe as in Fig. 2.4 and Fig. 2.5. The calculated result and the measured voltage at port 3 with connecting 150Ω to both port 2 and port 3 and applying continuous sinusoidal power wave of 10dBm to port 1 is shown in Fig. 2.6. The calculated values and the measured values are in good agreement. Since port 3 resistor value 150Ω is included in the termination resistor values used in the S-parameter estimation, the voltage evaluations for the new values of port 3 resistor, 100Ω and 270Ω are also shown in Figures 2.7 and 2.8. Here, the voltages between the measured and the estimated have an error of approximately 2dB but they are fairly in good agreement.

2.3.3 Analysis of error in estimated S-parameters

In the previous subsection, the imaginary part of S_{33} shows slight difference. This is considered to come from the extra stray capacitance at port 3 when the network analyzer is connected through an extra connector which does not exist when the termination resisters are connected to obtain the data used in the estimation stage. Assuming the stray capacitance value of 1.2pF, the result of the estimation is shown in Fig. 2.9 when adding -1.2pF capacitor in parallel with the termination resisters at port 3 to cancel the effect of stray capacitor. The error in the imaginary part of S_{33} is significantly reduced and other S-parameter values are not affected. From this result, the estimation of the effect of the stray capacitance is strongly supported.

2.4 Chapter Conclusion

An estimation method of the n-port S-parameters for reciprocal circuits has been presented. In this method, known loads are connected in turn to one port that is hard to access with a measurement probe and reflection and transmission characteristics between the remaining ports are measured. S-parameters are obtained by solving a linear least-squares equation and a quadratic equation only. 3 values of loads are required to solve the equation. Theoretically, the number of measurements does not increase as the number of ports increases when we count the operation measuring through directly measured ports with connecting a known load to indirectly measured ports as one measurement.

In addition, applying this method to derive the S-parameters of an immunity test system which can be seen as a 3-port circuit, the validity of this method has been confirmed. As the result, it has been shown that there is a sign ambiguity for the S-parameters between the hard-to-access port and other ports but the port voltage amplitudes calculated using them are valid. The port voltages have also been calculated using the estimated S-parameters and shown good agreement with the port voltages measured. In this method, there is no need to connect a network analyzer to one of the ports of the target circuit. So, it is effective for measuring the circuit which has a port with uncommon ground.

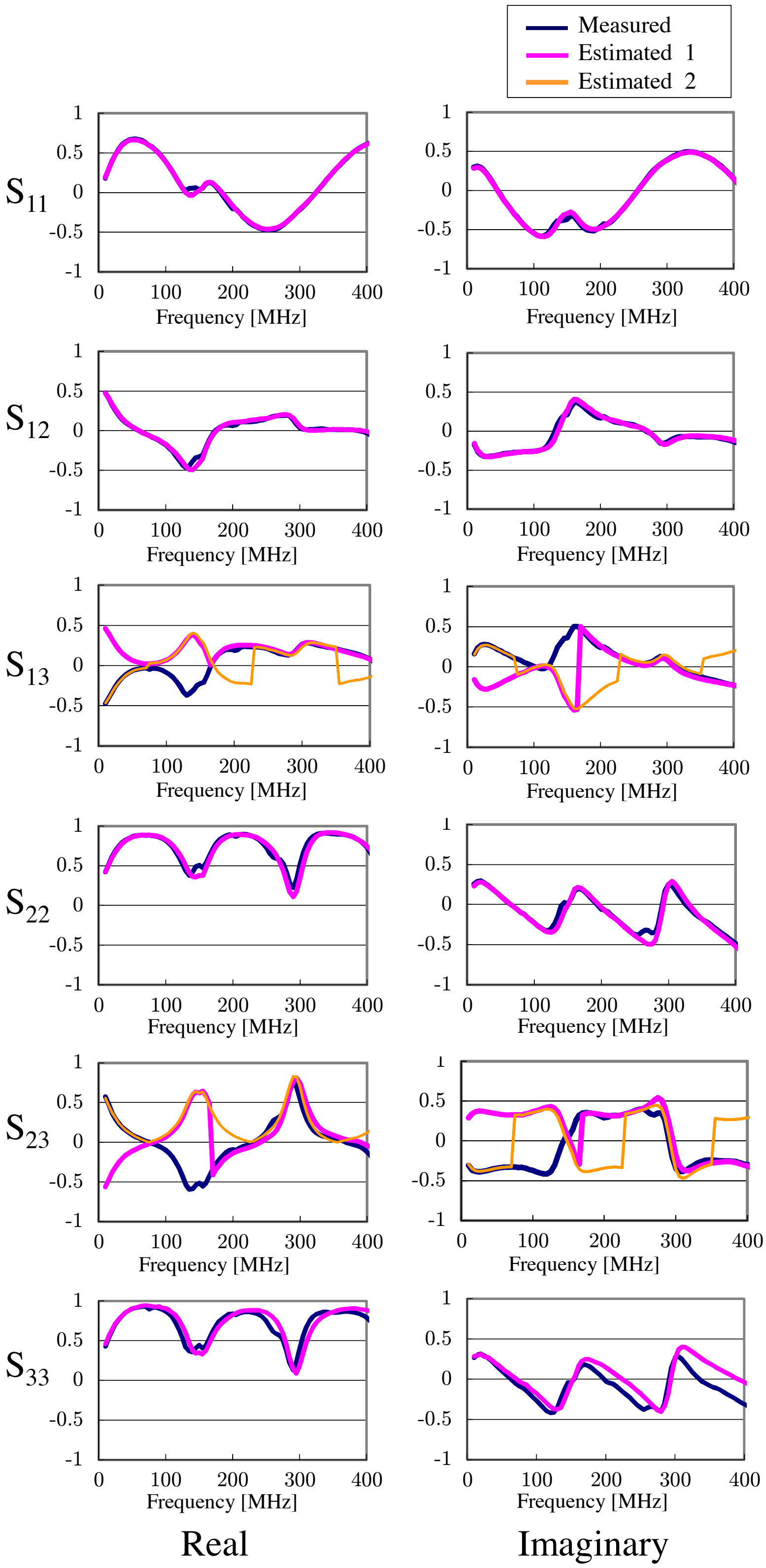


Figure 2.3: Measured and estimated S-parameters.

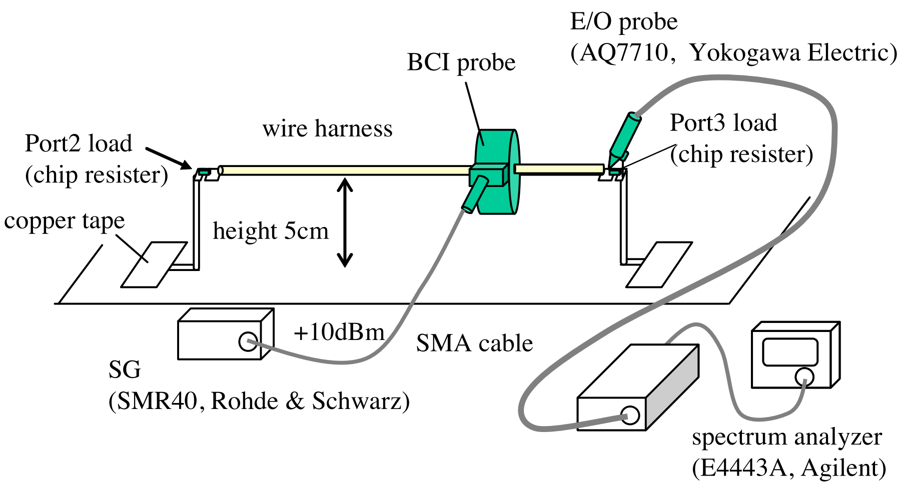


Figure 2.4: Immunity testing system using BCI probe - voltage measurement

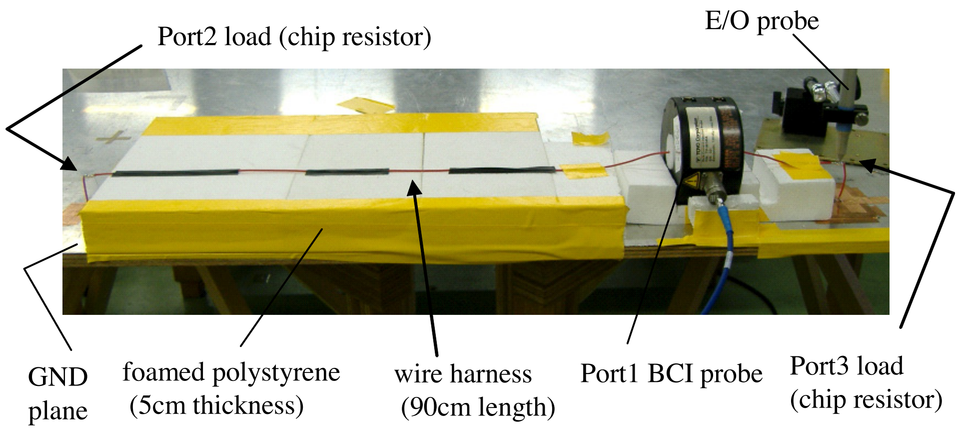


Figure 2.5: Picture of immunity testing system using BCI probe - voltage measurement

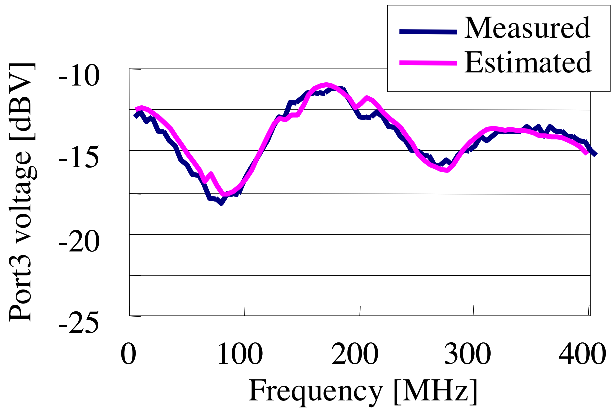


Figure 2.6: Port 3 voltage with 150 load.

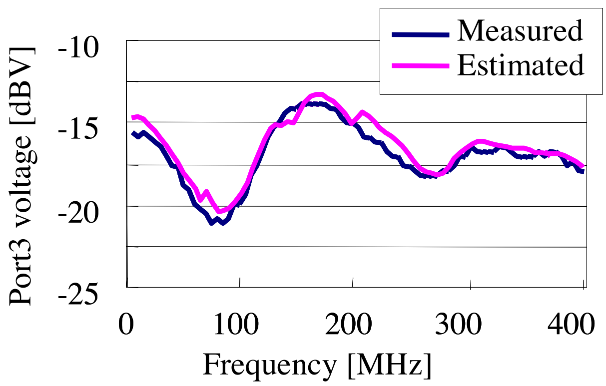


Figure 2.7: Port 3 voltage with 100 load.

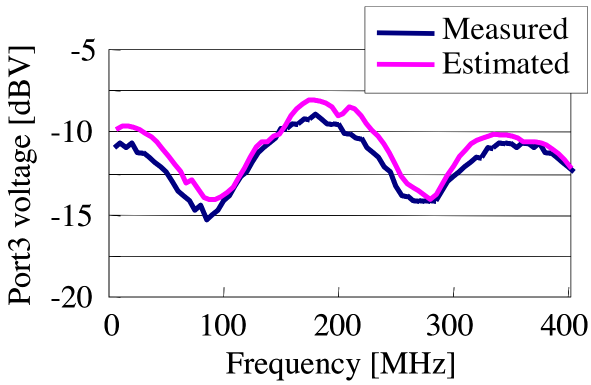


Figure 2.8: Port 3 voltage with 270 load.

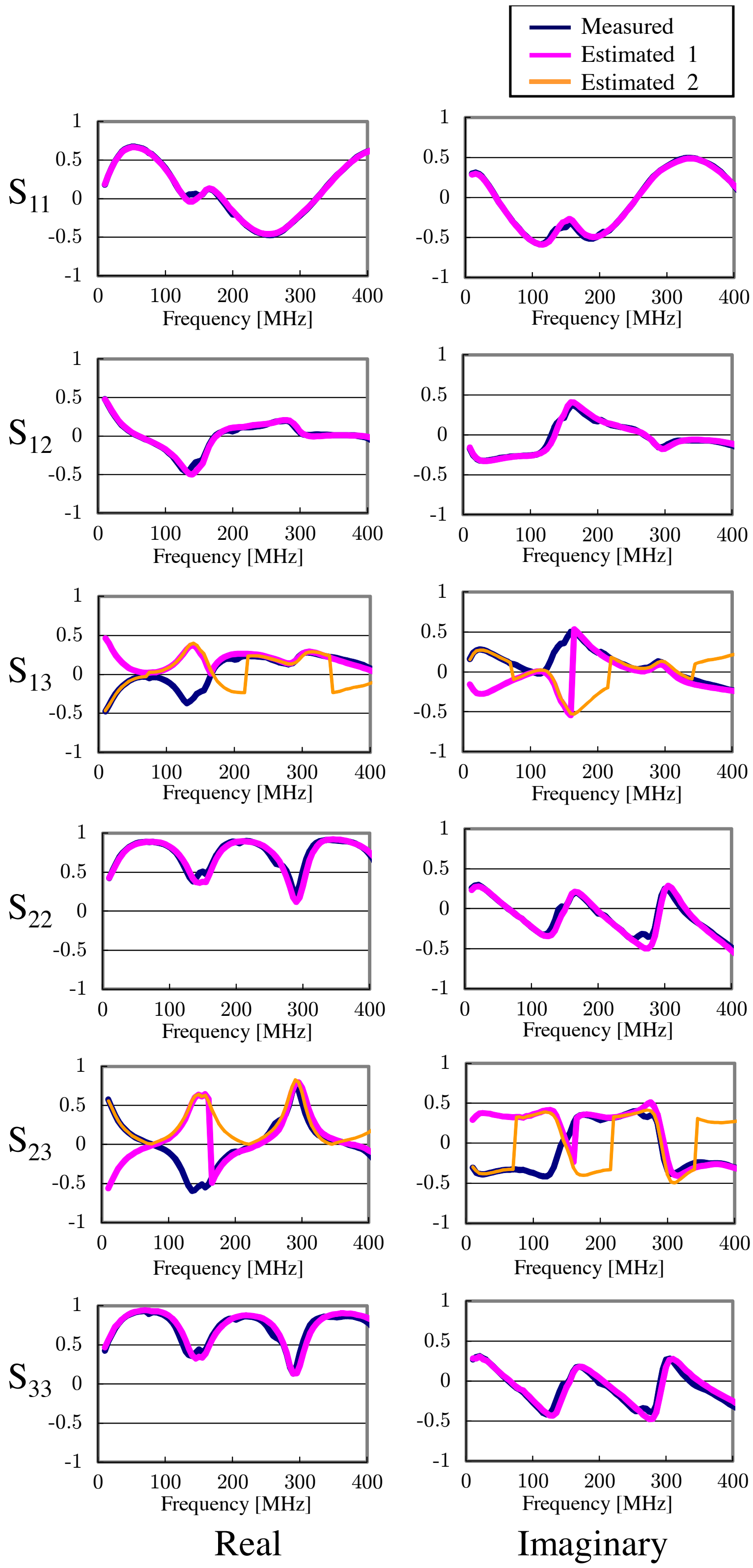


Figure 2.9: Measured and estimated S-parameters with load value correction of 1.2pF capacitance.

Chapter 3

Estimation of 3 port S-parameters with 1 port measurements

The basic idea of the S-parameter estimation method using equation decomposition and simultaneous solution introduced in the previous chapter can be applied to the circuits with arbitrary combination of direct and indirect measurement ports. To show its potential and limitation, the method is applied to three-port S-parameters for reciprocal circuits in this chapter. Here, several known loads are connected to two of the ports and reflection characteristic of the remaining port is measured. As in the previous chapter, S-parameters are obtained by solving a linear system equations and quadratic equations only. In addition, a method for determining the port voltages from the estimated S parameters is also described. Applying this method to estimate the S-parameters of an immunity test system, validity of this method has been confirmed.

3.1 Estimation Method

A reciprocal 3 port circuit shown in Fig. 3.1 has port 2 and port 3 which are hard to access with measurement probes. We measure the S-parameter of the remaining port 1 with attaching loads to port 2 and port 3 and estimate the whole 3-port S-parameters from the measured 1-port S-parameter.

The S-matrix $\mathbf{S}$ of the circuit can be expressed using the incident power wave $\mathbf{a}$ and reflected power wave $\mathbf{b}$ as follows:

$$\mathbf{b} = \mathbf{S}\mathbf{a} \tag{3.1}$$

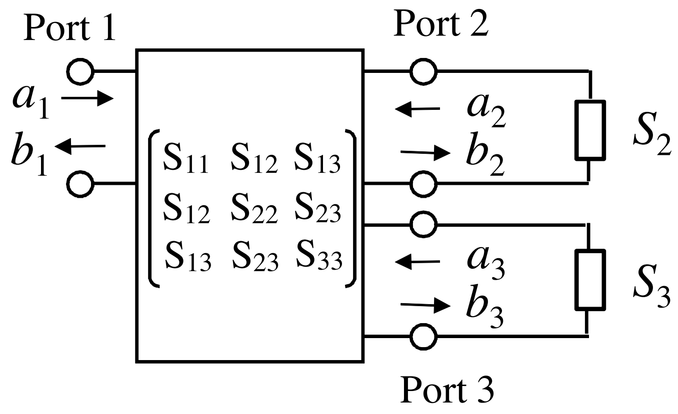


Figure 3.1: S-parameter measurement arrangement

where

$$\mathbf{S} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{bmatrix} \tag{3.2}$$

$$\mathbf{a} = [a_1, a_2, a_3]^T, \quad \mathbf{b} = [b_1, b_2, b_3]^T \tag{3.3}$$

From the relation between $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{S}$, the reflection waves can be expressed as:

$$b_1 = S_{11} a_1 + \begin{bmatrix} S_{12} & S_{13} \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix} \tag{3.4}$$

and

$$\begin{bmatrix} b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} S_{12} \\ S_{13} \end{bmatrix} a_1 + \begin{bmatrix} S_{22} & S_{23} \\ S_{23} & S_{33} \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix}. \tag{3.5}$$

When the loads with reflection parameters S_2 and S_3 are connected to port 2 and port 3 respectively, the incident waves and the reflecting waves satisfy:

$$\begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} S_2 & 0 \\ 0 & S_3 \end{bmatrix} \begin{bmatrix} b_2 \\ b_3 \end{bmatrix} \tag{3.6}$$

Substituting (3.5) to this and solving for $[a_2, a_3]^T$ we have:

$$\begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = \left(\mathbf{1} - \begin{bmatrix} S_2 & 0 \\ 0 & S_3 \end{bmatrix} \begin{bmatrix} S_{22} & S_{23} \\ S_{23} & S_{33} \end{bmatrix} \right)^{-1} \begin{bmatrix} S_2 & 0 \\ 0 & S_3 \end{bmatrix} \begin{bmatrix} S_{12} \\ S_{13} \end{bmatrix} a_1 \tag{3.7}$$

where $\mathbf{1}$ denotes the identity matrix of dimension 2. Substituting this to (3.4) yields:

$$b_1 = \hat{S} a_1 \tag{3.8}$$

where

$$\hat{S} = S_{11} + \begin{bmatrix} S_{12} & S_{13} \end{bmatrix} \left(\mathbf{1} - \begin{bmatrix} S_2 & 0 \\ 0 & S_3 \end{bmatrix} \begin{bmatrix} S_{22} & S_{23} \\ S_{23} & S_{33} \end{bmatrix} \right)^{-1} \begin{bmatrix} S_2 & 0 \\ 0 & S_3 \end{bmatrix} \begin{bmatrix} S_{12} \\ S_{13} \end{bmatrix} \tag{3.9}$$

corresponds to the measured S-parameter (reflection coefficient) of the directly measured port. This equation is the fundamental nonlinear estimation equation for the S-parameters.

To linearize the equation, we expand and reduce this expression to obtain:

$$\begin{bmatrix} 1 & S_2 S_3 & S_2 & S_3 & S_2 S_3 \hat{S} & S_2 \hat{S} & S_3 \hat{S} \end{bmatrix} \mathbf{m} = \hat{S} \tag{3.10}$$

where

$$\mathbf{m} = \begin{bmatrix} m_1 \\ m_2 \\ m_3 \\ m_4 \\ m_5 \\ m_6 \\ m_7 \end{bmatrix} = \begin{bmatrix} S_{11} \\ \begin{pmatrix} S_{11} S_{22} S_{33} + 2 S_{12} S_{13} S_{23} \\ -S_{11} S_{23}^2 - S_{12}^2 S_{33} - S_{13}^2 S_{22} \end{pmatrix} \\ S_{12}^2 - S_{11} S_{22} \\ S_{13}^2 - S_{11} S_{33} \\ S_{23}^2 - S_{22} S_{33} \\ S_{22} \\ S_{33} \end{bmatrix} \tag{3.11}$$

This is the basic linear equation to estimate the three port S-parameters with 1 port measurement. Since this consists of one equation for 7 unknowns, it does not have unique solution as it is. However, the number of equations can be increased to or above the number of unknowns by measuring $\hat{\mathbf{S}}$ several times with changing the values of load S_2 and S_3. By solving the simultaneous linear equations, the value of $\mathbf{m}$ can be obtained. From the components of $\mathbf{m}$, m_1, m_6 and m_7 directly gives the value of S_{11}, S_{22} and S_{33}, respectively. Applying them to m_3, m_4 and m_5, the quadratic equations can be solved to obtain the values of S_{12}, S_{13} and S_{23}, respectively.

At the time, each of the solutions for S_{12}, S_{13} and S_{23} has uncertainty in the sign, which creates totally 6 combinations of the signs of them. But using m_2 to limit the combination, finally 4 combinations of the signs can exist. This sign ambiguity comes from the fact that even if a $1 : -1$ transformer is attached to one or both of port 2 and port 3, it is unobservable from the other ports, that is, S_{11}, S_{22} and S_{33} are not affected by inverting the polarity of port 2 and/or port 3. This ambiguity in signs of component $S_{pq}, p \neq q$ is inherent one which is similar to the sign ambiguity of Y and Z parameters referred in [13]. However, when calculating the port voltage using the estimated S parameters, this sign ambiguity does not affect the voltage calculation result as proved in the following section.

3.2 Port Voltages

3.2.1 Calculation

In this section, a method to calculate the port voltages using the estimated S-matrix is presented.

When we denote the voltage vector and the current vector of the ports shown in Fig. 3.1 as $\mathbf{V}$ and $\mathbf{I}$ respectively, $\mathbf{a}$ and $\mathbf{b}$ in (3.1) satisfy:

$$\mathbf{a} = \mathbf{R}_0^{-\frac{1}{2}}\mathbf{V} + \mathbf{R}_0^{\frac{1}{2}}\mathbf{I} \tag{3.12}$$

$$\mathbf{b} = \mathbf{R}_0^{-\frac{1}{2}}\mathbf{V} - \mathbf{R}_0^{\frac{1}{2}}\mathbf{I} \tag{3.13}$$

where

$$\mathbf{V} = [V_1, V_2, V_3]^T \tag{3.14}$$

$$\mathbf{I} = [I_1, I_2, I_3]^T \tag{3.15}$$

$$\mathbf{R}_0 = \ \mathrm{diag}[R_{01}, R_{02}, R_{03}]. \tag{3.16}$$

Here, $\mathbf{R}_0$ is a diagonal matrix consisting of the set of reference impedance for each port. When the vectors of the voltage source and the internal impedance connected to each port are denoted as $\mathbf{E}_S$ and $\mathbf{Z}_L$, their relation with $\mathbf{I}$ and $\mathbf{V}$ can be described as:

$$\mathbf{E}_S = \mathbf{Z}_L\mathbf{I} + \mathbf{V} \tag{3.17}$$

where

$$\mathbf{E}_S = [E_{S1}, E_{S2}, E_{S3}]^T, \tag{3.18}$$

$$\mathbf{Z}_L = \ \mathrm{diag}[Z_1, Z_2, Z_3]. \tag{3.19}$$

Therefore, if we define the normalized values of $\mathbf{V}$, $\mathbf{E}_s$ and $\mathbf{Z}_L$ as:

$$\mathbf{v} = \mathbf{R}_0^{-\frac{1}{2}}\mathbf{V} \tag{3.20}$$

$$\mathbf{e}_s = \mathbf{R}_0^{-\frac{1}{2}}\mathbf{E}_s \tag{3.21}$$

$$\mathbf{z}_L = \mathbf{R}_0^{-\frac{1}{2}}\mathbf{Z}_L\mathbf{R}_0^{-\frac{1}{2}}, \tag{3.22}$$

the equation to obtain the values of normalized voltage $\mathbf{v}$ be:

$$\left\{(\mathbf{1}-\mathbf{S})+(\mathbf{1}+\mathbf{S})\mathbf{z}_L^{-1}\right\}\mathbf{v} = (\mathbf{1}+\mathbf{S})\mathbf{z}_L^{-1}\mathbf{e}_s \tag{3.23}$$

where $\mathbf{1}$ denotes the identity matrix.

3.2.2 Uniqueness of the absolute values

In this section, it is proved that the sign ambiguity of the S-parameters estimated as in the previous section 3.1 does not affect the absolute value calculations for the port voltages.

Let $\mathbf{S}$ in (3.2) denote the S-matrix estimated adopting the identical signs for the square root calculations for S_{12}, S_{13} and S_{23}, where the actual polarity of the signs are decided by the 2nd component of $\mathbf{m}$ in (3.11). Then, adopting the opposite signs for two of the three S parameters gives three distinct S-matrices:

$$\dot{\mathbf{S}} = \begin{bmatrix} S_{11} & S_{12} & -S_{13} \\ S_{12} & S_{22} & -S_{23} \\ -S_{13} & -S_{23} & S_{33} \end{bmatrix} \tag{3.24}$$

$$\ddot{\mathbf{S}} = \begin{bmatrix} S_{11} & -S_{12} & S_{13} \\ -S_{12} & S_{22} & -S_{23} \\ S_{13} & -S_{23} & S_{33} \end{bmatrix} \tag{3.25}$$

$$\dddot{\mathbf{S}} = \begin{bmatrix} S_{11} & -S_{12} & -S_{13} \\ -S_{12} & S_{22} & S_{23} \\ -S_{13} & S_{23} & S_{33} \end{bmatrix}. \tag{3.26}$$

Using the transformation matrices $\mathbf{K}$ and $\mathbf{J}$ defined as:

$$\mathbf{K} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{3.27}$$

and

$$\mathbf{J} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \tag{3.28}$$

the S-matrices above can be expressed as:

$$\dot{\mathbf{S}} = \mathbf{KSK} \tag{3.29}$$

$$\ddot{\mathbf{S}} = \mathbf{JSJ} \tag{3.30}$$

$$\dddot{\mathbf{S}} = \mathbf{KJSJK}. \tag{3.31}$$

Now, let the port voltage calculated from $\dot{\mathbf{S}}$ be $\dot{\mathbf{v}}$, then they satisfy the equation as (3.23):

$$\left\{(\mathbf{1} - \dot{\mathbf{S}}) + (\mathbf{1} + \dot{\mathbf{S}})\mathbf{z}_L^{-1}\right\} \dot{\mathbf{v}} = (\mathbf{1} + \dot{\mathbf{S}})\mathbf{z}_L^{-1}\mathbf{e}_s. \tag{3.32}$$

Applying (3.29) and $\mathbf{KK} = \mathbf{1}$ to this equation gives:

$$\left\{\mathbf{K}(\mathbf{1} - \mathbf{S})\mathbf{K} + \mathbf{K}(\mathbf{1} + \mathbf{S})\mathbf{K}\mathbf{z}_L^{-1}\right\} \dot{\mathbf{v}} = \mathbf{K}(\mathbf{1} + \mathbf{S})\mathbf{K}\mathbf{z}_L^{-1}\mathbf{e}_s. \tag{3.33}$$

Since both $\mathbf{K}$ and $\mathbf{z}_L^{-1}$ are diagonal matrices, and thus commutative,

$$\mathbf{K}\left\{(\mathbf{1} - \mathbf{S}) + (\mathbf{1} + \mathbf{S})\mathbf{z}_L^{-1}\right\} \mathbf{K}\dot{\mathbf{v}} = \mathbf{K}(\mathbf{1} + \mathbf{S})\mathbf{z}_L^{-1}\mathbf{K}\mathbf{e}_s. \tag{3.34}$$

By left multiplying $\mathbf{K}$ to the both sides:

$$\left\{(\mathbf{1} - \mathbf{S}) + (\mathbf{1} + \mathbf{S})\mathbf{z}_L^{-1}\right\} \mathbf{K}\dot{\mathbf{v}} = (\mathbf{1} + \mathbf{S})\mathbf{z}_L^{-1}\mathbf{K}\mathbf{e}_s. \tag{3.35}$$

Since only loads are connected to port 2 and 3, $E_{S2} = E_{S3} = 0$, and thus the 2nd and 3rd components of $\mathbf{e}_s$ becomes 0, which makes:

$$\mathbf{K}\mathbf{e}_s = \mathbf{e}_s. \tag{3.36}$$

Applying this to the previous equation gives:

$$\left\{(\mathbf{1} - \mathbf{S}) + (\mathbf{1} + \mathbf{S})\mathbf{z}_L^{-1}\right\} \mathbf{K}\dot{\mathbf{v}} = (\mathbf{1} + \mathbf{S})\mathbf{z}_L^{-1}\mathbf{e}_s. \tag{3.37}$$

Comparing this equation to (3.23), all terms are same excepting $\mathbf{K}\dot{\mathbf{v}}$ on the left hand side. So, we can derive:

$$\mathbf{K}\dot{\mathbf{v}} = \mathbf{v}. \tag{3.38}$$

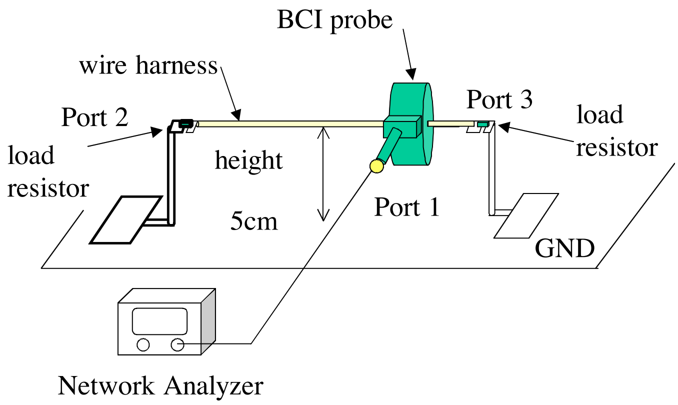


Figure 3.2: Immunity testing system using BCI probe

Left multiplying $\mathbf{K}$ to the both sides of this and denoting $\mathbf{v} = [v_1, v_2, v_3]^T$, we have:

$$\dot{\mathbf{v}} = \mathbf{K}\mathbf{v} \tag{3.39}$$

$$= \begin{bmatrix} v_1 \\ v_2 \\ -v_3 \end{bmatrix}. \tag{3.40}$$

Therefore, the port voltages calculated using $\dot{\mathbf{S}}$ have same absolute values as those using $\mathbf{S}$. The above operations can similarly be applied to the port voltages calculated using $\ddot{\mathbf{S}}$ or $\dddot{\mathbf{S}}$. So, the amplitude of port voltages calculated from the estimated S-parameters are unique despite the sign ambiguities. In addition, equations (3.29), (3.30) and (3.31) express the operation to invert the polarities of port 3 and port 2, respectively.

3.3 Evaluation of the Estimated Result

3.3.1 Comparison with measurement

In this section, we pick up a BCI testing system in Fig. 3.2 as a sample 3 port circuit and assume its port 2 and 3 are hard to access with measurement probes. Here, 3 values of termination resisters are used, namely, 75Ω, 150Ω and 300Ω to be connected to both port 2 and port 3. 3 port S-parameters of the system is estimated by measuring the reflection parameters of port 1 for all combinations of the termination resisters. We use all 9 equations thus obtained, and using the least square method to obtain the solution for the 7 unknowns of $\mathbf{m}$. The estimated S-parameters are shown in Fig. 3.3 with the values directly measured with a network analyzer. Here, only the results of taking positive sign for S_{12} and S_{13} square root calculations are shown. From this result, S_{12}, S_{13} and S_{23} show the sign ambiguity described in the previous section. Neglecting this sign ambiguity, the estimation and the measurement show good agreement in the frequency range. The S-parameters concerning port 1 shows especially good agreement since it is the port measured to obtain the data for estimation. S-parameters concerning only port 2 and port 3 have slight increase in errors as the frequency increases.

3.4 Chapter Conclusion

An estimation method of the 3 port S-parameters for reciprocal circuits has been proposed in this chapter. In the proposed method, known loads are connected to two of the ports that are hard to access with a measurement probe and the reflection characteristics of the remaining one port is measured. S-parameters are obtained by solving a linear system equations and quadratic equations only. In addition, applying this method to derive the S-parameters of an immunity test system which can be viewed as a 3 port circuit, the validity of this method has been confirmed. As the result, it was shown that there is a sign ambiguity for the S-parameters between the hard-to-access ports and other port. However, it was proved that the calculation using the estimated S-parameter gives the valid port voltage amplitudes despite of the sign ambiguity.

Also, it can be deduced through the procedure in this chapter that the estimation method introduced in the previous chapter can be applied to the circuits with arbitrary combination of direct and indirect measurement ports. However, it is necessary not only to expand the equation size but also to reproduce the whole procedure of constructing the equation according to the port configuration. Therefore, the general theoretical analysis in the framework of linear algebra can not be provided through this estimation method. Another estimation method which enables theoretical analysis in the framework of linear algebra will be proposed in the next chapter.

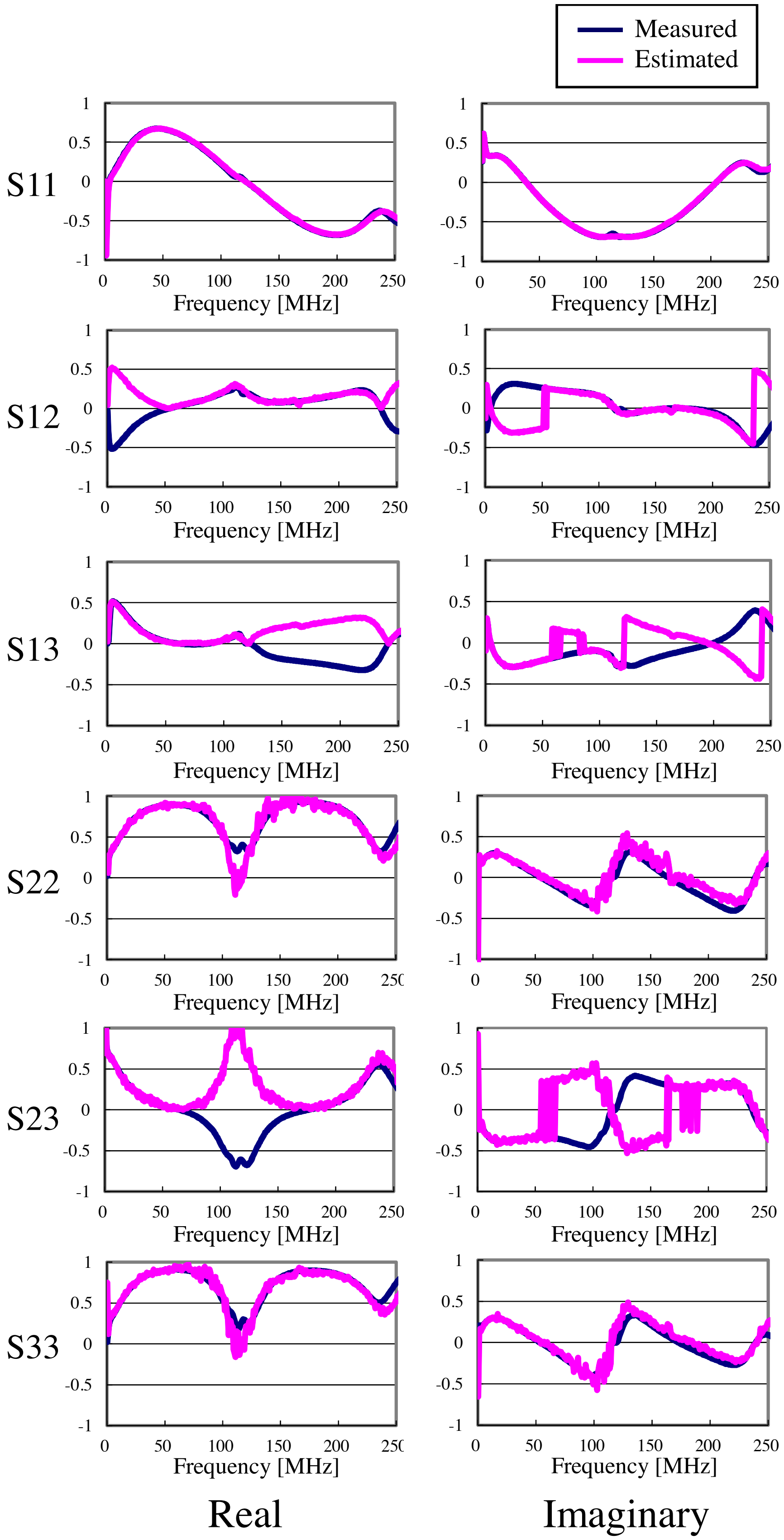


Figure 3.3: Measured and estimated S-parameters.

Chapter 4

Estimation of $2r$ port S-parameters with r port measurements

A novel estimation method of the $2r$ port S-parameters for reciprocal circuits with r port measurements is presented. This method is 'submatrix elimination and successive solution.' The estimation method introduced in the previous chapters can also be applied to the same reciprocal circuits but its requirements to reconstruct the equations disables the theoretical analysis in the framework of linear algebra. On the other hand, the method proposed in this chapter retains the same matrix equations with just modifying their matrix dimension size. This enables the theoretical analysis in the framework of linear algebra.

This method can be typically applied to estimate the S-parameters of both a multiport connection and a multiport device as follows. The multiport device is connected to one end of the multiport connection and a network analyzer is connected to its another end. Here, the two ends of the multiport connection are assumed to have different ground levels. Only measurements of reflection and transmission characteristics among the ports of the measured end of the connection are required for the estimation. Both S-parameters for the multiport connection and the multiport device are estimated by solving several linear equations and square root calculations only. The validity of the proposed method is also confirmed by simple experimental results.

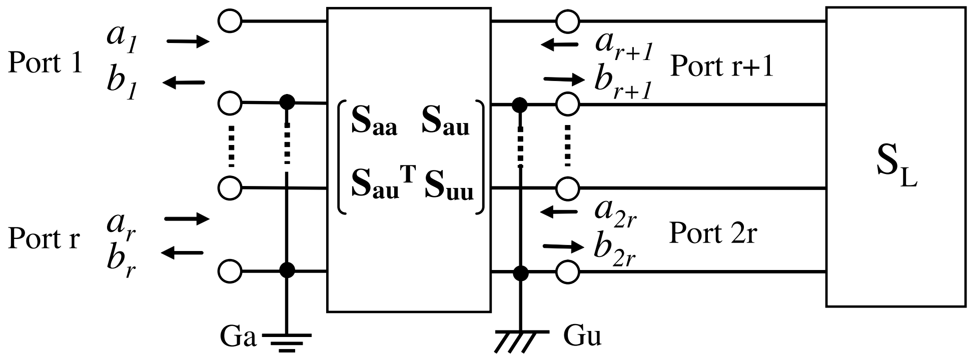


Figure 4.1: S-parameter port arrangement with non-common ground.

4.1 Modeling with Multiport S-parameter Circuit Blocks and Flow of Estimation

The target system of this chapter can be treated with an S-parameter model shown in Fig.4.1. The reciprocal $2r$-port circuit in the figure corresponds to the multiport connection having r ports in its ourput end that are hard to access with measurement probes. We measure the S-parameters of remaining r ports in its input end attaching known loads to the r output ports and estimate the whole $2r$-port S-parameters from the measured r-port S-parameters.

As the first step, we disconnect the multiport device and figure out the transfer S-parameters of the multiport connection by attaching known loads to its output end and measuring the S-parameters at its input end. Then, as the second step, by connecting the multiport device to the output end of the multiport connection and measuring the S parameters at its input end, we can calculate the S-parameters of the multiport device by de-embedding the transfer S-parameters of the multiport connection obtained in the first step from the measured S-parameters.

4.2 Estimation Method for Multiport Connection S-parameters

In this section, estimation method for the S-parameters of the multiport connection is described by dividing them into block matrices and solve them one by one.

4.2.1 Reduced S-matrix

The S-parameter matrix $\mathbf{S}$ can be written in block matrix form as follows where suffix 'a' indicates the ports in the input end of the multiport connection that are easily accessed with measurement probes and suffix 'u' indicates the ports in the output end of the multiport connection that are hard to access with measurement probes.

The incident wave $\mathbf{a} = \left[\mathbf{a_a}^{\mathrm{T}}\ \mathbf{a_u}^{\mathrm{T}}\right]^{\mathrm{T}}$ and the reflected wave $\mathbf{b} = \left[\mathbf{b_a}^{\mathrm{T}}\ \mathbf{b_u}^{\mathrm{T}}\right]^{\mathrm{T}}$ satisfy:

$$\begin{bmatrix} \mathbf{b_a} \\ \mathbf{b_u} \end{bmatrix} = \begin{bmatrix} \mathbf{S_{aa}} & \mathbf{S_{au}} \\ \mathbf{S_{au}^{\mathrm{T}}} & \mathbf{S_{uu}} \end{bmatrix} \begin{bmatrix} \mathbf{a_a} \\ \mathbf{a_u} \end{bmatrix} \tag{4.1}$$

where:

$$\mathbf{S_{aa}},\ \mathbf{S_{au}},\ \mathbf{S_{uu}} \in \mathbb{C}^{\mathrm{r}\times\mathrm{r}}, \mathbf{a_a}, \mathbf{b_a}, \mathbf{a_u}, \mathbf{b_u} \in \mathbb{C}^{r\times 1}.$$

When we connect loads with known S-parameter matrix of $\mathbf{S_L}$ to the u-ports, the following holds:

$$\mathbf{a_u} = \mathbf{S_L}\mathbf{b_u}. \tag{4.2}$$

By denoting the measured r port S-parameters as $\hat{\mathbf{S}}$, it follows that:

$$\mathbf{b_a} \ = \ \hat{\mathbf{S}}\mathbf{a_a}. \tag{4.3}$$

Using the equations (4.1) to (4.3), we can obtain:

$$\hat{\mathbf{S}} = \mathbf{S_{aa}} + \mathbf{S_{au}}\mathbf{S_L}\left(\mathbf{I} - \mathbf{S_{uu}}\mathbf{S_L}\right)^{-1}\mathbf{S_{au}^{\mathrm{T}}}. \tag{4.4}$$

This is the fundamental nonlinear estimation equation for the S-parameters in the form of matrix quadratic equation in terms of 3 submatrices which has the same form as (2.7) in chapter 2. However, we cannot use the same method to linearize the equation as in the previous chapter since $(\mathbf{I} - \mathbf{S_{uu}}\mathbf{S_L})$ is a matrix here.

So, in this chapter, to linearize the estimation equation, we develop a novel S-parameter estimation method using submatrix elimination and successive solution. In the following sections, we increase the number of equations using the measurements attaching new known loads. This enables eliminating $\mathbf{S_{uu}}$, then, $\mathbf{S_{au}}$ to obtain a matrix quadratic equation interms of $\mathbf{S_{aa}}$. Then, it is linearized and solved. Next, we obtain a matrix quadratic equation interms of $\mathbf{S_{au}}$, then, linearize it and solve it. Lastly, all solutions and equations are used to obtain $\mathbf{S_{uu}}$.

4.2.2 Elimination of $\mathbf{S_{uu}}$

In case of $\mathbf{S_L} = \mathbf{O}$, from (4.4), $\hat{\mathbf{S}} = \mathbf{S_{aa}}$ which enables the value of $\mathbf{S_{aa}}$ be directly obtained from the measured values. However, it is difficult to construct the known loads that keep matching throughout the target frequency range for the actual target electronic circuits. Therefore, we construct a method to measure and estimate using known loads not with zero reflection coefficients. This makes $|\mathbf{S_L}| \neq \mathbf{0}$ in general.

In case of $|\mathbf{S_L}| \neq \mathbf{0}$, taking the inverses of the both sides of (4.4). As the result, we have:

$$\mathbf{S_{uu}} = \mathbf{S_L^{-1}} - \mathbf{S_{au}^T} \left(\hat{\mathbf{S}} - \mathbf{S_{aa}} \right)^{-1} \mathbf{S_{au}}. \tag{4.5}$$

We conduct several measurement with several known loads attached to the hard-to-access u-ports. We denote the reflection parameter matrix $\mathbf{S_L}$ for the i-th measurement as $\mathbf{S_L}^{(i)}$ and the measured S-parameter matrix $\hat{\mathbf{S}}$ as $\hat{\mathbf{S}}^{(i)}$. Taking the difference of each side of (4.5) for the i-th and j-th measurements:

$$\mathbf{S_{au}^T} \left(\hat{\mathbf{S}}^{(\mathbf{i})} - \mathbf{S_{aa}} \right)^{-1} \mathbf{S_{au}} - \mathbf{S_{au}^T} \left(\hat{\mathbf{S}}^{(\mathbf{j})} - \mathbf{S_{aa}} \right)^{-1} \mathbf{S_{au}} = \mathbf{S_L^{(i)-1}} - \mathbf{S_L^{(j)-1}}.$$

It follows that:

$$\mathbf{S_{au}^T} \left\{ \left(\hat{\mathbf{S}}^{(\mathbf{i})} - \mathbf{S_{aa}} \right)^{-1} - \left(\hat{\mathbf{S}}^{(\mathbf{j})} - \mathbf{S_{aa}} \right)^{-1} \right\} \mathbf{S_{au}} = \mathbf{S_L^{(i)-1}} - \mathbf{S_L^{(j)-1}}.$$

$$\mathbf{S_{au}^T} \left(\hat{\mathbf{S}}^{(\mathbf{i})} - \mathbf{S_{aa}} \right)^{-1} \left\{ \left(\hat{\mathbf{S}}^{(\mathbf{j})} - \mathbf{S_{aa}} \right) - \left(\hat{\mathbf{S}}^{(\mathbf{i})} - \mathbf{S_{aa}} \right) \right\} \left(\hat{\mathbf{S}}^{(\mathbf{j})} - \mathbf{S_{aa}} \right)^{-1} \mathbf{S_{au}} = \mathbf{S_L^{(i)-1}} - \mathbf{S_L^{(j)-1}}$$

Finally, we have:

$$\left(\hat{\mathbf{S}}^{(\mathbf{i})} - \mathbf{S_{aa}} \right)^{-1} \left(\hat{\mathbf{S}}^{(\mathbf{j})} - \hat{\mathbf{S}}^{(\mathbf{i})} \right) \left(\hat{\mathbf{S}}^{(\mathbf{j})} - \mathbf{S_{aa}} \right)^{-1} = \mathbf{S_{au}^{T-1}} \left(\mathbf{S_L^{(i)-1}} - \mathbf{S_L^{(j)-1}} \right) \mathbf{S_{au}^{-1}}. \tag{4.6}$$

4.2.3 Estimation formula for $\mathbf{S_{aa}}$

Taking the inverses of the both sides of (4.6) and substituting $i = 1$, $j = 2$ gives:

$$\left(\hat{\mathbf{S}}^{(\mathbf{2})} - \mathbf{S_{aa}} \right) \left(\hat{\mathbf{S}}^{(\mathbf{2})} - \hat{\mathbf{S}}^{(\mathbf{1})} \right)^{-1} \left(\hat{\mathbf{S}}^{(\mathbf{1})} - \mathbf{S_{aa}} \right) = \mathbf{S_{au}} \left(\mathbf{S_L^{(1)-1}} - \mathbf{S_L^{(2)-1}} \right)^{-1} \mathbf{S_{au}^T}. \tag{4.7}$$

We conduct one more measurement and express the obtained values with upper suffix (3). Taking the inverses of the both sides of (4.6) and substituting $i = 1$, $j = 3$ gives:

$$\left(\hat{\mathbf{S}}^{(\mathbf{3})} - \mathbf{S_{aa}} \right) \left(\hat{\mathbf{S}}^{(\mathbf{3})} - \hat{\mathbf{S}}^{(\mathbf{1})} \right)^{-1} \left(\hat{\mathbf{S}}^{(\mathbf{1})} - \mathbf{S_{aa}} \right) = \mathbf{S_{au}} \left(\mathbf{S_L^{(1)-1}} - \mathbf{S_L^{(3)-1}} \right)^{-1} \mathbf{S_{au}^T} \tag{4.8}$$

Here, we choose the values of the loads to satisfy:

$$\mathbf{S_L^{(3)-1}} - \mathbf{S_L^{(1)-1}} = q\left(\mathbf{S_L^{(2)-1}} - \mathbf{S_L^{(1)-1}}\right) \tag{4.9}$$

where q be an arbitrary complex constant of non-zero.

From (4.8) and (4.9):

$$\left(\hat{\mathbf{S}}^{(3)} - \mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(3)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right) = \frac{1}{q}\mathbf{S_{au}}\left(\mathbf{S_L^{(1)-1}} - \mathbf{S_L^{(2)-1}}\right)^{-1}\mathbf{S_{au}^T}. \tag{4.10}$$

Substituting (4.10) times q from (4.7):

$$\begin{aligned}&\left(\hat{\mathbf{S}}^{(2)} - \mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(2)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right)\\ &- q\left(\hat{\mathbf{S}}^{(3)} - \mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(3)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right) = 0.\end{aligned} \tag{4.11}$$

Now, denoting

$\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} = \left(\hat{\mathbf{S}}^{(2)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}$, $\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1} = \left(\hat{\mathbf{S}}^{(3)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}$

and expanding (4.11), we have:

$$\begin{aligned}&\hat{\mathbf{S}}^{(2)}\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1}\hat{\mathbf{S}}^{(1)} - q\hat{\mathbf{S}}^{(3)}\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\hat{\mathbf{S}}^{(1)} - \left(\hat{\mathbf{S}}^{(2)}\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} - q\hat{\mathbf{S}}^{(3)}\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\right)\mathbf{S_{aa}}\\ &-\mathbf{S_{aa}}\left(\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1}\hat{\mathbf{S}}^{(1)} - q\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\hat{\mathbf{S}}^{(1)}\right) + \mathbf{S_{aa}}\left(\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} - q\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\right)\mathbf{S_{aa}} = \mathbf{0}.\end{aligned} \tag{4.12}$$

This is a matrix quadratic equation known as algebraic Riccati equation in terms of complex symmetric matrix $\mathbf{S_{aa}}$ with all its coefficient matrices also being complex symmetric.

However, (4.12) can be further organized and factorized as:

$$\left\{\left(\hat{\mathbf{S}}^{(2)}\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} - q\hat{\mathbf{S}}^{(3)}\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\right) - \mathbf{S_{aa}}\left(\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} - q\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\right)\right\}\left(\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right) = \mathbf{0} \tag{4.13}$$

Here, for $\left|\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right| = 0$ be satisfied, for example in terms of two-dimensional matrix, it follows that:

$$\begin{vmatrix}\hat{S}_{11}^{(1)} - S_{11} & \hat{S}_{12}^{(1)} - S_{12}\\ \hat{S}_{12}^{(1)} - S_{12} & \hat{S}_{22}^{(1)} - S_{22}\end{vmatrix} = \left(\hat{S}_{11}^{(1)} - S_{11}\right)\left(\hat{S}_{22}^{(1)} - S_{22}\right) - \left(\hat{S}_{12}^{(1)} - S_{12}\right)^2 = 0.$$

which means the components of S-matrix have specific mutual relation that is not appropriate.

Therefore, we assume $\left|\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right| \neq 0$. Then, from (4.13),

$$\left(\hat{\mathbf{S}}^{(2)}\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} - q\hat{\mathbf{S}}^{(3)}\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\right) - \mathbf{S_{aa}}\left(\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} - q\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\right) = \mathbf{0}.$$

Therefore,

$$\mathbf{S_{aa}} = \left(\hat{\mathbf{S}}^{(2)}\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} - q\hat{\mathbf{S}}^{(3)}\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\right)\left(\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} - q\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\right)^{-1}. \tag{4.14}$$

This is the fundamental formula to estimate $\mathbf{S_{aa}}$.

4.2.4 Estimation formula for $\mathbf{S_{au}}$

Substituting $\mathbf{S_{aa}}$ obtained in the previous section to equation (4.7) makes it a matrix quadratic equation in terms of $\mathbf{S_{au}}$.

Hereafter, to satisfy the requirement of (4.9) easily, two values of loads are chosen with the reflection parameters of $s_l^{(1)}$ and $s_l^{(2)}$ to construct the set of loads:

$$\mathbf{S_L^{(1)}} = \operatorname{diag}\left[s_l^{(1)}, \cdots, s_l^{(1)}\right], \ \mathbf{S_L^{(2)}} = \operatorname{diag}\left[s_l^{(2)}, \cdots, s_l^{(2)}\right] \tag{4.15}$$

Then, another measurement is conducted with a set of loads using another value of load $s_l^{(3)}$:

$$\mathbf{S_L^{(4)}} = \operatorname{diag}\left[s_l^{(3)}, s_l^{(2)}, \cdots, s_l^{(2)}\right] \tag{4.16}$$

Substituting $i = 1$, $j = 4$ and $i = 1$, $j = 2$ to (4.7) and taking the difference yields:

$$\begin{aligned}
&\left(\hat{\mathbf{S}}^{(4)} - \mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(4)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right) - \left(\hat{\mathbf{S}}^{(2)} - \mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(2)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right)\\
&= \mathbf{S_{au}}\left\{\left(\mathbf{S_L^{(1)-1}} - \mathbf{S_L^{(4)-1}}\right)^{-1} - \left(\mathbf{S_L^{(1)-1}} - \mathbf{S_L^{(2)-1}}\right)^{-1}\right\}\mathbf{S_{au}^T}\\
&= \mathbf{S_{au}}\operatorname{diag}(c, 0, \cdots, 0)\mathbf{S_{au}^T}\\
&= c\begin{bmatrix} s_{t11}^2 & s_{t11}s_{t21} & \cdots & s_{t11}s_{tr1}\\ s_{t21}s_{t11} & s_{t21}^2 & \cdots & s_{t21}s_{tr1}\\ \vdots & \vdots & \ddots & \vdots\\ s_{tr1}s_{t11} & s_{tr1}s_{t21} & \cdots & s_{tr1}^2 \end{bmatrix}.
\end{aligned} \tag{4.17}$$

where

$$c = \left(\frac{1}{s_l^{(1)}} - \frac{1}{s_l^{(3)}}\right)^{-1} - \left(\frac{1}{s_l^{(1)}} - \frac{1}{s_l^{(2)}}\right)^{-1}, \tag{4.18}$$

and (i, j) component of $\mathbf{S_{au}}$ is denoted as s_{tij}. By comparing the $(1, 1)$ elements in both sides of (4.17), s_{t11} can be solved as a square root. Then, dividing the other elements in the 1st row by s_{t11}, values of the components $s_{t21}, \ldots, s_{tr1}$ can be obtained.

Since repeating the similar operations to obtain the remaining elements introduces extra sign ambiguity in the square root calculation as in [21], we make the following operations instead.

We conduct two more measurements introducing 'thru' load sets $\mathbf{S_L^{(5)}}$ and $\mathbf{S_L^{(6)}}$ as defined below and denote the obtained values with the corresponding suffixes. For

$\mathbf{S}_{\mathbf{L}}^{(5)}$, we connect a known impedance between ports $r+1$ and $r+2$ and another impedance between ports ports $r+3$ and $r+4$, and so on. For $\mathbf{S}_{\mathbf{L}}^{(6)}$, we connect a known impedance between ports $r+2$ and $r+3$ and another impedance between ports $r+4$ and $r+5$, and so on.

A thru-load with series impedance Z has the S-parameter as:

$$\mathbf{S_t} = \frac{1}{Z+2R_0}\begin{bmatrix} Z & 2R_0 \\ 2R_0 & Z \end{bmatrix}.$$

where R_0 is the reference impedance of the S-parameters. So, denoting the S-parameter of each thru-load with corresponding suffix, we can express $\mathbf{S}_{\mathbf{L}}^{(5)}$ and $\mathbf{S}_{\mathbf{L}}^{(6)}$ as follows:

For some positive integer p,

when $r = 2p$,

$$\mathbf{S}_{\mathbf{L}}^{(5)} = \begin{bmatrix} \mathbf{S}_{\mathbf{t}1} & & 0 \\ & \ddots & \\ 0 & & \mathbf{S}_{\mathbf{t}p} \end{bmatrix}, \mathbf{S}_{\mathbf{L}}^{(6)} = \begin{bmatrix} 1 & & & & 0 \\ & \mathbf{S}_{\mathbf{t}p+1} & & & \\ & & \ddots & & \\ & & & \mathbf{S}_{\mathbf{t}2p-1} & \\ 0 & & & & 1 \end{bmatrix}.$$

when $r = 2p+1$,

$$\mathbf{S}_{\mathbf{L}}^{(5)} = \begin{bmatrix} \mathbf{S}_{\mathbf{t}1} & & & 0 \\ & \ddots & & \\ & & \mathbf{S}_{\mathbf{t}p} & \\ 0 & & & 1 \end{bmatrix}, \mathbf{S}_{\mathbf{L}}^{(6)} = \begin{bmatrix} 1 & & & 0 \\ & \mathbf{S}_{\mathbf{t}p+1} & & \\ & & \ddots & \\ 0 & & & \mathbf{S}_{\mathbf{t}2p-1} \end{bmatrix}.$$

We may use identical impecance for all thru-loads including mere short. In that case, all $\mathbf{S}_{\mathbf{t}x}$ become identical. In any case, both $\mathbf{S}_{\mathbf{L}}^{(5)}$ and $\mathbf{S}_{\mathbf{L}}^{(6)}$ become tridiagonal matrices.

By substituting $i = 1$, $j = 2$ to (4.7) and $i = 5$, $j = 6$ to (4.6) then multiplying the latter to the former from the right, we have:

$$\left(\hat{\mathbf{S}}^{(2)} - \mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(2)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(5)} - \mathbf{S_{aa}}\right)^{-1}\left(\hat{\mathbf{S}}^{(6)} - \hat{\mathbf{S}}^{(5)}\right)\left(\hat{\mathbf{S}}^{(6)} - \mathbf{S_{aa}}\right)^{-1}$$
$$= \mathbf{S_{au}}\left(\mathbf{S}_{\mathbf{L}}^{(1)-1} - \mathbf{S}_{\mathbf{L}}^{(2)-1}\right)^{-1}\left(\mathbf{S}_{\mathbf{L}}^{(5)-1} - \mathbf{S}_{\mathbf{L}}^{(6)-1}\right)\mathbf{S}_{\mathbf{au}}^{-1}. \tag{4.19}$$

By denoting the left hand side of this equation as $\mathbf{L}$ and the term concerning the load in the right hand side as :

$$\mathbf{D} = \left(\mathbf{S}_{\mathbf{L}}^{(1)-1} - \mathbf{S}_{\mathbf{L}}^{(2)-1}\right)^{-1}\left(\mathbf{S}_{\mathbf{L}}^{(5)-1} - \mathbf{S}_{\mathbf{L}}^{(6)-1}\right),$$

from (4.19) we have a homogeneous Sylvester equation:

$$\mathbf{L}\mathbf{S}_{\mathbf{au}} - \mathbf{S}_{\mathbf{au}}\mathbf{D} = \mathbf{0}. \tag{4.20}$$

Here, since $\mathbf{S}_{\mathbf{L}}^{(1)}$ and $\mathbf{S}_{\mathbf{L}}^{(2)}$ are diagonal matrices and $\mathbf{S}_{\mathbf{L}}^{(5)}$ and $\mathbf{S}_{\mathbf{L}}^{(6)}$ are tridiagonal matrices, $\mathbf{D}$ becomes a tridiagonal matrix. By extracting the 1st column and using the tridiagonal property of $\mathbf{D}$, it follows:

$$\mathbf{L}\begin{bmatrix} s_{t11} \\ s_{t21} \\ \vdots \\ s_{tr1} \end{bmatrix} - \begin{bmatrix} s_{t11} \\ s_{t21} \\ \vdots \\ s_{tr1} \end{bmatrix} d_{11} - \begin{bmatrix} s_{t12} \\ s_{t22} \\ \vdots \\ s_{tr2} \end{bmatrix} d_{21} = \mathbf{0}. \tag{4.21}$$

where d_{ij} denotes the (i,j) component of $\mathbf{D}$.

Since we already solved $[s_{t11}, s_{t21}, \ldots, s_{tr1}]^{\mathrm{T}}$, we can calculate the values of $[s_{t12}, s_{t22}, \ldots, s_{tr2}]^{\mathrm{T}}$ from the equation above.

Likewise, by extracting the 2nd column and using the tridiagonal property of $\mathbf{D}$, it follows:

$$\mathbf{L}\begin{bmatrix} s_{t12} \\ s_{t22} \\ \vdots \\ s_{tr2} \end{bmatrix} - \begin{bmatrix} s_{t11} \\ s_{t21} \\ \vdots \\ s_{tr1} \end{bmatrix} d_{12} - \begin{bmatrix} s_{t12} \\ s_{t22} \\ \vdots \\ s_{tr2} \end{bmatrix} d_{22} - \begin{bmatrix} s_{t13} \\ s_{t23} \\ \vdots \\ s_{tr3} \end{bmatrix} d_{32} = \mathbf{0}. \tag{4.22}$$

Since we already solved $[s_{t11}, s_{t21}, \ldots, s_{tr1}]^{\mathrm{T}}$ and $[s_{t12}, s_{t22}, \ldots, s_{tr2}]^{\mathrm{T}}$, we can calculate the values of $[s_{t13}, s_{t23}, \ldots, s_{tr3}]^{\mathrm{T}}$ from the equation above.

Repeating the similar calculations for the remaining columns of the left hand side of equation (4.20), we can obtain all the components of $\mathbf{S}_{\mathbf{au}}$.

In the above procedure, as the result of the square root operation for s_{t11}, we obtain two versions of $\mathbf{S}_{\mathbf{au}}$ with opposite signs. This inherent ambiguity in signs comes from that we do not assume common ground between a-ports and u-ports as described in [19] [20]. It is similar to the admittance or impedance parameter sign ambiguity in [13].

4.2.5 Estimation formula for $\mathbf{S}_{\mathbf{uu}}$

Althoug substituting $\mathbf{S}_{\mathbf{aa}}$ and $\mathbf{S}_{\mathbf{au}}$ obtained in the previous sections and one of the measured $\hat{\mathbf{S}}$ to (4.5) gives an estimation for $\mathbf{S}_{\mathbf{uu}}$, we should use least square estimation using all the measured $\hat{\mathbf{S}}$ so far to decrease the error in estimation. Transforming

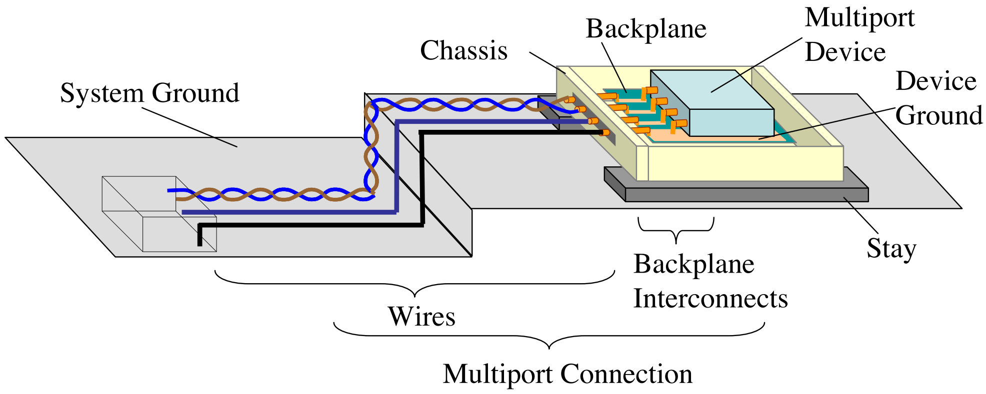


Figure 4.2: Multiport connection and multiport device in typical circuit system configuration.

equation (4.4), we have:

$$\left(\hat{\mathbf{S}}^{(\mathbf{i})} - \mathbf{S_{aa}}\right) \mathbf{S}_{\mathbf{au}}^{\mathrm{T}-1} \mathbf{S_{uu}} = \left(\hat{\mathbf{S}}^{(\mathbf{i})} - \mathbf{S_{aa}}\right) \mathbf{S}_{\mathbf{au}}^{\mathrm{T}-1} \mathbf{S}_{\mathbf{L}}^{(\mathbf{i})-\mathbf{1}} - \mathbf{S_{au}}. \tag{4.23}$$

for the i-th load set. Making the equations above for all loads and solving them in the least square sense gives us the solution for $\mathbf{S_{uu}}$.

From the operations so far, we have the whole S parameters for the multiport connection.

4.3 Estimation Method for Multiport Device S-parameters

If we denote the S-matrix for a multiport device as $\mathbf{S_I}$, we can obtain the relation for it by replacing $\mathbf{S_L}$ with $\mathbf{S_I}$ in (4.5) since it occupies the same position in the circuit as $\mathbf{S_L}$ in Fig.4.1. So,

$$\mathbf{S_I} = \left(\mathbf{S_{uu}} + \mathbf{S}_{\mathbf{au}}^{\mathrm{T}} \left(\hat{\mathbf{S}}_{\mathbf{I}} - \mathbf{S_{aa}}\right)^{-1} \mathbf{S_{au}}\right)^{-1}. \tag{4.24}$$

We conduct a measurement with connecting target mulltiport device to the output end of the multiport connection and obtain $\hat{\mathbf{S}}_{\mathbf{I}}$. Applying the connection S-parameters $\mathbf{S}$ estimated in the previous section and the $\hat{\mathbf{S}}_{\mathbf{I}}$ to (4.24), we can estimate the S parameter of the target multiport device. The sign ambiguities of $\mathbf{S_{au}}$ are canceled as the result of the matrix multiplication operation in (4.24).

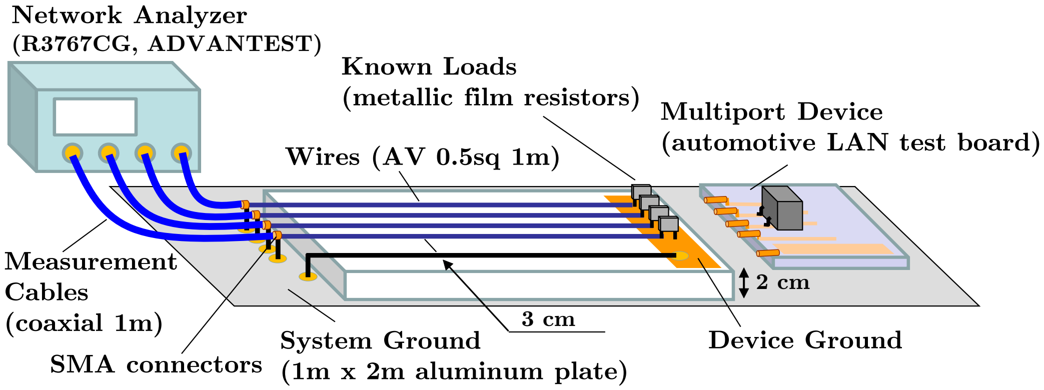


Figure 4.3: S-parameter measurement configuration.

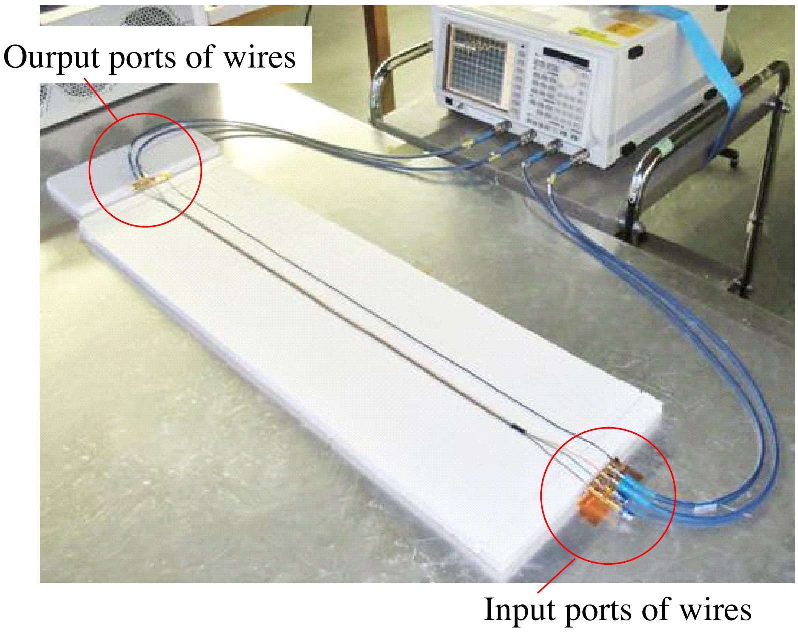


Figure 4.4: Picture of connection wire partial S-parameter measurement.

4.4 Uniqueness of the Absolute Values of Port Voltages

If we connect a multiport device with known S-parameters, the port voltages can be calculated using the estimated multiport connection S-parameters. Since the estimated S-parameters have a sign ambiguity, but the same analysis shown in section 2.2.2 can be applied and it can be shown that the absolute values of the port voltages are unique regardless of which sign is adopted.

4.5 Evaluation of the Estimation Method with Experiment

A typical electric circuit system is shown in Fig.4.2 that has wires and a backplane interconnects in chassis mounting a device. The wires and the interconnects together consists a multiport connection and the device consists a multiport device. The system can be treated with an S-parameter model shown in Fig.4.1.

In this section, the effectiveness of the S parameter estimation method proposed in this chapter is evaluated in terms of an actual measurement data obtained by the experiment with a simple implementation of the system in Fig.4.2 using four closely placed connection wires and a ground wire placed in the distance of 3cm from the other wires as shown in Fig. 4.3. At the output end u-ports of the connection wires, there placed a strip of copper tape as device ground that was separated from the bottom aluminum plate as system ground. The S-parameter measurement was conducted using a network analyzer with 0.83308MHz interval and the data in range of 1 to 200MHz is used in this section.

For the estimation, four input ports of the connection wires were measured as a-ports. We used the sets of known loads shown in table 4.1. In the calculation, not the measured frequency characteristics but the nominal values of the known loads were used, that is, reflection parameter value of 1 for open, -1 for short, theoretical reflection parameter in reference to 50Ω for 100Ω and 270Ω, etc.

Some results of the estimated 8-port connection S-parameters are shown in Fig. 4.5. To obtain the measured values using 4-port network analyzer, we chose port

Table 4.1: Load sets used in connection S-parameter estimation.

Load set #	Load values: Port 5	Port 6	Port 7	Port 8	Equations applied: $\mathbf{S_{aa}}$	$\mathbf{S_{au}}$	$\mathbf{S_{uu}}$
(1)	100Ω	100Ω	100Ω	100Ω	(4.14)	(4.17) (4.20)	(4.23)
(2)	open	open	open	open	(4.14)	(4.17) (4.20)	(4.23)
(3)	short	short	short	short	(4.14)	-	(4.23)
(4)	270Ω	100Ω	100Ω	100Ω	-	(4.17)	(4.23)
(5)	thru short		thru short		-	(4.20)	(4.23)
(6)	open	thru short		open	-	(4.20)	(4.23)

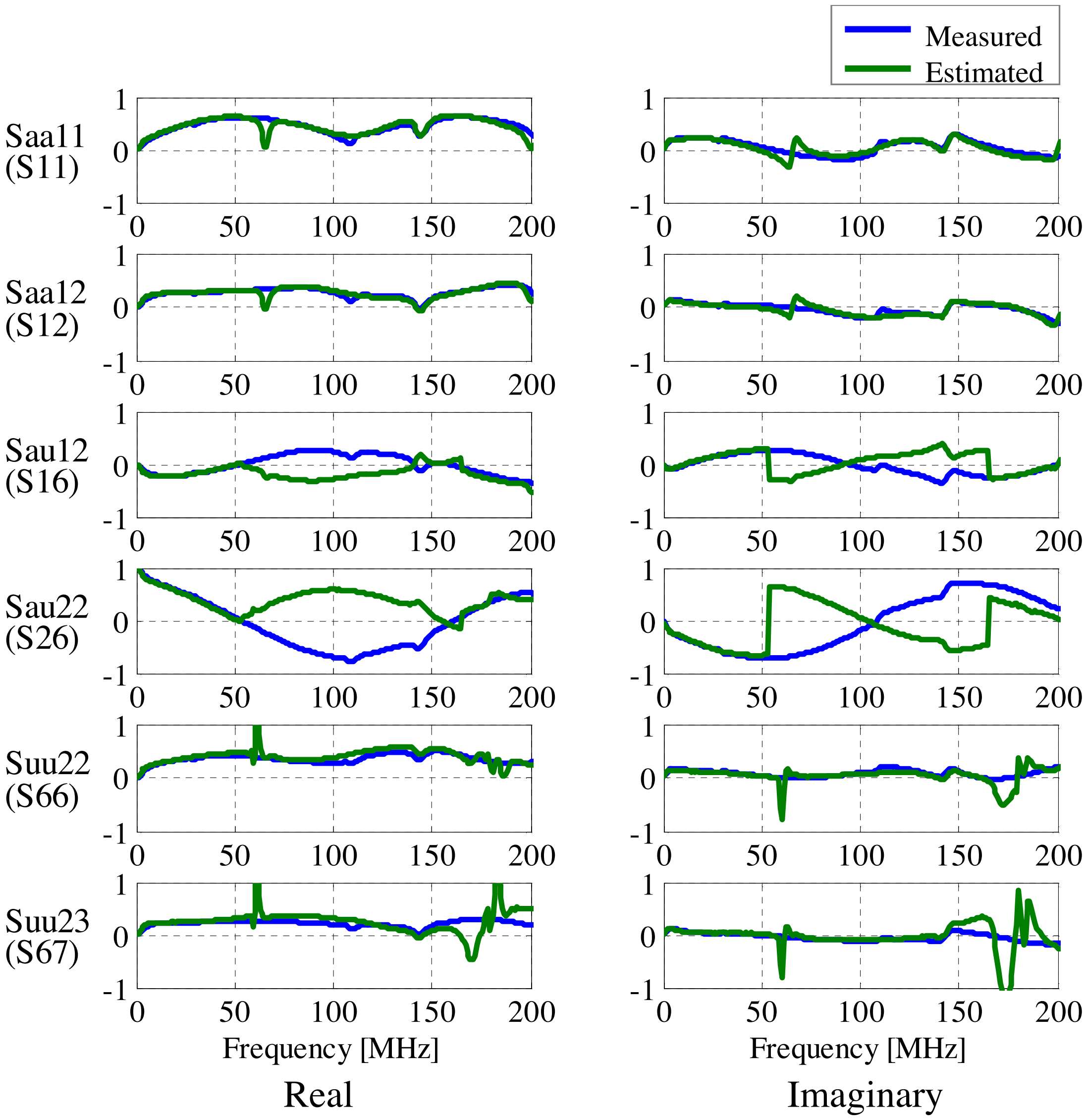


Figure 4.5: Measured and estimated values of connection S-parameters (excerpted).

1 and port 2 out of the input a-ports of the wires and port 6 and port 7 out of the output u-ports of the wires to connect to the network analyzer and terminated other ports with 50Ω terminators as shown inf Fig. 4.4. Through this measurement configuration, we have added an extra path through the network analyzer's common ground between the system ground and the device ground, which might have introduced some disturbances to the measured S-parameters. In the estimation process, the positive result of the square root operation was taken for s_{t11} in equation (4.17). As the result, the value for $\mathbf{S_{au}}$ shows sign ambiguity as expected. The estimated values have spikes around 70MHz. Since the lengths of the wires (multiport connections) are all 1m, the spikes are estimated to have come from the errors in

measurements caused by the quarter-wave resonances in the wires. Except for these spikes, results of the measured and the estimated S-parameters show good agreement below 150MHz neglecting the sign ambiguity in $\mathbf{S_{au}}$.

Next, we connected an automotive LAN test board as a sample multiport device to the output u-ports of the connection wires and measured the input a-ports of them to obtain $\hat{\mathbf{S}}_\mathbf{I}$ values. Some results of the estimated 4-port S-parameters for the target device are shown in Fig. 4.6. Here, Port 1 through to Port 4 of the 4-port device were connected to Port 5 through to Port 8 of the 8-port connection wires, respectively. The estimated values are plotted as 'Estimated (no penalty)' and show large fluctuations above 50MHz with large deviation from the Measured values. We will try to alleviate this fluctuation in the next section. The sign ambiguity in $\mathbf{S_{au}}$ does not affect the results as can be seen.

4.6 Method to Improve the Frequency Smoothness of Multiport Device S-parameters

So far, we treat the S-parameters and their manipulations in one frequency point basis. However, sometimes the instability feature of inverse problem becomes not negligible and in that situation treating the S-parameters as continuous functions of frequency is effective. For this purpose, instead of using (4.24), we can use an equation obtained from (4.4):

$$\left\{\left(\hat{\mathbf{S}}_\mathbf{I} - \mathbf{S_{aa}}\right)\mathbf{S}_\mathbf{au}^{\mathrm{T}-1}\mathbf{S_{uu}} + \mathbf{S_{au}}\right\}\mathbf{S_I} = \left(\hat{\mathbf{S}}_\mathbf{I} - \mathbf{S_{aa}}\right)\mathbf{S}_\mathbf{au}^{\mathrm{T}-1}. \tag{4.25}$$

Denoting the right hand side of the equation above as $\mathbf{S_{rh}}$ and the coefficient matrix in its left hand side as $\mathbf{S_e}$, the equation to solve $\mathbf{S_I}$ for all the frequency sampling points simultaneously can be written as:

$$\begin{bmatrix} \mathbf{S}_{\mathbf{e}(1)} & & & 0 \\ & \mathbf{S}_{\mathbf{e}(2)} & & \\ & & \ddots & \\ 0 & & & \mathbf{S}_{\mathbf{e}(w)} \end{bmatrix} \begin{bmatrix} \mathbf{S}_{\mathbf{I}(1)} \\ \mathbf{S}_{\mathbf{I}(2)} \\ \vdots \\ \mathbf{S}_{\mathbf{I}(w)} \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{\mathbf{rh}(1)} \\ \mathbf{S}_{\mathbf{rh}(2)} \\ \vdots \\ \mathbf{S}_{\mathbf{rh}(w)} \end{bmatrix}. \tag{4.26}$$

where the suffix indicates the frequency points and w denotes the number of frequency points. The above equation has no relation between the frequency points. It is known that to alleviate the instability in an inverse problem, we can use the "smoothing matrix" $\mathbf{H}$ of dimension w to introduce quadratic penalty to large second derivatives in a scalar data series [23]- [25] :

$$\mathbf{H} = \begin{bmatrix} 1 & -2 & 1 & & & & & & \\ -2 & 5 & -4 & 1 & & & \text{\Large 0} & & \\ 1 & -4 & 6 & -4 & -1 & & & & \\ & & & \ddots & & & & & \\ & & 1 & -4 & 6 & -4 & -1 & & \\ & & & & & \ddots & & & \\ & & & & 1 & -4 & 6 & -4 & -1 \\ & & \text{\Large 0} & & & 1 & -4 & 5 & -2 \\ & & & & & & 1 & -2 & 1 \end{bmatrix}$$

To apply this for the $\mathbf{S_I}$ matrix data series, denoting the right hand side of equation (4.26) as $\bar{\mathbf{S}}_{\mathbf{rh}}$ and the coefficient matrix in its left hand side as $\bar{\mathbf{S}}_{\mathbf{e}}$, we solve the following equation of least-square with quadratic penalty:

$$\left(\bar{\mathbf{S}}_{\mathbf{e}}^{*}\,\bar{\mathbf{S}}_{\mathbf{e}} + \lambda^{\mathbf{2}}\mathbf{P}\right) \begin{bmatrix} \mathbf{S}_{\mathbf{I}(1)} \\ \mathbf{S}_{\mathbf{I}(2)} \\ \vdots \\ \mathbf{S}_{\mathbf{I}(w)} \end{bmatrix} = \bar{\mathbf{S}}_{\mathbf{e}}^{*}\,\bar{\mathbf{S}}_{\mathbf{rh}} \tag{4.27}$$

where

$$\mathbf{P} = \mathbf{H} \otimes \mathbf{I_r}, \tag{4.28}$$

asterisk '$*$' indicates Hermitian conjugate, λ is an appropriate real constant, '$\otimes$' indicates Kronecker product [26] and $\mathbf{I_r}$ is an identity matrix of dimension r.

If we have an a priori knowledge about the smoothness of the device S-parameter, we can have an appropriate result by adjusting the largeness of λ.

4.7 Evaluation of the Improvement in the Frequency Smoothness with Experimental Values

The method in the previous section is evaluated by applying it to the experiment data obtained in section 4.5. For the quadratic penalty, λ value of 2000 was applied.

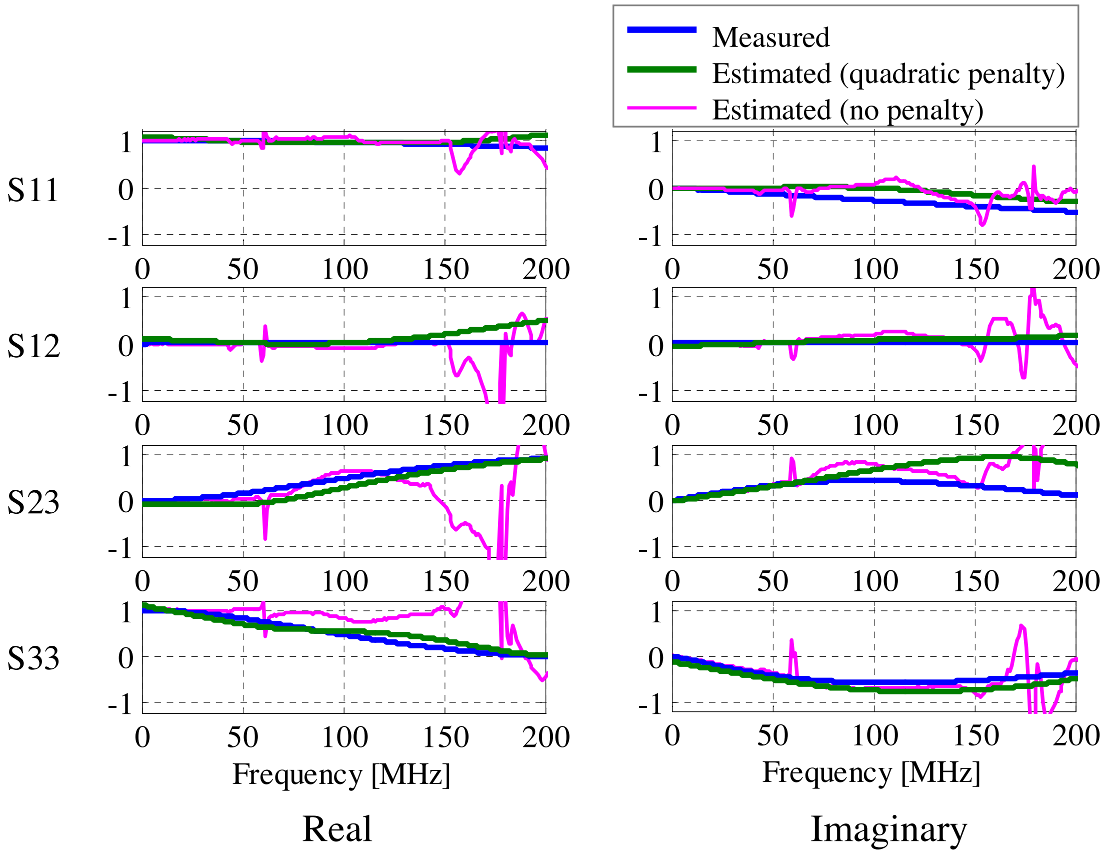


Figure 4.6: Measured and estimated values of device S-parameters (excerpted).

This relieves the fluctuation of the estimated values and the deviation from the measured values is significantly reduced throughout the evaluated frequencies. As the result, the estimated values now show good agreement with the measured values below 100MHz.

4.8 Chapter Conclusion

An estimation method of submatrix elimination and successive solution for the 2r-port S-parameters for reciprocal circuits with direct measurements of r ports has been presented. The fundamental nonlinear estimation equation was given in the form of a matrix quadratic equation. Then, the equation was transforms into a set of matrix linear equations. To do this, the number of equations were increased through extra measurements with connecting known loads in turn to the indirect measurement ports. This approach enables the theoretical analysis in the framework of linear algebra. As the result, it was shown that at least 6 sets of known loads are required to solve the whole connection S-parameters and they include 2 sets of thru-loads to calculate the transfer characteristics except for the 4-port circuit case which requires 5 sets of known loads including 1 set of thru-load. Theoretically, the number of measurements does not increase as the number of ports increases when we count the operation measuring through directly measured ports with connecting a set of known load circuits to indirectly measured ports as one measurement.

The estimation method was presented assuming the estimations for the S-parameters of both a multiport connection and a multiport device. The multiport device is connected to the output end of the multiport connection and a network analyzer is connected to its input end. Here, the ground for the multiport device and the ground for the input end of the connection as the system ground are not assumed to be common. The measurements of reflection and transmission characteristics among the ports of input end of the connection are only required while several sets of loads are connected to the output end of the connection for the estimation, which makes this method effective in case that connecting measurement probes directly to the multiport device is physically difficult or it might change the high frequency environments of the device. Both multiport connection and device S-parameters are estimated by solving several linear equations and square root calculations only. It was also shown that to obtain the device S-parameter smooth enough in frequency, introducing quadratic penalty with smoothing matrix is effective to cancel the instability of the inverse problem. The validity of the proposed method has also been confirmed by simple experimental results.

This study shows the ground configurations specified above reduces the sign ambiguities in the multiport connection S-parameters to one while they are unlimited without specifying the ground configuration as in our previous study. Also, the original results of this study includes the method to estimate the S-parameters of the multiport device and the proof that they do not have sign ambiguities in spite of the sign ambiguities in the multiport connection S-parameters.

Chapter 5

Improved estimation of 4 port S-parameters with 2 port measurements

As a specific implementation of the method described in the previous chapter, an estimation method for the S-parameters of the 2-port circuit that is connected to far ends of the 4-port connection circuit is presented in this chapter. A calculation technique to improve the estimation accuracy is also presented, which can be applied to this target without extra calculation or measurement costs.

To obtain the 2-port S-parameters, first, the 4-port S-parameters for the connection circuit are estimated using indirect measurements. Several known loads are connected to the 2 ports at the far ends of the connection circuit instead of the target 2-port circuit in turn and the reflection and transmission characteristics among the near end 2 ports of the connection circuit are measured. Next, 2-port S-parameters of the target circuit are estimated from the measured S-parameters and the estimated S-parameters for the connection circuit.

By applying this method to estimate the S-parameters of the circuit contained in a high-voltage (HV) unit mounted on electric-driven vehicles with power cables in an EM simulation model, the validity of the proposed method is also confirmed. This utilizes an advantage of this method that it is not necessary to connect measurement instruments to the far end ports of the connection circuit.

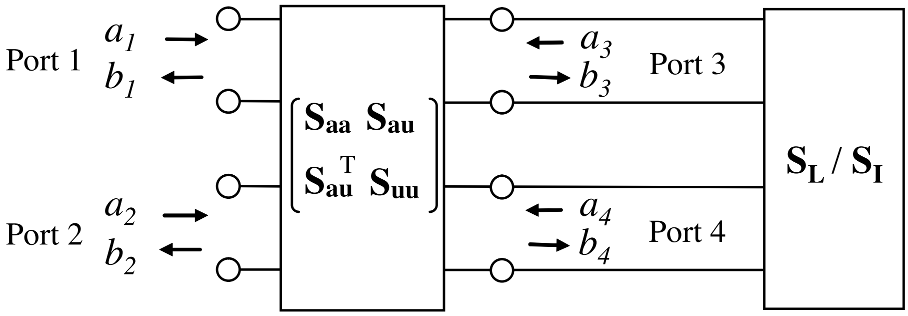


Figure 5.1: S-parameter port arrangement of 4-port connection circuit and 2-port target circuit

5.1 Estimation Method

5.1.1 Reduced S-matrix

The target system of this chapter has two ports that can be directly measured with an instrument and two internal ports that can be connected to known loads but are hard to measure directly with an instrument. This can be viewed as a reciprocal 4-port connection circuit shown in Fig.5.1 which has two ports that are hard to access with measurement probes. We measure the S-parameters of the remaining two ports by attaching known loads to the 2 hard-to-access ports and estimate the whole 4-port S-parameters from the measured 2-port S-parameters.

The S-parameter matrix $\mathbf{S}$ can be written in block matrix form as follows where suffix a indicates the ports for direct measurements and suffix u indicates the ports for indirect measurements. The incident wave $\mathbf{a} = \left[\mathbf{a_a}^{\mathrm{T}}\ \mathbf{a_u}^{\mathrm{T}}\right]^{\mathrm{T}}$ and the reflected wave $\mathbf{b} = \left[\mathbf{b_a}^{\mathrm{T}}\ \mathbf{b_u}^{\mathrm{T}}\right]^{\mathrm{T}}$ satisfy:

$$\begin{aligned} \begin{bmatrix} \mathbf{b_a} \\ \mathbf{b_u} \end{bmatrix} &= \mathbf{S} \begin{bmatrix} \mathbf{a_a} \\ \mathbf{a_u} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{S_{aa}} & \mathbf{S_{au}} \\ \mathbf{S_{au}^{\mathrm{T}}} & \mathbf{S_{uu}} \end{bmatrix} \begin{bmatrix} \mathbf{a_a} \\ \mathbf{a_u} \end{bmatrix} \end{aligned} \tag{5.1}$$

where:

$$\mathbf{S_{aa}},\ \mathbf{S_{au}},\ \mathbf{S_{uu}} \in \mathbb{C}^{2\times2},\ \mathbf{a_a}, \mathbf{b_a}, \mathbf{a_u}, \mathbf{b_u} \in \mathbb{C}^{2\times1}.$$

When we connect loads with known S-parameter values of $\mathbf{S_L}$ to the u-ports, the following holds:

$$\mathbf{a_u} = \mathbf{S_L}\mathbf{b_u}. \tag{5.2}$$

By denoting the measured 2-port S-parameters as $\hat{\mathbf{S}}$, it follows that:

$$\mathbf{b_a} = \hat{\mathbf{S}}\mathbf{a_a}. \tag{5.3}$$

Rewriting Eq.(5.1), we have:

$$\hat{\mathbf{S}} = \mathbf{S_{aa}} + \mathbf{S_{au}}\mathbf{S_L}\left(\mathbf{I} - \mathbf{S_{uu}}\mathbf{S_L}\right)^{-1}\mathbf{S_{au}^T}. \tag{5.4}$$

Here, $\mathbf{I}$ denotes the identity matrix.

In case of $\mathbf{S_L} = \mathbf{O}$, from Eq.(5.4), $\hat{\mathbf{S}} = \mathbf{S_{aa}}$ which enables the value of $\mathbf{S_{aa}}$ be directly obtained from the measured values. In the following subsections, we consider the general case of $|\mathbf{S_L}| \neq \mathbf{0}$.

5.1.2 Elimination of $\mathbf{S_{uu}}$ and estimation formula for $\mathbf{S_{aa}}$

Detailed deduction of the equations in this subsection can be found in chapter 4. The outline is shown here again.

From Eq.(5.4). we have:

$$\mathbf{S_{uu}} = \mathbf{S_L^{-1}} - \mathbf{S_{au}^T}\left(\hat{\mathbf{S}} - \mathbf{S_{aa}}\right)^{-1}\mathbf{S_{au}}. \tag{5.5}$$

We conduct multiple measurements with several known loads attached to the hard-to-access u-ports. We denote the loads' S-matrix $\mathbf{S_L}$ for the i-th measurement as $\mathbf{S_L}^{(i)}$ and the measured S-matrix $\hat{\mathbf{S}}$ as $\hat{\mathbf{S}}^{(i)}$. Taking the differences of each side of Eq.(5.5) for the i-th and j-th measurements and simplifying them yields:

$$\left(\hat{\mathbf{S}}^{(\mathrm{i})} - \mathbf{S_{aa}}\right)^{-1}\left(\hat{\mathbf{S}}^{(\mathrm{j})} - \hat{\mathbf{S}}^{(\mathrm{i})}\right)\left(\hat{\mathbf{S}}^{(\mathrm{j})} - \mathbf{S_{aa}}\right)^{-1} = \mathbf{S_{au}^{T-1}}\left(\mathbf{S_L^{(i)-1}} - \mathbf{S_L^{(j)-1}}\right)\mathbf{S_{au}^{-1}}. \tag{5.6}$$

Taking the inverses of the both sides of Eq.(5.6) gives:

$$\left(\hat{\mathbf{S}}^{(\mathrm{j})} - \mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(\mathrm{j})} - \hat{\mathbf{S}}^{(\mathrm{i})}\right)^{-1}\left(\hat{\mathbf{S}}^{(\mathrm{i})} - \mathbf{S_{aa}}\right) = \mathbf{S_{au}}\left(\mathbf{S_L^{(i)-1}} - \mathbf{S_L^{(j)-1}}\right)^{-1}\mathbf{S_{au}^T}. \tag{5.7}$$

We conduct one more measurement and express the obtained values with upper suffix (3) under the condition that the values of the loads satisfy:

$$\mathbf{S_L^{(3)-1}} - \mathbf{S_L^{(1)-1}} = q\left(\mathbf{S_L^{(2)-1}} - \mathbf{S_L^{(1)-1}}\right) \tag{5.8}$$

where q is an arbitrary non-zero constant.

Using Eq.(5.7) and the relation of Eq.(5.8) and denoting
$\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} = \left(\hat{\mathbf{S}}^{(2)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}$ and $\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1} = \left(\hat{\mathbf{S}}^{(3)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}$,
we have:

$$\mathbf{S_{aa}} = \left(\hat{\mathbf{S}}^{(2)}\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} - q\hat{\mathbf{S}}^{(3)}\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\right)\left(\mathbf{\Delta}\hat{\mathbf{S}}^{(2)-1} - q\mathbf{\Delta}\hat{\mathbf{S}}^{(3)-1}\right)^{-1} \tag{5.9}$$

where $\left|\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right| \neq 0$ is assumed. This is the fundamental formula to estimate $\mathbf{S_{aa}}$.

5.1.3 Estimation formula for $\mathbf{S_{au}}$

Solving the first column vector of $\mathbf{S_{au}}$

Substituting $\mathbf{S_{aa}}$ obtained in the previous section to Eq.(5.7) makes it a matrix quadratic equation in terms of $\mathbf{S_{au}}$. Since it is not directly solvable, we will solve it in a stepwise manner.

Hereafter, to satisfy the requirement of Eq.(5.8) easily, three values of terminating loads are chosen with the reflection parameters of s_1, s_2 and s_3 to construct the pairs of loads:

$$\mathbf{S_L}^{(1)} = \mathrm{diag}[s_1, s_1],\ \mathbf{S_L}^{(2)} = \mathrm{diag}[s_2, s_2],\ \mathbf{S_L}^{(3)} = \mathrm{diag}[s_3, s_3]. \tag{5.10}$$

Then, another measurement is conducted with loads:

$$\mathbf{S_L}^{(4)} = \mathrm{diag}[s_3, s_2]. \tag{5.11}$$

Substituting $i = 1$, $j = 4$ and $i = 1$, $j = 2$ to Eq.(5.7) and taking the difference yields:

$$\begin{aligned}
&\left(\hat{\mathbf{S}}^{(4)} - \mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(4)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right) - \left(\hat{\mathbf{S}}^{(2)} - \mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(2)} - \hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right) \\
&= \mathbf{S_{au}}\left\{\left(\mathbf{S_L}^{(1)-1} - \mathbf{S_L}^{(4)-1}\right)^{-1} - \left(\mathbf{S_L}^{(1)-1} - \mathbf{S_L}^{(2)-1}\right)^{-1}\right\}\mathbf{S_{au}^T} \\
&= \mathbf{S_{au}}\begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix}\mathbf{S_{au}^T} \\
&= c\begin{bmatrix} s_{t11}^2 & s_{t11}s_{t21} \\ s_{t21}s_{t11} & s_{t21}^2 \end{bmatrix},
\end{aligned} \tag{5.12}$$

where

$$c = \frac{s_1^2\,(s_2 - s_3)}{(s_3 - s_1)\,(s_2 - s_1)} \tag{5.13}$$

and the i, j components in $\mathbf{S_{au}}$ are denoted as s_{tij}. By comparing the $(1, 1)$ elements in both sides of Eq.(5.12), s_{t11} can be solved as a square root. Then, dividing the 2nd element in the 1st row by s_{t11}, the value of s_{t21} can be obtained. However, we have to introduce a sign ambiguity to the solution by the square root operation.

So far we obtained the first column vector of $\mathbf{S_{au}}$. The second column vector has to be solved keeping the consistency with the first column vector. This can be achieved by either of the following two methods.

Solving the second column vector of $\mathbf{S_{au}}$ using thru-load measurement

Multiplying Eq.(5.6) to Eq.(5.7) from the right yields:

$$\begin{aligned}&\left(\hat{\mathbf{S}}^{(2)}-\mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(2)}-\hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)}-\mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(\mathrm{i})}-\mathbf{S_{aa}}\right)^{-1}\left(\hat{\mathbf{S}}^{(\mathrm{j})}-\hat{\mathbf{S}}^{(\mathrm{i})}\right)\left(\hat{\mathbf{S}}^{(\mathrm{j})}-\mathbf{S_{aa}}\right)^{-1}\\&=\mathbf{S_{au}}\left(\mathbf{S_L}^{(1)-1}-\mathbf{S_L}^{(2)-1}\right)^{-1}\left(\mathbf{S_L}^{(\mathrm{i})-1}-\mathbf{S_L}^{(\mathrm{j})-1}\right)\mathbf{S_{au}}^{-1}.\end{aligned}\tag{5.14}$$

By denoting the left hand side of this equation as $\mathbf{L_h}$ and the term concerning the load in the right hand side as :

$$\mathbf{D}=\left(\mathbf{S_L}^{(1)-1}-\mathbf{S_L}^{(2)-1}\right)^{-1}\left(\mathbf{S_L}^{(\mathrm{i})-1}-\mathbf{S_L}^{(\mathrm{j})-1}\right).\tag{5.15}$$

From Eq.(5.14) we have:

$$\mathbf{L_h}\mathbf{S_{au}}-\mathbf{S_{au}}\mathbf{D}=\mathbf{0}.\tag{5.16}$$

By extracting the first column, it follows:

$$\begin{bmatrix}L_{h11}-D_{11} & L_{h12}\\ L_{h21} & L_{h22}-D_{11}\end{bmatrix}\begin{bmatrix}s_{t11}\\ s_{t21}\end{bmatrix}-D_{21}\begin{bmatrix}s_{t12}\\ s_{t22}\end{bmatrix}=\mathbf{0}.\tag{5.17}$$

Since we already solved s_{t11} and s_{t21} in Eq.(5.12), we can calculate the values of s_{t12} and s_{t22} from the above equation. Thus, all the components of $\mathbf{S_{au}}$ are solved. However, to enable the calculation, non-zero value of D_{21} is required, which then requires at least one of $\mathbf{S_L}^{(\mathrm{i})}$ or $\mathbf{S_L}^{(\mathrm{j})}$ has non-diagonal components that means a load connecting the terminals of u-ports, i.e. "thru load." This means that by relating the u-ports using the thru load, the sign ambiguities can be reduced.

Solving the second column vector of $\mathbf{S_{au}}$ using the terminating load measurements

Using $\mathbf{S_L}^{(3)}$ instead of $\mathbf{S_L}^{(2)}$ in Eq.(5.12) we have:

$$\begin{aligned}&\left(\hat{\mathbf{S}}^{(4)}-\mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(4)}-\hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)}-\mathbf{S_{aa}}\right)-\left(\hat{\mathbf{S}}^{(3)}-\mathbf{S_{aa}}\right)\left(\hat{\mathbf{S}}^{(3)}-\hat{\mathbf{S}}^{(1)}\right)^{-1}\left(\hat{\mathbf{S}}^{(1)}-\mathbf{S_{aa}}\right)\\&=\mathbf{S_{au}}\left\{\left(\mathbf{S_L}^{(1)-1}-\mathbf{S_L}^{(4)-1}\right)^{-1}-\left(\mathbf{S_L}^{(1)-1}-\mathbf{S_L}^{(3)-1}\right)^{-1}\right\}\mathbf{S_{au}}^{\mathrm{T}}\\&=\mathbf{S_{au}}\begin{bmatrix}0 & 0\\ 0 & -c\end{bmatrix}\mathbf{S_{au}}^{\mathrm{T}}\\&=-c\begin{bmatrix}s_{t12}^2 & s_{t12}s_{t22}\\ s_{t22}s_{t12} & s_{t22}^2\end{bmatrix},\end{aligned}\tag{5.18}$$

By applying similar operations as in Sect.5.1.3, we can solve the second column vector with sign ambiguities. Since we have to keep the sign consistency of the solution with the first column vector we use Eq.(5.17) to check the consistency and invert the sign of the solution if the equation is not satisfied.

Since $\mathbf{S_{au12}}$ and $\mathbf{S_{au22}}$ are indirectly calculated using the estimated values of $\mathbf{S_{au11}}$ and $\mathbf{S_{au21}}$ in this method while they are directly calculated from the measured values in the previous subsection, this method is estimated to provide better accuracy although it requires a little more calculation.

When the estimation methods will be extended to the target circuits with more ports than assumed in this chapter, the measurements and calculations required for the second method in this subsection becomes much more than those for the first method although the second method provides better accuracies.

In the above procedures, as the result of the square root operation for s_{t11}, we obtain two versions of $\mathbf{S_{au}}$ with opposite signs. This inherent ambiguity in signs comes from that we do not assume common ground between a-ports and u-ports as described in [19] [20] [28]. It is similar to the admittance or impedance parameter sign ambiguity in [13].

5.1.4 Estimation formula for $\mathbf{S_{uu}}$

By substituting $\mathbf{S_{aa}}$ and $\mathbf{S_{au}}$ obtained in the previous sections and one of the measured $\hat{\mathbf{S}}$ to the above, we obtain the estimation for $\mathbf{S_{uu}}$ from Eq.(5.5).

Least square equation can also be used by collecting more than r equations to reduce the error as follows:

$$\begin{bmatrix} \mathbf{I} \\ \mathbf{I} \\ \vdots \\ \mathbf{I} \end{bmatrix} \mathbf{S_{uu}} = \begin{bmatrix} \mathbf{S_L^{(1)-1}} - \mathbf{S_{au}^T} \left(\hat{\mathbf{S}}^{(1)} - \mathbf{S_{aa}}\right)^{-1} \mathbf{S_{au}} \\ \mathbf{S_L^{(2)-1}} - \mathbf{S_{au}^T} \left(\hat{\mathbf{S}}^{(2)} - \mathbf{S_{aa}}\right)^{-1} \mathbf{S_{au}} \\ \vdots \\ \mathbf{S_L^{(p)-1}} - \mathbf{S_{au}^T} \left(\hat{\mathbf{S}}^{(p)} - \mathbf{S_{aa}}\right)^{-1} \mathbf{S_{au}} \end{bmatrix} \tag{5.19}$$

From the operations so far, we have the estimated values for $\mathbf{S_{aa}}$, $\mathbf{S_{au}}$ and $\mathbf{S_{uu}}$, thus, the whole 4-port connection S-parameters. However, there are inherent ambiguities of signs in terms of the blocks of $\mathbf{S_{au}}$ and $\mathbf{S_{au}^T}$.

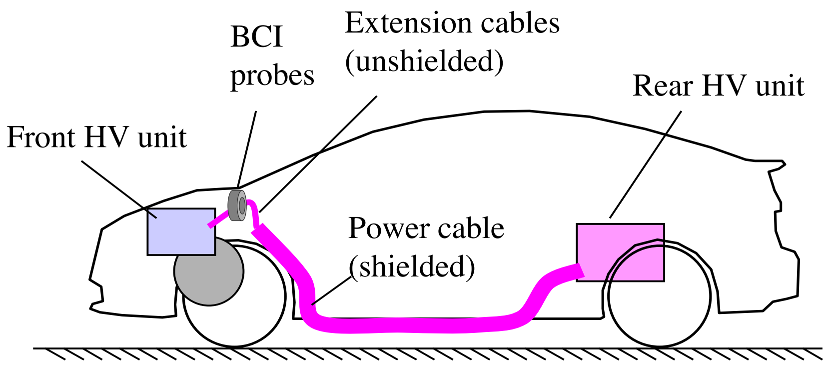


Figure 5.2: BCI probes are inserted to the short unshielded cables extending the internal conductors of the shielded power cable. In the typical structure, this can be done at the connector of the front HV unit.

5.1.5 Estimation formula for S-parameters of internal 2-port target circuit

If we denote the 2-port S-matrix for the internal high-voltage circuit as $\mathbf{S_I}$, we can obtain the relation for it by replacing $\mathbf{S_L}$ with $\mathbf{S_I}$ in Eq.(5.5) since it occupies the same position in the circuit as $\mathbf{S_L}$ in Fig.5.1. So, we have:

$$\mathbf{S_I} = \left(\mathbf{S_{uu}} + \mathbf{S_{au}^T} \left(\hat{\mathbf{S}} - \mathbf{S_{aa}}\right)^{-1} \mathbf{S_{au}}\right)^{-1}. \tag{5.20}$$

Applying the estimated $\mathbf{S}$ and the $\hat{\mathbf{S}}$ measured with normal connection of internal high-voltage circuit to the above equation, we can estimate the $\mathbf{S_I}$. The sign ambiguities of $\mathbf{S_{au}}$ are canceled as the result of the matrix multiplication operation in Eq.(5.20).

5.2 Evaluation of the Estimation Method with Numerical Experiment

5.2.1 Measurement Configuration for Internal Circuit in HV Unit using BCIs

A typical configuration of high voltage systems in electric driven vehicles is shown in Fig.5.2. It consists of two HV units that are mounted at separate locations in

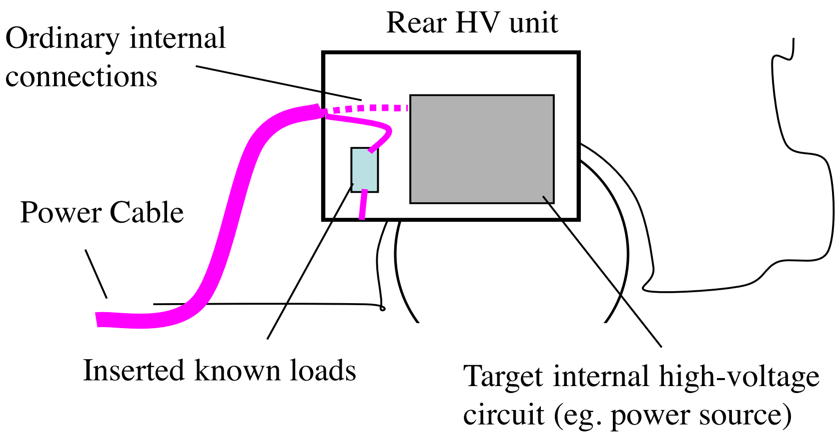


Figure 5.3: In the rear HV unit, power cable connections to the target internal high-voltage 2-port circuit are detached and routed to insert the known loads used for the estimation data measurement.

a vehicle and are connected to each other with power cables. The rear HV unit contains an internal high-voltage circuit, typically a power source. To measure the S-parameters of the internal high-voltage circuit, we insert a BCI probe to each of the power cables at their connectors on the front HV unit. As the first step, we figure out the connection S-parameters between the ports of the BCI probes and the power cable's connection points to the target internal high-voltage circuit by inserting known loads to the connection points (Fig.5.3) and measuring the S-parameters at the BCI probes. Then, as the second step, by returning to the ordinary connection to the internal high-voltage circuit and measuring the S-parameters at the BCI probes, we can calculate the S-parameters of the internal high-voltage circuit using the connection S-parameters obtained in the first step.

The measurement configuration for the HV unit described above has two BCI ports that can be measured with an instrument and two internal ports that can be connected to known loads but are hard to measure directly with an instrument. This can be viewed as a reciprocal 4-port connection circuit shown in Fig.5.1 which has two ports that are hard to access with measurement probes.

5.2.2 Results of the numerical experiment

In this section, the effectiveness of the S-parameter estimation method proposed in this chapter is evaluated using an EM simulation modeling the automotive immunity

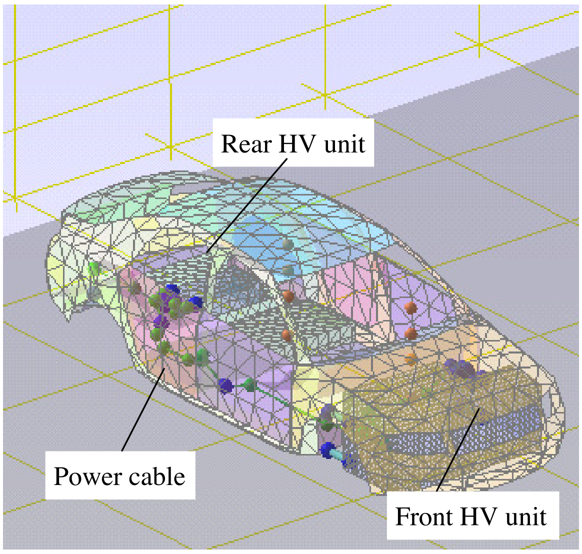
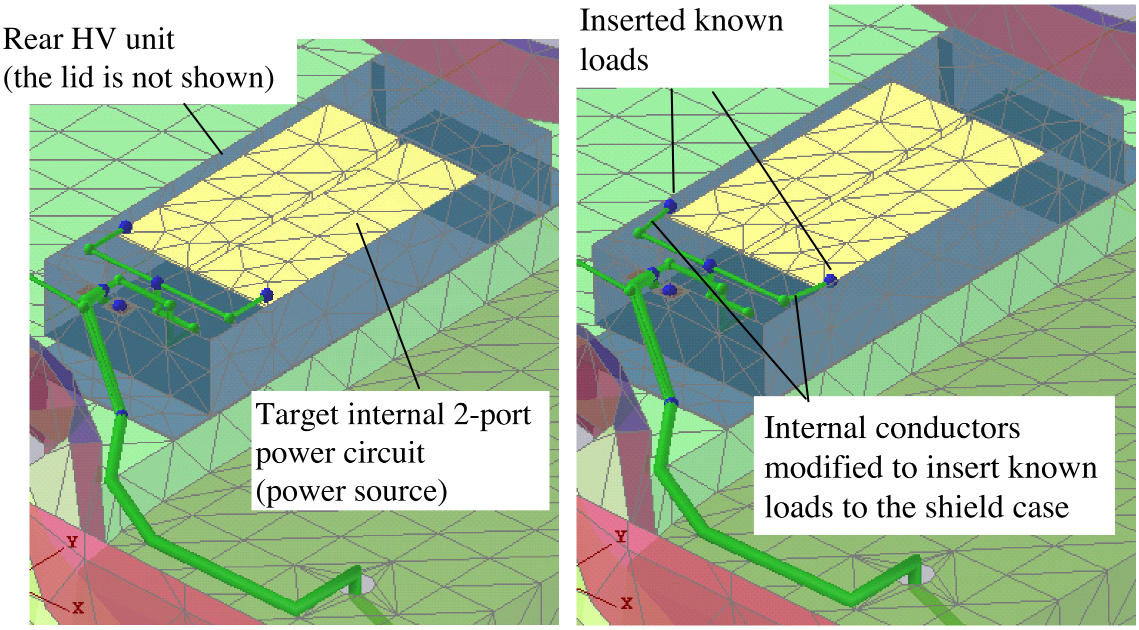


Figure 5.4: Simulation model for a vehicle with HV units. 2-port target circuit(power source) is contained in rear HV unit.

testing shown in Fig. 5.4. A generic sedan type electrically driven vehicle model is used. The vehicle has a front HV unit in its engine compartment and a rear HV unit in the luggage room (Fig. 5.5). The power cable and BCIs are configured as described in Sect.5.2.1, where the BCI was modeled with the S-parameters obtained from actual measurements. EMC studio [29] is used as the EM simulator to calculate the frequency characteristics in the range of 20 to 200 MHz.

For the estimation, only the two ports at the BCIs have been virtually 'measured.' As the known loads to connect to the u-ports, we used resistors of 500Ω and 1kΩ and open impedance also. The load combinations and the corresponding applied estimation equations are shown in Table 5.1. For the calculation of $\mathbf{S_{aa}}$, we connected identical loads between the u-ports and the shield case, which can be regarded as a ground as in Fig. 5.5(b). For the calculation of $\mathbf{S_{au}}$, we connected 500Ω to one of the ports and 1kΩ to the other to obtain the first column of $\mathbf{S_{au}}$ using Eq.(5.12). Then, 500Ω is connected between the two port terminals as a thru load to calculate the remaining components for $\mathbf{S_{au}}$ using Eq.(5.17). For the calculation of $\mathbf{S_{uu}}$, we tried the measured values for any of the load combinations used so far to build least square equations and all of them gave the similar results. The results of using the combinations excluding the thru load are shown in the following.

The calculations for $\mathbf{S_{au}}$ were made using the methods described in Sects.5.1.3 and 5.1.3. Both results are shown below. The results for the estimated 4-port



(a) Ordinary connection (b) Inserting known loads

Figure 5.5: Internal structure of the rear HV unit.

connection S-parameters are shown in Figs.5.6 and 5.7. Here, the positive result of the square root operation was taken for s_{t11} in Eq.(5.12). As the result, the values for $\mathbf{S_{au}}$ show sign ambiguities as expected. In Fig. 5.7, results of the EM simulation and the estimation show good agreement neglecting the sign ambiguity in $\mathbf{S_{au}}$ while in Fig. 5.6, they do not show good agreement in $\mathbf{S_{au12}}$ and $\mathbf{S_{au22}}$ which leads to the large differences in $\mathbf{S_{uu12}}$ and $\mathbf{S_{uu22}}$ through the calculation process. This is caused since in Fig. 5.6 $\mathbf{S_{au11}}$ and $\mathbf{S_{au21}}$ are directly calculated from the measured values while $\mathbf{S_{au12}}$ and $\mathbf{S_{au22}}$ are indirectly calculated using the estimated values of

Table 5.1: Load sets used for 4-port connection S-parameter estimation.

Load	Load values		Equations applied		
set #	Port 3	Port 4	$\mathbf{S_{aa}}$	$\mathbf{S_{au}}$	$\mathbf{S_{uu}}$
1	open	open	(5.9)	(5.12)(5.18)(5.17)	(5.19)
2	500Ω	500Ω	(5.9)	(5.12)(5.17)	(5.19)
3	1kΩ	1kΩ	(5.9)	(5.18)(5.17)	(5.19)
4	500Ω	1kΩ	-	(5.12)(5.18)(5.17)	(5.19)
5	500Ω thru		-	(5.17)	-

$\mathbf{S_{au11}}$ and $\mathbf{S_{au21}}$ and the measured values. This is assumed to increase the errors in $\mathbf{S_{au12}}$ and $\mathbf{S_{au22}}$ estimations especially above 70MHz since the amplitudes of $\mathbf{S_{au}}$ components are small and easily affected in this example.

The 2-port S-parameters for the target internal power circuit are shown in Figs.5.8 and 5.9. Here, Port 1 and Port 2 of the 2-port circuit are connected to Port 3 and Port 4 of the 4-port connection circuit, respectively. Reflecting the characteristics of the estimated 4-port connection S-parameters, in Fig. 5.8, the estimated values for $\mathbf{S_{12}}$ and $\mathbf{S_{21}}$ show large glitches and the estimated value for $\mathbf{S_{22}}$ shows considerable difference from the reference EM simulation value. On the other hand, in Fig. 5.8, the estimation and reference EM simulation show good agreement below 150MHz although the differences increase as the frequency becomes higher and the real part of $\mathbf{S_{11}}$ and $\mathbf{S_{22}}$ have the maximum difference of about 0.2 at 200MHz. The sign ambiguity in $\mathbf{S_{au}}$ does not affect the results in either case as can be seen.

5.3 Chapter Conclusion

An estimation method for the S-parameters of 4-port circuits with 2 port direct measurements is presented. The estimation equations are almost the same as in the previous chapter except for a method to improve the estimation accuracy by introducing symmetry to the equation for the transmission coefficients of the S-parameters utilizing that the number of indirect measurement ports is 2. The improvement can be introduced to this target without extra calculation or measurement costs. The comparison of different calculation methods for the transmission coefficients was provided as well as the proof that a thru load is required to calculate the transmission coefficients.

The estimation method was assumed to be applied for the estimation of 2-port S-parameters of the circuit connected with a 4-port connection circuit. In this method, to obtain the S-parameters, first, the 4-port S-parameters for the connection circuit are estimated using indirect measurement. Several known loads are connected to the indirectly measured 2 ports in turn and reflection and transmission characteristics among the directly measured 2 ports are measured. Therefore, there is no need to connect measurement instruments to the ports that are connected to the known loads. So, it is effective for measuring the circuit that has ports with an uncommon ground. The connection S-parameters are obtained by solving several linear equations and square root calculations only. This eliminates the needs to solve a set of nonlinear equations. Validity of this method has been confirmed by applying this method to estimate the S-parameters of the circuit contained in a high-voltage (HV) unit mounted on electric-driven vehicles in EM simulation model.

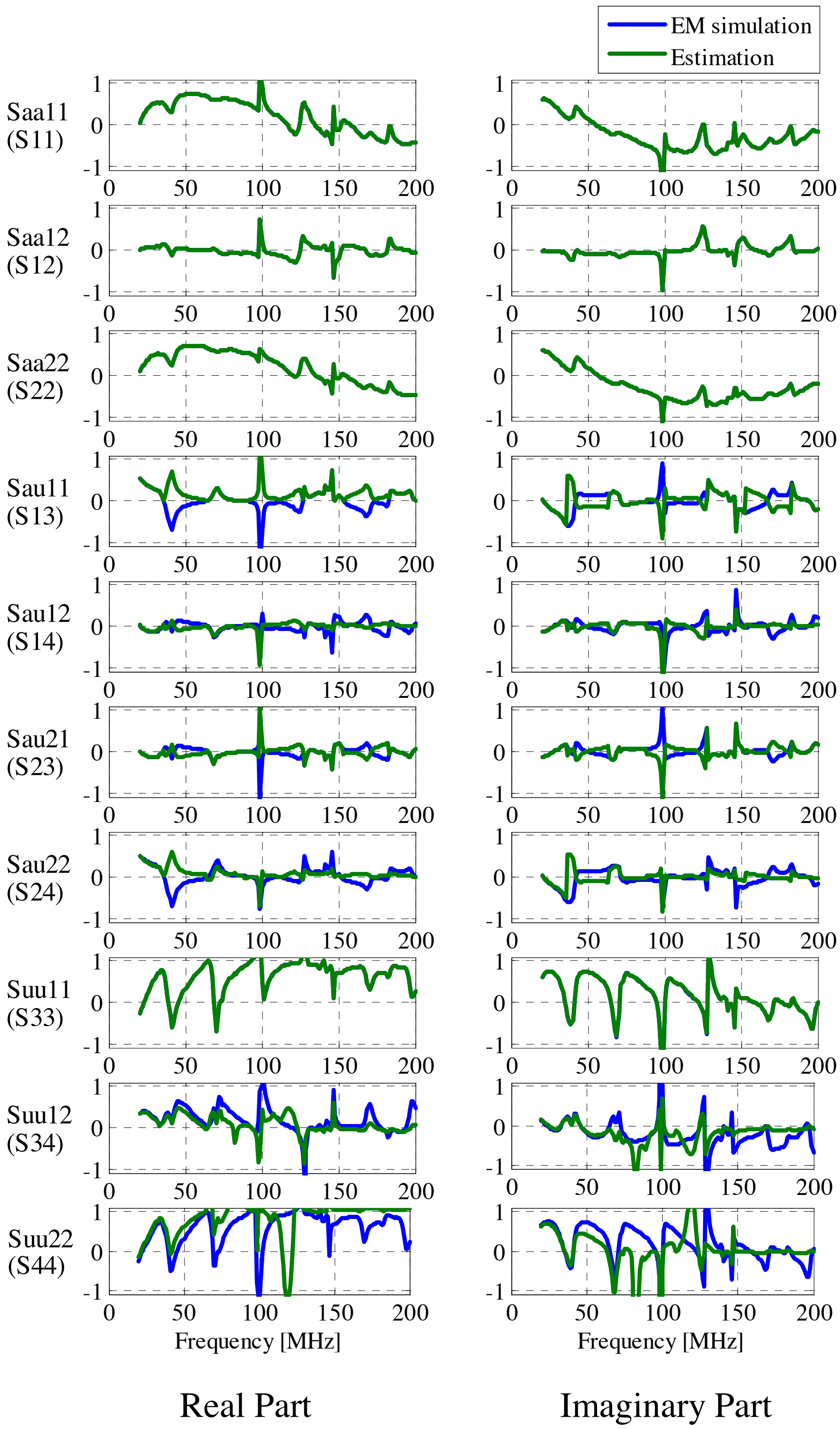


Figure 5.6: Simulated and estimated values of 4-port connection circuit(power cable) S-parameters, where the estimation used the method in Sect.5.1.3.

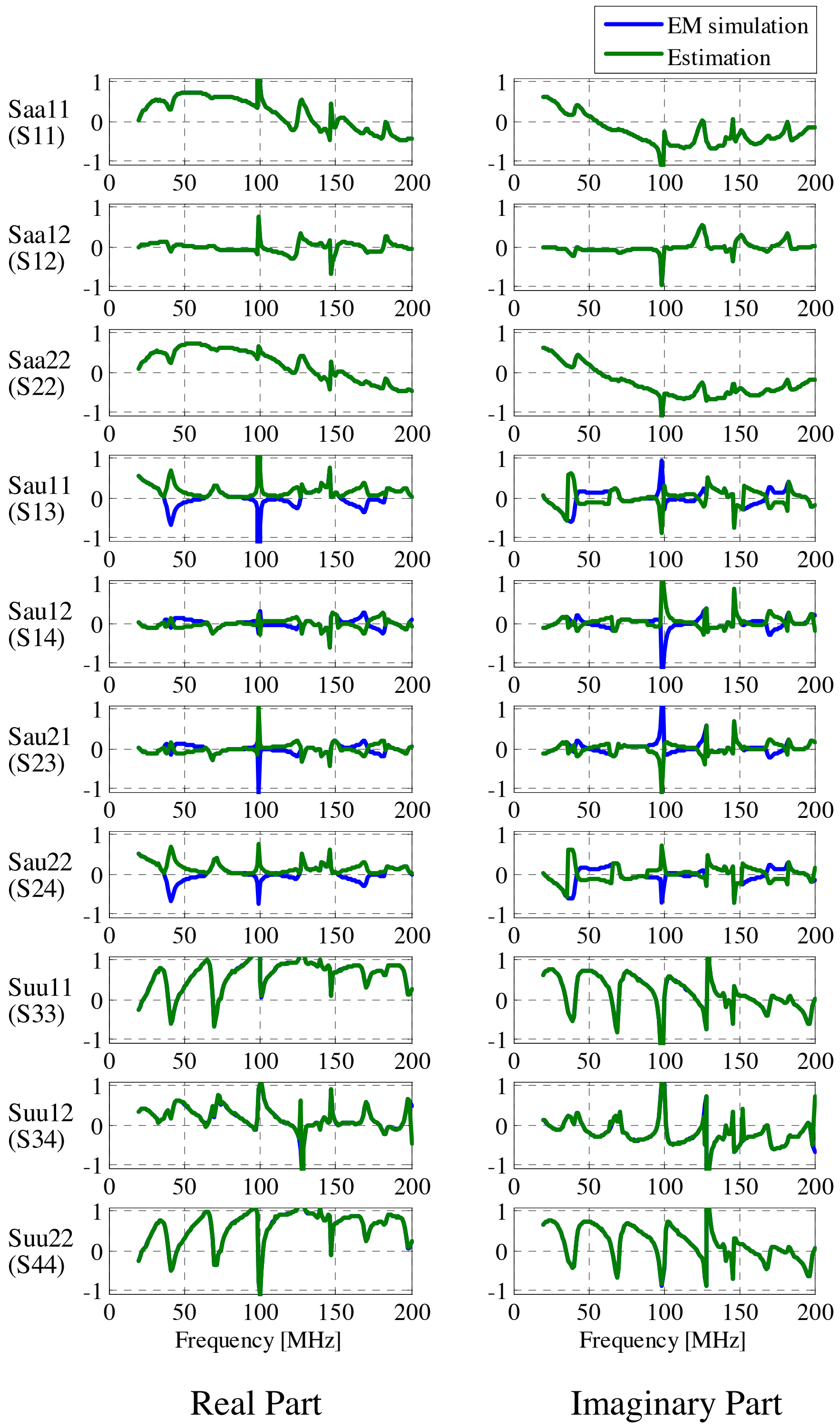


Figure 5.7: Simulated and estimated values of 4-port connection circuit(power cable) S-parameters, where the estimation used the method in Sect.5.1.3.

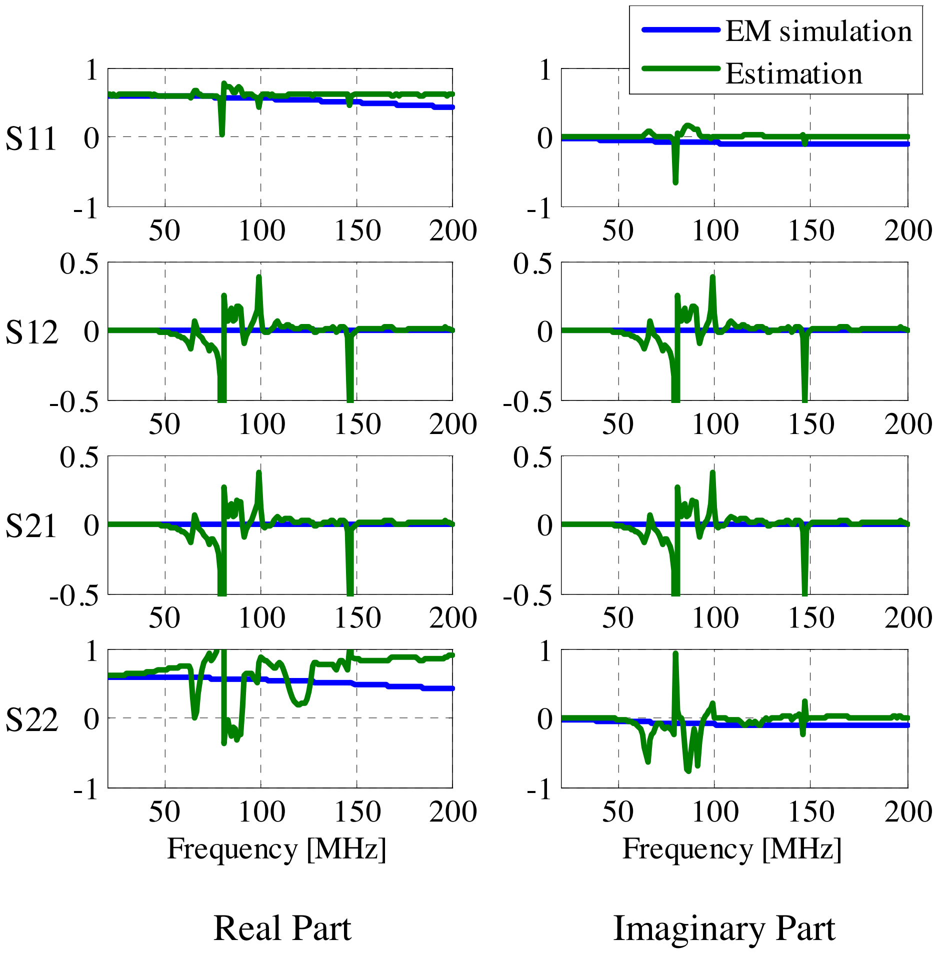


Figure 5.8: Simulated and estimated values of 2-port circuit(power source) S-parameters, where the estimation used the method in Sect.5.1.3.

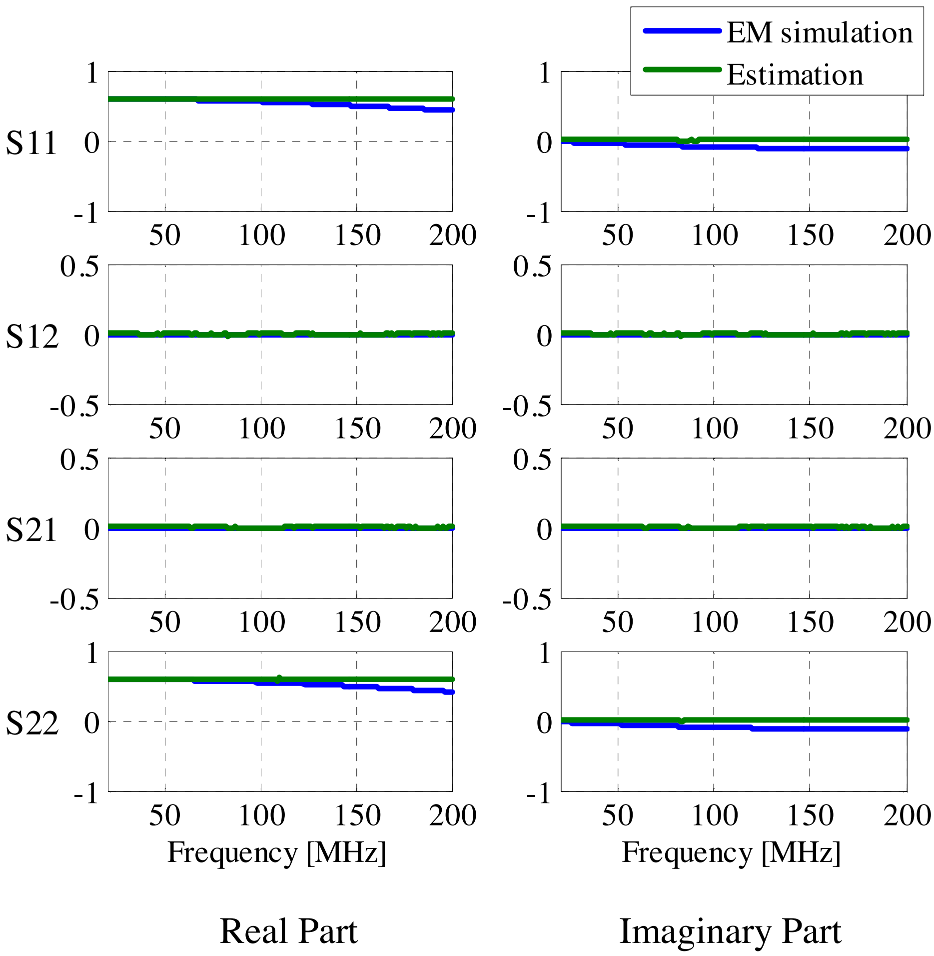


Figure 5.9: Simulated and estimated values of 2-port circuit(power source) S-parameters, where the estimation used the method in Sect.5.1.3.

Chapter 6

Conclusions and future work

Two approaches have been presented in this thesis to estimate the complete S-parameters of multiport reciprocal circuits with directly measuring some of the ports and connecting known loads to the remaining hard-to-access ports (indirectly measured ports). The essential difference resides in how to linearize the fundamental nonlinear estimation equation in the form of a matrix quadratic equation.

1. Method of 'equation decomposition and simultaneous solution':

   An estimation method of the n-port S-parameters for reciprocal circuits with $n-1$ port measurements has been presented. In the estimation procedure, all circuit equations are decomposed and recombined to form a linear equation for all S-parameters, then, simultaneously solved to obtain the whole solution at once. In this method, known loads are connected in turn to one port that is hard to access with a measurement probe and reflection and transmission characteristics between the remaining ports are measured. S-parameters are obtained by solving a linear least-squares equation and a quadratic equation only. 3 values of loads are required to solve the equation. Theoretically, the number of measurements does not increase as the number of ports increases when we count the operation measuring through directly measured ports with connecting a known load to indirectly measured ports as one measurement.

   Applying this method to derive the S-parameters of an immunity test system which can be seen as a 3-port circuit, the validity of this method has been confirmed. As the result, it has been shown that there is a sign ambiguity for the S-parameters between the hard-to-access port and other ports but the port voltage amplitudes calculated using them are valid. The port voltages have also been calculated using the estimated S-parameters and shown good agreement with the port voltages measured. In this method, there is no need to connect a network analyzer to one of the ports of the target circuit. So, it is effective for measuring the circuit which has a port with uncommon ground.

Also, to show its potential and limitation, the estimation method was applied to the 3-port S-parameters with 1-port measurement. It also showed a good agreement between an example measurement and the estimation. 7 measurements are required this time to obtain the solution. It can be deduced that the estimation method can be applied to the circuits with arbitrary combination of direct and indirect measurement ports. However, it has been clarified that it is necessary to repeat the whole procedure of constructing the equation to apply this method to a new arbitrary configuration of the whole ports and directly measured ports, therefore the general theoretical analysis in the framework of linear algebra can not be provided through this estimation method.

2. Method of 'submatrix elimination and successive solution':

   Also, an estimation method of the $2r$ port S-parameters for reciprocal circuits with r port measurements has been presented. To obtain the analytical solutions in the framework of linear algebra, a novel method to solve the fundamental nonlinear estimation equation has been developed. The submatrices of the S-matrix are eliminated with increasing the number of equations using extra measurements to obtain a matrix quadratic equation interms of one submatrix. Then, it is transformed into a linear equation and solved. This process is repeated to solve each submatrix in successive manner. This novel second method was developed here since applying the first method to these targets requires extensive equation decomposition operations that requires the operation itself to be expressed in an algorithm. This second method enabled the theoretical analysis in the framework of linear algebra. As the result, it was shown that at least 6 sets of known loads are required to solve the whole connection S-parameters and they include 2 sets of thru-loads to calculate the transfer characteristics except for the 4-port circuit case which requires 5 sets of known loads including 1 set of thru-load. Theoretically, the number of measurements does not increase as the number of ports increases when we count the operation measuring through directly measured ports with connecting a set of known load circuits to indirectly measured ports as one measurement.

   The estimation method has been applied for the S-parameters of a multiport connection. A multiport device is connected to one end of the multiport connection and a network analyzer is connected to the other end. Here, the ground for the multiport device and the ground for the measurement end of the connection as the system ground are not assumed to be common. The measurements of reflection and transmission characteristics among the ports of input end of the connection are only required while several sets of loads are connected to the output end of the connection for the estimation, which makes this method effective in case that connecting measurement probes directly to the multiport device is physically difficult or it might change the high frequency environments of the device. Both multiport connection and device S-parameters are estimated by solving several linear equations and square root calculations only. This eliminates the needs to solve a set of nonlinear equations. It was also shown that to obtain the device S-parameter smooth enough in frequency, introducing quadratic penalty with smoothing matrix is effective to cancel the instability of the inverse problem. The validity of the proposed method has also been confirmed by simple experimental results.

This study shows the ground configurations specified above reduces the sign ambiguities in the multiport connection S-parameters to one while they are unlimited without specifying the ground configuration as in our previous study. Also, the original results of this study includes the method to estimate the S-parameters of the multiport device and the proof that they do not have sign ambiguities in spite of the sign ambiguities in the multiport connection S-parameters.

As a specific application of this estimation method, an estimation method for the 2-port S-parameters of the circuit contained in a high-voltage (HV) unit mounted on electric-driven vehicles has also been presented. In this method, to obtain the S-parameters, first, the 4-port S-parameters for the connection circuit are estimated using partial measurement. Several known loads are connected to 2 ports in a HV unit in turn and reflection and transmission characteristics among the remaining 2 ports are measured. The comparison of different calculation approaches for the transfer characteristics was provided as well as the proof that a thru load is required to calculate the transfer characteristics. By applying this method to estimate the S-parameters of a HV unit in EM simulation model, validity of this method has been confirmed.

The future study includes the method to select the appropriate values of the known loads to be attached to the output ports of the multiport connection and also the extension of the above methods to estimating the S-parameter of circuits with arbitrary number of total ports and directly measured ports.

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Chapter

List of publications

Publications related to this thesis

Journal paper:

1. Noboru Maeda, S. Fukui, K. Ichikawa, Y. Sakurai, T. Sekine and Y. Takahashi, “S Parameter Estimation of n Port Reciprocal Circuits with $n-1$ Port Measurements,” IEICE Trans. Electro. (Japanese Edition), Vol.J96-C, No.12, pp.463-470, Dec. 2013.

Refereed conference papers:

1. Noboru Maeda, S. Fukui, K. Ichikawa, Y. Sakurai, T. Sekine and Y. Takahashi, “An Estimation Method for the n Port S Parameters with n-1 Port Measurements,” EMC Europe 2013, pp.348–353, Sep. 2013.

2. Noboru Maeda, S. Fukui, T. Sekine and Y. Takahashi, “An Estimation Method for the 3 Port S-parameters with 1 Port Measurements,” 21st European Conference on Circuit Theory and Design, Sep. 2013.

3. Noboru Maeda, S. Fukui, T. Naoi, K. Ichikawa, T. Sekine and Y. Takahashi, “Mathematics of 2r-Port S-parameter Estimation by the r-Port Measurements,” EDAPS 2013, Dec. 2013.

4. Noboru Maeda, S. Fukui, T. Murakami, T. Naito, T. Sekine and Y. Takahashi, “S-parameter Estimation for the Components in Automotive High-voltage Units with Partial Measurements,” EMC’14/Tokyo, May 2014.

5. Noboru Maeda, S. Fukui, T. Sekine and Y. Takahashi, “S-parameter Estimation for a Multiport Connection and a Multiport Device with Non-common Ground,” EMC Europe 2014, Sep. 2014.

Non-refereed conference papers:

1. Noboru Maeda, S. Fukui, K. Ichikawa, Y. Sakurai, T. Sekine and Y. Takahashi, “Estimation of the 3-port S Parameters with 2-port Measurements and Its Application to the Immunity Testing System,” IEICE Technical Report, EMCJ2012-98, pp.81-85, Dec. 2012 (in Japanese).

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4. Noboru Maeda, S. Fukui, T. Naoi, K. Ichikawa, T. Sekine and Y. Takahashi, “Estimation for 2r-port S-parameters by the r-Port Measurements,” IEICE Technical Report, EMCJ2013, 113(368), pp.55-60, Dec. 2013 (in Japanese).

About the author

Noboru Maeda was born in Aichi, Japan in 1961. In 1984, he received B.E. degree in Electrical Engineering from Kyoto University, Kyoto, Japan and joined the electronics application group in Denso Corporation. He received M.S. degree in Computer Science from Stanford University, CA, U.S.A. in 1990. In 1994, he joined the electronic circuit group in Nippon Soken, Inc. He has been engaged in research on electric vehicle circuits, wireless and wired communications and EMC analysis for automobiles.