Vulnerability and Connectivity Evaluation of Road Network by Topological Analytics

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Chapter 1

Introduction

1.1. Background and Research Objective

In recent years, the impact of natural disasters such as floods, landslides caused by heavy rains and typhoons has been increasing because of climate and social changes such as global warming, depopulation, land use change and so on. Also, there is always a risk of earthquakes in Japan. Thus, increasing societal resilience is an important issue. Normal urban activities rely heavily on transport systems, especially the road network, which are not completely disaster-proof. After a disaster, the road network actually becomes more important than other transportation modes (IATSS, 2000) because of the extensive road coverage and the network's robustness in maintaining the connectivity of urban systems. Furthermore, people rely on road network more in depopulated areas where the public transport system is insufficient and transportation modes are limited. To construct robustness of road networks, it is important to identify critical locations which have a significant impact on the overall performance in the network failure and disruption. Lots of studies have been carried out to evaluate the robustness of the road network, and they can be categorised into two study groups; network reliability analysis and network vulnerability analysis. The network reliability analysis in general evaluates a network based on the expected loss of the system under given probability of disruptions (eg, Bell and Iida, 1997), whereas vulnerability analysis evaluates the network by the consequence of the event regardless of its occurrence probability (eg, Berdica, 2002; D'Este and Taylor, 2003).

Conventional network reliability evaluation methods use the probability of link disruption caused by various hazards such as landslide, rock falling, serious accidents and so on is adequate if it is available. However, the estimation of such probability is in general very difficult, since such hazard does not occur frequently and, moreover, the probability of occurrence may depend on geographical, meteorological and social conditions. For these issues, vulnerability analysis evaluates the magnitude of the disaster impact regardless of the occurrence probability. Vulnerability is an indicator of network weakness from the loss of accessibility, connectivity, demand and so on. Some of vulnerability evaluation methods use the demand data such OD traffic volume information and attractiveness of locations. When the traffic demand data is available from the traffic assignment or survey, it is evaluated based on that demand data. However, in reality, there are many cases where such accurate demand data is not available, like at disaster or in the future planning stage. The problem is that the evaluation results depend heavily on probability and OD data that are difficult to obtain accurately. Also, almost all evaluation methods of reliability and vulnerability requires a calculation with high computational loads such as shortest path search, path enumeration or traffic assignment. Therefore, the scale or detail of the network to analyse is limited.

To relax these limitations, this study attempts to evaluate vulnerability and connectivity of road network by using topological indicators. One advantage of this approach is to be computationally tractable for large networks because traffic assignment and shortest path search are not required. The calculation of the network topological indicator has low computational loads and the evaluation result can be obtained easily and quickly. Another advantage is that it does not need any assumptions such as perfect information (if deterministic user equilibrium is assumed) and perfect control (system optimal be all users following the administrator) because it focuses only on the network structure. In addition, the road network topology is analysed without using probability or demand data.

The proposed method which is tractable for large-scale networks enables evaluation independent of the detail level of network. The vulnerability and connectivity evaluation including small city roads in wide-scale network has a potential to identify critical vulnerable parts that cannot be identified in the aggregated networks. Moreover, because the proposed method evaluates networks that do not depend on demand data, it can identify the critical parts in future network plans that are particularly difficult to estimate the demand. Thus the proposed method is helpful for the road network improvement policy.

The objective of this study is to add a new perspective to the field of road network evaluation by showing analytics using network topological indicator that can extract critical parts that was difficult to find due to the limitations of conventional methods. Specifically, the contribution of this thesis are,

- Proposal for road network evaluation method using network topological indicators,
- Confirmation of suitability of the proposed method to road network evaluation by comparing with the conventional methods,
- Confirmation of the easiness for applying onto the large-scaled road networks,
- Verification of the usefulness of the proposed method via practical application results.

1.2. Organisation of Thesis

The outline of the thesis is shown in Figure 1.1. This thesis has two topics, vulnerability analytics and connectivity analytics.

Chapter 1 explains the background and research objective. Chapter 2 introduces road network evaluation methods related to this study that are classified into each concept. Based on the issues shown by the literature review, the study focus indicates the position of this study in the road network evaluation research field. Chapter 3 organises the challenges and objectives of evaluation using each weighted network. The weighted network is represented matrix considering the measured values of traffic function as weights. As the methods for weighted network analytics, network topological indicators are introduced. Chapter 4 proposes a network vulnerability evaluation method by using topological indicators. The suitability of the proposed method for road network evaluation is confirmed by comparison with the conventional method. Moreover, the proposed method is applied to a practical road network. The interpretation of the evaluation results is discussed. Chapter 5 proposes a network connectivity evaluation method by using topological indicators. Same as Chapter 4, the proposed method is compared with the conventional connectivity evaluation method and the method is further applied to practical networks. As one example of the application, the evaluation results by several different weights are verified. Both two proposed methods in Chapters 4 and 5 are verified to work on large-scaled road networks. In Chapter 6, the connectivity evaluation method is applied to road networks for different years to analyse the impact of road improvements. Chapter 7 concludes this study and indicates future works.

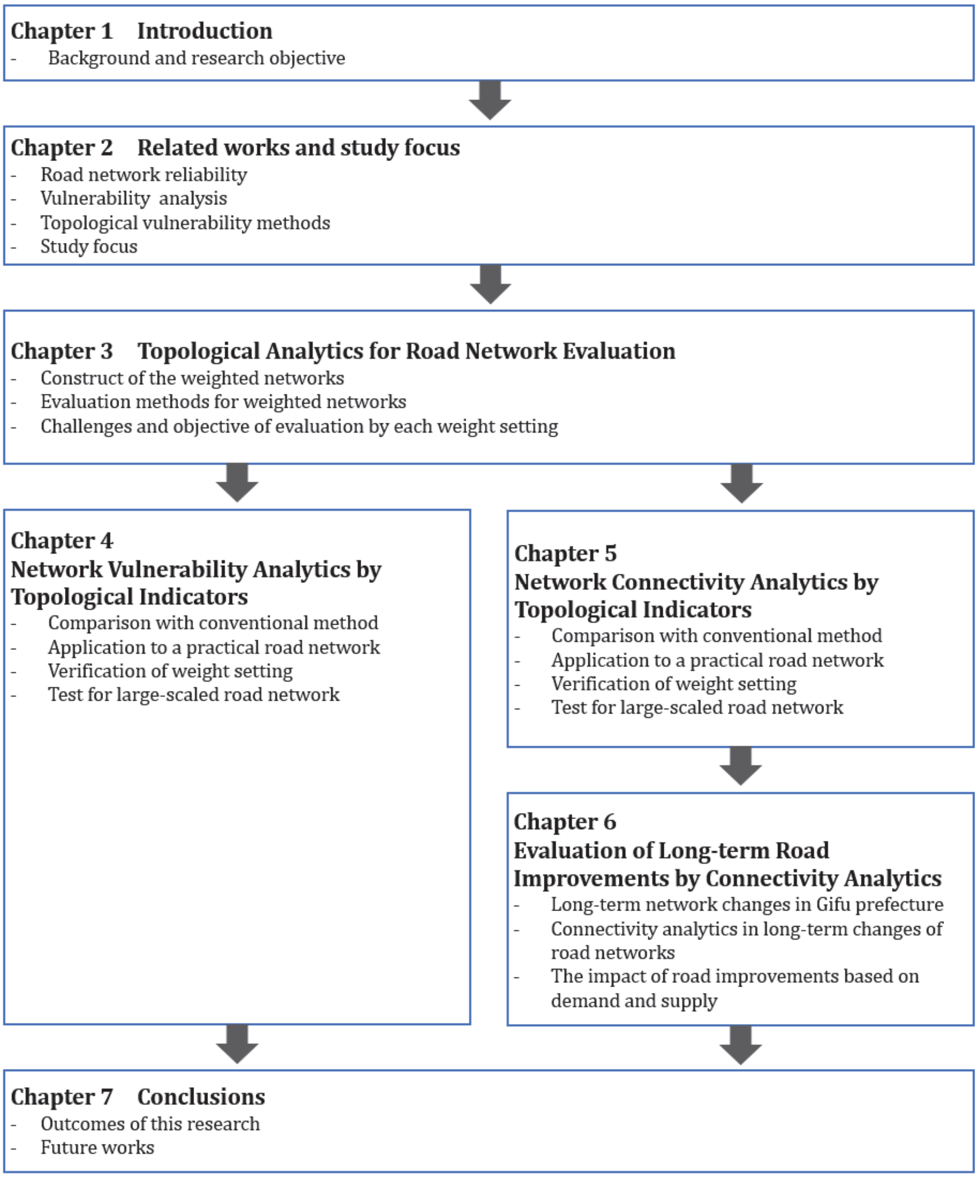

Figure 1.1 Outline of thesis

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Chapter 2

Related Works and Study Focus

This study attempts to propose a method to evaluate a connectivity improvement of road networks so that the network can be robust against possible natural/man-made disasters. For this purpose, it is necessary to identify sufficient and insufficient parts in terms of connectivity in road networks. Various researches related to network evaluation methods have been proposed. Among them, this research proposes effective and efficient methods that can easily and quickly evaluate in any size of network. In this chapter, these researches are summarised, and the focus of this study is described.

2.1. Related Works

2.1.1. Performance evaluation metrics of transportation system

Many metrics have been proposed to analyse and evaluate the impact of disaster on transportation system. The disaster includes natural disaster, human error like traffic accidents and man-made attacks. Faturechi and Miller-Hooks (2015) classified such metrics into seven general indicators, risk, vulnerability, reliability, robustness, flexibility, survivability and resilience. Among them, this research evaluates the road network connectivity based on reliability and vulnerability metrics to construct a robust road network system. Identifying critical and susceptible parts on the connectivity is useful information for efficient and effective road improvement for robustness. Among the seven metrics of transportation system mentioned above, the definition and qualitative conceptualisation literatures of robustness, vulnerability and reliability are summarised in Table 2.1. The following sections will review existing studies on reliability and vulnerability.

Table 2.1 The definition and qualitative literatures of three metrics (Faturechi and Miller-Hooks, 2015)

Concept	General definition
Robustness	Ability to withstand or absorb disturbances and remain intact when exposed to disruptions
Reliability	Probability that a system remains operative at a satisfactory level post-disaster
Vulnerability	Susceptibility of the system to threats and incidents causing operational degradation

2.1.2. Road network reliability

Various methods and theories have been developed to measure network reliability with a view to improving the robustness of transportation systems. As a method to evaluate the performance of a transportation system without considering traffic demand, Wakabayashi and Iida (1992) proposed *connectivity reliability*, which is the probability that a specific pair of nodes remains connected when a network is subject to disruptions with given probabilities. Connectivity reliability gained significance following the catastrophic Kobe earthquake in 1995. The connectivity reliability considers whether the node is connected or not regardless of the quality of connection.

However, in discussing transport system the level of service, such as travel time or congestion level, should be considered. For this purpose, *travel time reliability* and *capacity reliability* have been proposed. Travel time reliability is the probability that a trip can reach its destination within a given period at a given time of day (Bell and Iida, 1997). Federal Highway Administration in the United States uses travel time reliability as a key performance index. This index is evaluated by experienced network users and defines the level of service offered by the road network. Travel time variability can be considered as an indicator of demand satisfaction in congested situation. Fosgerau and Karlström (2010) considered the value of reliability to be the value of a change in the standard deviation of trip duration. This paper revealed that standard deviation of a distribution of trip durations has relationship with the cost of scheduling model formulated as an opportunity cost per minute of starting early and a greater cost per minute of finishing late relative to some fixed deadline. However, there are assumptions that the decision maker knows that travel time distribution. Also, travel time fluctuates by time-of-day and day-of-week (van Lint and van Zuylen, 2005). Moreover, in addition to considering other factors such as weather and seasons, demand information is necessary to estimate travel time reliability. Many of travel time reliability studies are analysed based on uncertain source like that, and methods to relax them have been studied. They are introduced below. Most of travel time reliability researches assume probability distributions explicitly. In real life, however, such distribution might not be available. In order to relax this assumption, Ng et al. (2011) proposed an approach that only requires the specification of the first Nth moments of travel time and a set of finite intervals in which the random quantities are hypothesised to reside. In case that the uncertainty sources are statistically independent, the upper bounds on the tail probabilities that are valid under all conceivable probability distributions are obtained. The worst-case bounds of travel time reliability are found without exact probability distribution. Furthermore, reliability model with link disruption often assumes the degradation probability is independent among links. However, multiple links are frequently disrupted due to the same cause such as floods and earthquakes. Therefore, it can cause over-optimistic estimate. Sumalee and Watling (2003) proposed a method for estimating travel time reliability under dependent link failures, which operates by identifying those network states with large probabilities. This approach is designed for scenarios that have high probabilities, and Sumalee and Watling (2008) proposed cause-based approach that defines the independent link degradation probabilities under each cause of failure. The dependent link degradation under each failure is represented by the causal tree structure of the failures. As described, some studies have been researched to deal with uncertain demand and probability, relaxing assumptions is one of the main challenges in reliability analysis.

On the other hand, Chen et al. (2002) presented capacity reliability as a performance measure for a road network. It is defined as the probability that road network capacity is sufficient for the demand at a required level of service, while accounting for driver route choice. Capacity reliability is based on the concept of reserve capacity which indicates unused link capacity. This assumes that every OD pair will have uniform growth or decline in its OD demand pattern. To relax this assumptions, Chen et al. (2013) proposed two alternative approach, ultimate and practical capacity which allow non-uniform growth in the spatial distribution of the OD demand pattern. These approaches make it possible to evaluate both changes in demand volume and variations in demand pattern. Also, Sumalee and Kurauchi (2006) adopted the capacity reliability index that introduces the randomness of the link capacities to represent random effects of the disaster assuming emergency or resource activity after the initial period of the disaster. This enables to evaluate the performance of different traffic management strategies under the randomly degraded network condition after a major disaster.

There are other reliability metrics of transport system regardless of the road, for instance *encountered reliability* (Bell and Schmöcker, 2002) measures the probability of users trip successfully without encountering link disruption, and *travel demand satisfaction reliability* (Heydecker et al., 2007) defined by the probability that the road network can accommodate a given latent travel demand.

2.1.3. Vulnerability analytics

Reliability analysis mentioned above is calculated based on the probability estimation, like link disruption probability. The probability estimation is, however, often very difficult especially when we need to handle extremely rare events such as natural disasters, and the result heavily relies on its probability. For this reason, a measure that does not rely on probability estimation, called vulnerability, has been proposed. The vulnerability analysis evaluates the consequence of a disruption regardless of its probability. Vulnerability in the road transportation system is defined as the susceptibility to incidents that result in significant reductions in road network serviceability (Berdica, 2002). D'Este and Taylor (2003) also presented vulnerability as an indicator of "network weakness". This indicator measures the potential loss of node accessibility due to link disruptions based on Hansen (1957)'s measure of accessibility. The definition by Berdica (2002) and the definition by D'Este and Taylor (2003) are related. They are used for network failure of "short-term" (hours-days-weeks) and "long-term" (weeks-month-year), respectively (Taylor, 2017). In term of indicator proposal and application based on the vulnerability concept, Taylor et al. (2006) identified the critical sections by the changes in generalised travel cost and accessibility indicator caused by degradation between cities using the Australia main road network. Also, Jenelius et al. (2006) derives the link importance indices and site exposure indices based on the increase in generalised travel cost when links are closed. These indices are applied to the road network of northern Sweden.

Mattson and Jenelius (2015) also identified two approaches to vulnerability analysis: *system-based vulnerability analysis* and *topological vulnerability analysis*. The system-based vulnerability analysis considers the interaction between supply and demand found by a comprehensive transport model. The topological vulnerability analysis looks at the topology of the network only. Since this paper seeks for the connectivity of nodes, we adopt the latter approach. The topological vulnerability analysis does not need to use any traffic assignment or shortest path search algorithm but rather uses graph theory to analyse a network based on indicators describing the connections between the nodes. The topology vulnerable analysis which this paper belongs will be described in a later section. The studies of system-based vulnerability analysis are reviewed here.

Nagurney and Qiang (2007) defined the importance indices based on the reciprocals of the travel costs. This is the demand weighted generalisation of the network efficiency indicator. Moreover, Nagurney and Qiang (2012) demonstrates that well-defined system optimised network model by using general travel cost functions understand not only critical system part but also the underlying behaviour of decision-makers, the resulting flows, and incused costs in reality of demands for resources. About the vulnerability by the characteristics of target road network for studies, Balijepalli and Oppong (2014) applied four indicators; change in generalised cost measure (Taylor et al. 2006), network efficiency measure (Nagurney and Qiang, 2007), importance measure (Jenelius et al. 2006), network robustness index (Scott et al. 2006), and new vulnerability indicator which considers serviceability and link priority reflecting road rank to a dense network in urban areas. The results show that vulnerability indices based on distances are not suitable for the dense urban networks. The proposed vulnerable indicator means a relatively higher loss of capacity has a greater significance to the vulnerability measure than a similar road with relatively lower loss of capacity will have.

Another approach to identify vulnerable links is through the game theory, where a demon aims to maximise total driving time by failing links while drivers minimise their driving time subject to expected link failures (Bell, 2000). In the Nash mixed strategy equilibrium, the probability that a demon chooses a link to fail measures its vulnerability, as the demon would prefer to fail links that cause drivers' maximum loss. This takes into account both driver preference for links and the individual drivers' loss in the event of a failure. Stochastic user equilibrium is assumed to estimate drivers' preference for route choice and thus the method is computationally intensive. As methods using linear programming, Kurauchi et al. (2009) evaluated road network vulnerability by counting the number of distinct paths that do not share a link. Node pairs connected by few distinct paths are more vulnerable. Their vulnerability evaluation heavily relies on the level of detail and boundary situation of the network, and the computationally tractable method that can handle all roads without any approximation is preferable.

The optimisation techniques are also in mathematical modelling approach. Matisziw and Murray (2009) proposed an evaluation method of flow loss by link disruptions using an integer programming formulation. The application to the road network in Ohio shows that the result of integer program is equivalent to the result of path-based approach which requires enumeration of all paths. Also, Ho et al. (2013) formulates bi-level vulnerability analysis using a continuum traffic equilibrium model to identify vulnerable locations. It assumes traffic equilibrium under the degraded situation as a lower-level problem, and an upper-level problem finds the most vulnerable locations that reduces the accessibility index. The travel costs and traffic flows at the lower-level are determined by the vulnerable locations given from the upper-level. The result of numerical example test shows that accessibility which depends on land use pattern affects the road network vulnerability.

2.1.4. Topological vulnerability methods

Taylor (2017) mentioned a topological method to evaluate vulnerability identifies critical locations in the network where failure or disconnection will have the maximum impact on network performance.

Topological vulnerability analysis has mainly two aspects; one is a *network efficiency* and another is a *node centrality*. Latora and Marchiori (2001) defined network efficiency as a measure of information exchange which is the average across all node pairs of the reciprocal of the distance between each node pair. This achieves a maximum when using Euclidian distance. Mattson and Jenelius (2015) presented a global efficiency index, which indicates how direct the connections are between all node pairs by comparing the Euclidean distances with the shortest network distances.

Most of the centrality measures used for topological vulnerability are betweenness centrality (Freeman, 1979). In an idea of vulnerability that identify important locations, betweenness centrality, evaluating by the number of nodes or links included in the shortest path, has attracted many researchers. For example, Demšar et al. (2008) confirmed that node betweenness centrality and cut node methods are useful to identify critical locations of the network. Since node centrality is directly related to this study, existing works concerning about centrality measures will be discussed later.

2.1.5. Centrality measures

Centrality measures originated from the social sciences fields (Newman, 2010). Centrality is a value that indicates which node or link is 'central' in the network, and there are various definitions of centrality. The idea of centrality was firstly applied in social networks to understand human community structure in small groups (Bavelas, 1949). Subsequently, this concept has been adopted in various fields such as diffusions of infectious diseases, information and communication systems, economics, engineering and so on. Table 2.2 shows the summary and reference of representative centrality measures with reference to Newman (2010). In the table, x_i represents a centrality value of node i, N: the number of nodes, d_{ij}: distance between node i and node j, n_{st}^{i}: the number of shortest paths from s to t that traverses node i, g_{st}: the total number of shortest paths from s to t and a_{ij} : an element of the adjacency matrix **A**, k_j^{out}:the out degree of node j and α, β: the positive constants.

Table 2.2 Representative centrality measures

Centrality measure	Reference	Formulation	Definition
Degree Centrality	Proctor & Loomis (1951)	$x_i = \sum_j^n a_{ij}$	The number of links connected to the node.
Closeness Centrality	Beauchamp (1965)	$x_i = \frac{n}{\sum_j d_{ij}}$	The mean distane from a node to other nodes using the shortest path through a network between two vertices.
Eigenvector Centrality	Bonacich (1972)	$x_i = \sum_j A_{ij} x_j$	A node's importance in a network is increased by having connections to other nodes that are themselves important.
Betweenness Centrality	Freeman (1977)	$x_i = \sum_{st} \frac{n_{st}^i}{g_{st}}$	The extent to which a node lies on the shortest paths between other nodes.
Page Rank	Brin and Page (1998)	$x_i = \alpha \sum_j A_{ij} \frac{x_j}{k_j^{out}} + \beta$	Based on the concept of eigenvector centrality, their centrality divided by their out-degree.

The simplest measure is the degree centrality (DC) which is defined as the number of links connected to each node (Proctor and Loomis, 1951). An extension of the DC is eigenvector centrality (EC). EC (Bonacich, 1972) is the concept that node importance is increased by having connections to other nodes that are themselves important. As for centrality measures using the geographical distance, closeness centrality (CC) (Beauchamp, 1965) measures how close each node is to all other nodes and betweenness centrality (BC) (Freeman, 1977) determines the number of shortest paths that traverses a particular node or link.

For road network evaluations by centrality analysis, Duan et al. (2014) evaluated the stability of urban road network robustness by three different granularities, segment stroke and community level, using the degree and betweenness centrality. They found that the level of robustness varies by observation granularity, and that centrality measures can effectively represent the robustness of the network. For instance, the segment level is robust to degree centrality based attack, while the stroke level is extremely vulnerable to target attacks. Zhang et al. (2011) concluded that the betweenness centrality is the best measure among degree, closeness and betweenness centralities when partitioning a road network into several traffic analysis zones. Lämmer et al. (2006) showed that the frequency distribution of the betweenness centrality based on travel time follows the power law. This means that the number of important nodes with high betweenness centrality values are limited in the whole network, indicating a clear hierarchical order of the roads. Additionally, the distribution of travel time budget and reachable nodes under that travel time budget follows the scaling law. From this distribution, the existence of arterial roads dramatically expands the reachable area within a given travel time. The hierarchy of road networks is indicated by travel times. Jiang et al. (2004) adopted a special notation method where each node represents a street name and links are created if there is an intersection between two streets. Three centrality measures are applied to characterise the urban areas and identify important streets: degree, closeness and betweenness centrality. They clarified the characteristics of centrality measures, how degree centrality gives a sense of each street's integration with respect to its neighbouring street, how the closeness centrality reflects the way a street is integrated to all other streets, and how the betweenness centrality shows s bridge role of a street between other streets. Other studies also evaluate road networks with measures of centrality, degree, closeness, betweenness, straightness, and information (ex. Crucitti et al., 2006).

Centrality measures can give the results of all nodes in the network. Among them, the eigenvector centrality (EC) does not restrict to shortest paths and thus each node affects all of its neighbours simultaneously. EC is therefore ideally suited for "influence type" processes that simultaneously assume multiple "paths" such as spread of trends and information (Borgatti, 2005). Moreover, compared with CC and BC which require the shortest path search, EC with small computational load is suitable for the evaluation in large-scaled network which is one of the advantages of network topological analytics. The limitation of the EC is that the size of the eigenvector is undetermined, and it is often normalised with the length of the vector as one. It means that the values of EC are relatively scored and the values of EC can only be comparable within the same network. Another limitation is that the values are often extremely concentrated onto a few large hubs in a network. Martin et al. (2015) attempted to relax this problem by changing the expression of adjacency matrix. However, this limitation is not important for the road networks because road networks in general do not have large hubs.

Examples of EC being used in society are introduced here. The mechanism of EC is the origin of

Page Rank (PR) (Brin & Page, 1998) used in Google web search engine to generate lists of useful web pages. To evaluate web sites, PR has been modified to emphasise the importance of incoming links. A web site linked from many pages is very valuable. On the other hand, even the site links to many pages, it is strange that high centrality of the site transits to all connected pages. Thus, the measure of PR divides the centrality of those nodes by their out-degree. In this way, the measure which modified the EC is used in web search system. Furthermore, in research professionals' relationships using the co-author data of published papers, EC is more suited for finding key authors than other centrality measures. The reason is the authors are highly qualified and they have relations with other highly qualified researchers and then have a probability to publish good quality papers (Bihari & Pandia, 2015). Many other kinds of social networks have been conceptually or empirically analysed what type of centrality measures are suited (for example, Landherr et al., 2010; Borgatti, 2005). Centrality measures in social network have been well studied but much less work has been done on applications of EC for road networks.

2.1.6. Spectral partitioning method

Partitioning is one of the network topological indicator other than centrality. The links that have a significant impact on the road networks when that links are disrupted are recognised as critical links. Improving critical links lead to reduce the vulnerability of the network. This study attempts to find critical links for vulnerability of network by using spectral partitioning method in graph theory. Spectral partitioning divides the nodes of network efficiently based on specific functions characterised as weights. This involves findings clusters of nodes which have rich intra-cluster connections and poor inter-cluster connections. When such clusters do not overlap, their boundaries define the partitioning of network (Tsiatas et al., 2013). It is known that spectral analysis of the normalised graph Laplacian can reveal important structural properties of a network. In particular, the eigenvalues of the Laplacian matrix of a network are closely related its connectivity. Therefore, bounds for the smallest nonzero eigenvalue of the graph Laplacian provides information on how well connected the network is (Spielman, 2015). This is closely related to finding bottlenecks in a network, since a partition can often be found by dividing the network at its bottlenecks. Recently, spectral analysis has been used in various fields. For example, Ma et al. (2009) have utilised spectral bisection technique to group adjacent intersections with similar traffic flow characteristics into one sub-network. Also, Khan et al. (2016) have proposed an energy efficient network design and management system by using the spectral clustering approach to reduce energy consumption of network infrastructure.

2.2. Study Focus

As is mentioned in Chapter 1 and explained in the related works, it is essential to identify critical parts where failure or disruption will have the most important effect in the road networks. This is a valuable information for road improvement policy decision and leads to construct a robust network efficiently. To identify the critical parts, this study focuses on vulnerability and connectivity analytics. By D'Este and Taylor (2003), vulnerability of node is defined as *"A network node is vulnerable if loss of a small number of links significantly diminishes the accessibility of the node, as measured by a standard index of accessibility."* The potential impacts of nodes by link disruption and degradation on the whole of network is evaluated by vulnerability analysis.

As methods for evaluating the impact of link disruptions, reliability analysis shown in 2.1.2 evaluates by probability. However, it is difficult to estimate the disaster occurrence probability or link disruption probability accurately. Moreover, it is particularly difficult to understand user behaviour such as the travel time distribution in disaster and congestion situation. Therefore, some studies have been proposed methods to relax such uncertainties. In such a trend, vulnerability analysis that does not use the probability has been actively discussed in recent years. Vulnerability analysis evaluates the consequence of a disruption regardless of its probability. As mentioned in 2.1.3, there are many types of vulnerability indicators such as network efficiency and node centrality (topological) and generalised costs and mathematical models such as game theory (system-based). However, most of these studies require a high computational calculation load such as the shortest path search, enumerating routes and traffic assignment. Furthermore, it is difficult to obtain accurate demand data when the demand needs to be considered, and it may have assumptions such as perfect information and perfect control. Thus, methods that can evaluate the network without using route information are effective among the topological vulnerability analysis which can be evaluated by the network configuration. Nevertheless, there is no clear knowledge about the relationship between the evaluation by network topological indicators and traditional vulnerability analysis. By showing the usefulness of network topological indicators as road network evaluation methods, the value of that indicators can be confirmed. Another advantage of evaluation by network topological indicators which are not independent on the network size is that it can be applied to highly detailed road networks. As Duan et al. (2014) shows, the evaluation may differ depending on the resolution level of the road network. Hence, there are evaluations that can be obtained by analysing a network that includes roads of all ranks like small city roads.

Based on these backgrounds, the objective of this study is to add a new perspective to the field of road network evaluation by showing analytics using network topological indicators that can extract critical parts that was difficult to find due to the limitations of conventional methods. This study proposes the vulnerability analytics and connectivity analytics methods using spectral partitioning method and eigenvector centrality method. By comparing the proposed method with the traditional method, the usefulness of the proposed methods is verified. Because these methods do not require high computational calculation loads, it can be applied to large-scaled road networks.

It is very useful to identify critical parts that affect significantly to the vulnerability of road networks. When these critical nodes or links are disrupted, the evaluation measure of parts that are easily affected or not easily affected is connectivity analytics. The weak parts of connectivity in term of network topology should be heavily affected by failure and disruption in a disaster. Conversely, the effect of parts where network topological connectivity is stable will be small. Therefore, the connectivity analytics to understand areas with weakly and strongly connected is important.

This chapter summarised the studies of road network evaluation for each metrics, indicated the issues of them, and clarified the position and objective of this research. In addition, the reasons for adopting the two network topological indicators used for vulnerability and connectivity analysis in this study were described. In vulnerability and connectivity analysis, the one of the new point of this study is network topological analytics considering the measured values of traffic function by weight settings. For the interpretation of results, the challenges and objectives of the analytics with each weight are summarised in the next chapter.

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Chapter 3

Topological Analytics for Road Network Evaluation

3.1. Introduction

The road network is basically regarded and evaluated as their physical connection structure and the flow on the network. Many evaluation methods have been researched however the network topological analytics have advantages such as low calculation loads and no assumptions as mentioned in the section of Study Focus in Chapter2. Therefore, this research attempts to evaluate the road network by topological analytics. Chapter 3 proposes the road network evaluation methods based on the network topological analytics by considering the measured values of traffic function as weights. The process for adding the traffic function features value as weight will be described. Moreover, the vulnerability and connectivity indicators, which evaluate the road networks by analysing weighted network is introduced.

One advantage of analytics by network topological indicators using graph theory is that weights can be selected depending on what you want to evaluate. It is necessary to organised what can be evaluated by analysing the network considering the weight. In this study, various weights are applied according to the research objective. The challenges that are expected to be solved by weight setting and the objectives to be clarified by analysis and evaluation using weighted network are indicated.

3.2. Construction of weighted network

The weight values are given to each link in the network. Figure 3.1 shows undirected test network consisting of 4 nodes and 5 links. The values on each link indicate weights. By adopting various feature values such as distance, travel time, traffic volume, and so on as weights, weighted network analysis is based on each feature. Therefore, the weight settings changes depending on the objective of analysis.

In graph theory, a network is represented as a matrix, and the characteristics of the graph are understood by analysing the matrix. An adjacency matrix, degree matrix, Laplacian matrix and normalised Laplacian matrix using in this research are defined as follows. These matrices of the network in Figure 3.1 are shown as an example.

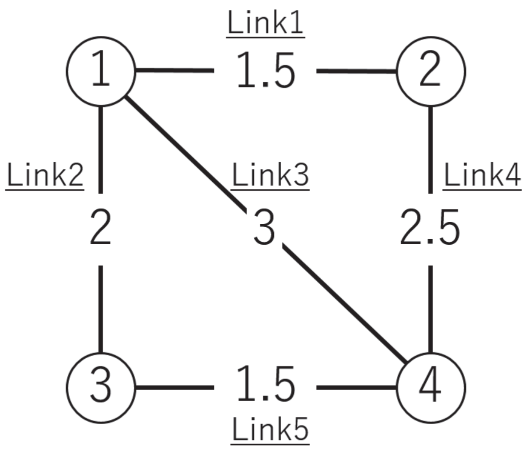

Figure 3.1 Undirected test network

Adjacency Matrix

An adjacency matrix represents the connection relationship on network. Consider an undirected network $G = (V, E, \mathbf{w})$, where V is a set of nodes, E is a set of links and $\mathbf{w}$ is a vector of link weights. Let $\mathbf{A}_G$ a weighted adjacency matrix for network G. The dimension of this matrix is $n \times n$, where $n = |V|$. The elements of $\mathbf{A}_G$ are

$$a_{uv} = \begin{cases} w_e & \text{if } e = (u, v) \in E \\ 0 & otherwise \end{cases} \tag{3.1}$$

where $w_e \in \mathbf{w}$, link $e = (u, v) \in E$ and nodes $u, v \in V$

The weight w_e corresponding to the link e is set to a non-negative value as network indicators. If the weights are not considered (i.e. unweighted graph), $w_e = 1$. For an undirected graph, the adjacency matrix is symmetric.

An adjacency matrix of test network is

$$\mathbf{A}_G = \begin{pmatrix} 0 & 1.5 & 2 & 3 \\ 1.5 & 0 & 0 & 2.5 \\ 2 & 0 & 0 & 1.5 \\ 3 & 2.5 & 1.5 & 0 \end{pmatrix}. \tag{3.2}$$

Degree Matrix

A degree matrix is a diagonal matrix composed of the degree of each node (i.e. the number of links flowing into/out from each node). In case of weighted network, a sum of connected link weights represents the value. The degree matrix $\mathbf{D}_G$ is a diagonal $n \times n$ matrix for network G. The diagonal elements are shown as follows and the other elements are zero.

$$d_{uv} = \begin{cases} \sum_{v \in \mathbf{V}} a_{uv} & if \;\; u = v \in V \\ 0 & otherwise \end{cases} \tag{3.3}$$

A degree matrix of test network is

$$\mathbf{D}_G = \begin{pmatrix} 6.5 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 3.5 & 0 \\ 0 & 0 & 0 & 7 \end{pmatrix} \tag{3.4}$$

Laplacian Matrix

A Laplacian matrix $\mathbf{L}_G$ for network G is defined as follows using the adjacency matrix and degree matrix.

$$\mathbf{L}_G = \mathbf{D}_G - \mathbf{A}_G \tag{3.5}$$

By definition. the column or row sum of a Laplacian matrix is zero.

A Laplacian matrix of test network is

$$\mathbf{L}_G = \begin{pmatrix} 6.5 & -1.5 & -2 & -3 \\ -1.5 & 4 & 0 & -2.5 \\ -2 & 0 & 3.5 & -1.5 \\ -3 & -2.5 & -1.5 & 7 \end{pmatrix} \tag{3.6}$$

Normalised Laplacian Matrix

A normalised Laplacian matrix $\mathbf{N}_G$ is normalised so that diagonal elements is 1. It is defined as,

$$\mathbf{N}_G = \mathbf{D}_G^{-0.5}\mathbf{L}_G\mathbf{D}_G^{-0.5} \tag{3.7}$$

A normalised Laplacian matrix of test network is

$$\mathbf{N}_G = \begin{pmatrix} 1 & -0.29 & -0.42 & -0.44 \\ -0.29 & 1 & 0 & -0.47 \\ -0.42 & 0 & 1 & -0.3 \\ -0.44 & -0.47 & -0.3 & 1 \end{pmatrix}. \tag{3.8}$$

Analysis using these matrices of weighted network shows the characteristics of network considering the measured values of traffic function.

3.3. Spectral Partitioning Method

This section describes the spectral partitioning method, one of the graph partitioning methods. The spectral partitioning method finds a cut set that divides the network evenly while minimising the weights included in the cut set. By applying this method to a weighted road network, the evaluations based on each viewpoint are performed. For example, the parts that are easy to become bottlenecks and the parts that have potential to be affected at the disaster. The relationship between the interpretation of these evaluations and the weight settings will be described later. In here, the derivation and characteristics of the spectral partitioning method is explained.

3.3.1. Derivation process of partition

Eigenvalue and Eigenvector of a Laplacian Matrix

Note that for undirected networks $\mathbf{A}_G$ is symmetric, and therefore $\mathbf{L}_G$ is also symmetric. According to the Spectral Theorem (see Spielman, 2015), when the elements of $\mathbf{L}_G$ is a real and symmetric matrix, there exist non-negative real $\lambda_1, \ldots, \lambda_n$, known as eigenvalues, and n mutually orthogonal unit vectors $\boldsymbol{\varphi}_1, \ldots, \boldsymbol{\varphi}_n$, known as eigenvectors. While the eigenvalues are unique, the eigenvectors are not. It follows from the definition of a Laplacian matrix (see von Luxburg, 2007) that

$$\begin{aligned}\mathbf{x}^T\mathbf{L}_G\mathbf{x} &= \sum_{u,v\in V} x_u(d_{uu} - a_{uv})x_v = \sum_{u\in V} x_u^2 d_{uu} - \sum_{u,v\in V} x_u x_v a_{uv} \\ &= 0.5(\sum_{u\in V} x_u^2 d_{uu} - 2\sum_{u,v\in V} x_u x_v a_{uv} + \sum_{v\in V} x_v^2 d_{vv}) \\ &= 0.5\sum_{u,v\in V} a_{uv}(x_u - x_v)^2 = 0.5\sum_{e=(u,v)\in E} w_e(x_u - x_v)^2 \geq 0\end{aligned} \tag{3.9}$$

where $\mathbf{x}$ is a vector of n elements.

Thus, a Rayleigh quotient of vector $\mathbf{x}$ for the Laplacian can be written as

$$\frac{\mathbf{x}^T\mathbf{L}_G\mathbf{x}}{\mathbf{x}^T\mathbf{x}}.$$

By definition, if $\boldsymbol{\varphi}$ is a unit eigenvector of the Laplacian, then the following equation holds,

$$\frac{\boldsymbol{\varphi}^T\mathbf{L}_G\boldsymbol{\varphi}}{\boldsymbol{\varphi}^T\boldsymbol{\varphi}} = \boldsymbol{\varphi}^T\mathbf{L}_G\boldsymbol{\varphi} = \lambda, \tag{3.10}$$

where λ is the corresponding eigenvalue.

Second smallest eigenvalue

Label the eigenvalues in an order of size as follows:

$$\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$$

It follows (see Spielman, 2015) that

$$\lambda_i = \min_{\mathbf{x}\perp\boldsymbol{\varphi}_1,\ldots,\boldsymbol{\varphi}_{i-1}} \frac{\mathbf{x}^T\mathbf{L}_G\mathbf{x}}{\mathbf{x}^T\mathbf{x}}, \tag{3.11}$$

where

$$\boldsymbol{\varphi}_i = \arg\min_{\mathbf{x}\perp\boldsymbol{\varphi}_1,\ldots,\boldsymbol{\varphi}_{i-1}} \frac{\mathbf{x}^T\mathbf{L}_G\mathbf{x}}{\mathbf{x}^T\mathbf{x}}, \tag{3.12}$$

implying that the eigenvectors are required to be mutually orthogonal. Therefore,

$$\lambda_2 = \min_{\mathbf{x}\perp\boldsymbol{\varphi}_1} \frac{\mathbf{x}^T\mathbf{L}_G\mathbf{x}}{\mathbf{x}^T\mathbf{x}} = \min_{\mathbf{x}\perp\mathbf{1}} \frac{\mathbf{x}^T\mathbf{L}_G\mathbf{x}}{\mathbf{x}^T\mathbf{x}}, \tag{3.13}$$

where $\boldsymbol{\varphi}_1 = \{1/\sqrt{n}, \ldots, 1/\sqrt{n}\}$, as the eigenvector corresponding to the smallest eigenvalue is a unit vector with all equal elements. As will be shown next, the second smallest eigenvalue of the Laplacian is intimately related to the strength of the connection between any sub-network and the rest of the network. When the Laplacian has two zero eigenvalues, there is a sub-network which is disconnected from the rest of the network.

Lower bound of network connectivity

Consider a sub-network $S \subset V$. One way to measure how connected S is to the rest of the network is to focus on the boundary of S, namely:

$$\partial(S) \equiv \{(u,v) \in E: u \in S, v \in V - S\} \tag{3.14}$$

This boundary is referred to as the cut, as it partitions the network into two sub-networks with node sets S and $V - S$ respectively.

Consider vector $\mathbf{x}_S$ with elements:

$$x_S(u) = \begin{cases} 1/m & \text{if } u \in S \\ 1/(m-n) & \text{if } u \in V - S \end{cases} \tag{3.15}$$

where $|\mathbf{S}| = m$. It is easy to see that$\mathbf{x}_S \perp \mathbf{1}$ since: $\boldsymbol{x}_S^T\mathbf{1} = 0$.

It follows from (3.9) and (3.13) that

$$\lambda_2 \le \frac{\mathbf{x}_S^T\mathbf{L}_G\mathbf{x}_S}{\mathbf{x}_S^T\mathbf{x}_S} = \frac{n}{m(n-m)}\sum\nolimits_{e\in\partial(S)} w_e \tag{3.16}$$

Let us define the right hand side of (3.16) as the connectivity of the sub-network to the rest of the network. Hence the second smallest eigenvalue provides a lower bound for the connectivity of any sub-network. If $\lambda_2 \gg 0$, all sub-network within a network must be well connected. Hence the second smallest eigenvalue of the Laplacian offers a measure of the connectedness of a network.

Normalised Laplacian

To measure the sub-network by the number of links rather than the number of nodes, normalised Laplacian shown in (3.9) is used.

By a variable transformation,

$$\mathbf{y} = \mathbf{D}_G^{-0.5}\mathbf{x}. \tag{3.17}$$

Then,

$$\frac{\mathbf{y}^T\mathbf{L}_G\mathbf{y}}{\mathbf{y}^T\mathbf{D}_G\mathbf{y}} = \frac{\mathbf{x}^T\mathbf{D}_G^{-0.5}\mathbf{L}_G\mathbf{D}_G^{-0.5}\mathbf{x}}{\mathbf{x}^T\mathbf{x}} = \frac{\mathbf{x}^T\mathbf{N}_G\mathbf{x}}{\mathbf{x}^T\mathbf{x}}. \tag{3.18}$$

Let the eigenvalues of the normalised Laplacian be

$$0 = \nu_1 \le \nu_2 \le \cdots \le \nu_n, \tag{3.19}$$

and $\mathbf{d}^{0.5}$ be a vector whose u^{th} element is $\sqrt{d_{uu}}$. The eigenvalues for $\nu_1 = 0$ is $\mathbf{d}^{0.5}$ since

$$\mathbf{N}_G\mathbf{d}^{0.5} = \mathbf{D}_G^{-0.5}\mathbf{L}_G{\mathbf{D}_G}^{-0.5}\mathbf{d}^{0.5} = \mathbf{D}_G^{-0.5}\mathbf{L}_G\mathbf{1} = \mathbf{D}_G^{-0.5}\mathbf{0} = \mathbf{0}. \tag{3.20}$$

Hence,

$$\nu_2 = \min_{\mathbf{x}\perp\mathbf{d}^{0.5}} \frac{\mathbf{x}^T\mathbf{N}_G\mathbf{x}}{\mathbf{x}^T\mathbf{x}} = \min_{\mathbf{y}\perp\mathbf{1}} \frac{\mathbf{y}^T\mathbf{L}_G\mathbf{y}}{\mathbf{y}^T\mathbf{D}_G\mathbf{y}}. \tag{3.21}$$

Spectral Partitioning

Consider the following network partitioning problem.

$$\min_{\mathbf{y}} \mathbf{y}^T\mathbf{L}_G\mathbf{y} \tag{3.22}$$

subject to

$$\mathbf{y} \in \{a,b\}^n \tag{3.23}$$

$$\mathbf{y}^T\mathbf{1} = 0 \tag{3.24}$$

$$\mathbf{y}^T\mathbf{D}_G\mathbf{y} = 1 \quad (3.25)$$

By the optimality condition, the coefficients a and b for obtaining the optimal solution are,

$$a = \left(\frac{(n-m)^2}{(n-m)^2 d(S) + m^2 d(V-S)}\right)^{0.5}, \quad (3.26)$$

$$b = -\left(\frac{m^2}{(n-m)^2 d(S) + m^2 d(V-S)}\right)^{0.5}. \quad (3.27)$$

As (3.21) omits (3.21, it is a relaxation of (3.22), following that

$$\nu_2 \leq \frac{\mathbf{y}_S^T\mathbf{L}_G\mathbf{y}_S}{\mathbf{y}_S^T\mathbf{D}_G\mathbf{y}_S} = \frac{n^2}{(n-m)^2 d(S) + m^2 d(V-S)} \sum\nolimits_{e\in\partial(S)} w_e, \quad (3.28)$$

where the elements of $\mathbf{y}_S$ are

$$y_S(u) = \begin{cases} a & \text{if } u \in S \\ -b & \text{if } u \in V-S \end{cases} \quad (3.29)$$

Hence the second smallest eigenvalue of the normalised Laplacian matrix provides a lower bound for a measure of the connection of sub-network S to the rest of network $S-V$. This is the normalised cut by Bandeira (2015). In this case, the normalisation reflects the number of links either side of the cut rather than the number of nodes.

Cheeger's cut, constant and inequality

The eigenvalues of adjacency matrix, Laplacian matrix and normalised Laplacian matrix enables to show the upper and lower bounds of indicators related to the connections. The literature on spectral analysis invariably refers to Cheeger's cut, constant and inequality. Cheeger's cut (see Bandeira, 2015) is defined as

$$h(S) = \frac{1}{\min[d(S), d(V-S)]} \sum\nolimits_{e\in\partial(\mathbf{S})} w_e. \quad (3.30)$$

The right hand side of (3.30) is referred by Spielman (2015) to conductance and is an alternative measure of how well sub-network S is connected to the rest of the network. The Cheeger's constant for network G with sub-network S that minimise $h(S)$ is

$$h_G = \min_{S\subset V} h(S). \quad (3.31)$$

From the Cheeger's inequality, the Cheeger's constant has its upper and lower bounds defined by the second smallest eigenvalue of the normalised Laplacian matrix as

$$0.5\nu_2 \leq h_G \leq \sqrt{2\nu_2}. \quad (3.32)$$

It can be shown that

$$\begin{aligned} \frac{n^2}{(n-m)^2 d(S) + m^2 d(V-S)} \sum\nolimits_{e\in\partial(S)} w_e &\leq \frac{d(V)}{d(S)d(V-S)} \sum\nolimits_{e\in\partial(S)} w_e \\ &= \left(\frac{1}{d(S)} + \frac{1}{d(V-S)}\right) \sum\nolimits_{e\in\partial(S)} w_e \leq 2h(S). \end{aligned} \quad (3.33)$$

From (3.17),

$$\nu_2 \leq \frac{n^2}{(n-m)^2 d(S) + m^2 d(V-S)} \sum\nolimits_{e\in\partial(S)} w_e \leq 2h(S). \quad (3.34)$$

To better understand inequality (3.33), consider

$$\frac{1}{(1-\theta)^2 d(S) + \theta^2 d(V-S)} \tag{3.35}$$

where $0 \leq \theta \leq 1$. This fraction achieves a maximum when $\theta = d(S)/d(V)$.
Hence,

$$\frac{n^2}{(n-m)^2 d(S) + m^2 d(V-S)} \leq \frac{d(V)}{d(S)d(V-S)}. \tag{3.36}$$

From (3.28) and (3.36) ,

$$\nu_2 \leq \frac{d(V)}{d(S)d(V-S)} \sum\nolimits_{e \in \partial(S)} w_e. \tag{3.37}$$

Let us define the right hand side of (3.37) as a connectivity of the sub-network to the rest of the network. From the meaning of minimising the right hand side, the second smallest eigenvalue designates the cut set minimising the sum of the weights on the boundary while dividing the sum of degrees into two equal parts.

Procedure to partition the network

A natural question to ask is, given ν_2, where in the network does the corresponding sub-network boundary $\partial(S)$ lie. Von Luxburg(2007) suggests that a natural basis for a boundary is the sign of the elements of $\boldsymbol{\varphi}_2$; nodes with a non-negative eigenvector lie on one side of the boundary while nodes with a negative eigenvector lies on the other. This research adopts this approach.
From the above, the procedure of the spectral partitioning method can be summarised as follows,

1. select a link weight that is appropriate for the research objective,
2. derive the normalised Laplacian matrix of the network to be analysed,
3. obtain the second smallest eigenvalue of the normalised Laplacian matrix. If the second smallest eigenvalue is 0, find the smallest non-zero eigenvalue, and
4. partition the network according to the sign of the eigenvector corresponding to the smallest non-zero eigenvalue.

The cut set is the set of links that partition the network based on the sum of the smallest weights such that the weight densities of each sub-network are approximately equal. Links in the cut set are critical as they are easy to disconnect the network.

3.3.2. Partition result on test network

The second smallest eigenvalue of the normalised Laplacian matrix of a test network find the cut set. The cut set in a test network divided into a subnetwork including node1, node3 and another subnetwork including node2, node4 (Figure 3.2). This cut is given by the second smallest eigenvalue of normalised Laplacian matrix as shown in (3.37), the contents of right hand side of (3.37) in this partition result are shown.

The right hand side of (3.37) minimise the sum of the weights on the partition while dividing the sum of degree evenly. The sum of weights in cut set of test network is

$$\sum\nolimits_{e\in\partial(S)} w_e = w_1 + w_3 + w_5 = 1.5 + 3 + 1.5 = 6.0, \tag{3.38}$$

the degree of each node is represented by a degree matrix shown in (3.2). Therefore, the sum of degree contained in each subnetwork are shown follows.

$$d(S) = 10, \tag{3.39}$$

$$d(V - S) = 11, \tag{3.40}$$

$$d(V) = 21. \tag{3.41}$$

The value of (3.41) do not depend the cut set because this is the sum of degree in whole of network. By the result from (3.39) to (3.41), the relationship of the sum of degree in two subnetwork is

$$\frac{d(V)}{d(S)d(V-S)} = 0.191, \tag{3.42}$$

the right hand side of (3.37) is

$$\frac{d(V)}{d(S)d(V-S)} \sum\nolimits_{e\in\partial(S)} w_e = 1.145 \tag{3.43}$$

The detailed of property that minimises the sum of the weights on the cut set while dividing the sum of degrees into two equal parts were shown using test network.

If the network is divided by different cut set, the value on the right hand side of (3.37) is as follows. Figure 3.3 shows position of the cut set determined arbitrarily by author. In this case, the sum of weight is small,

$$\sum\nolimits_{e\in\partial(S)} w_e = w_2 + w_4 = 2 + 1.5 = 3.5, \tag{3.44}$$

the sum of degree of two sub network is far from equal,

$$\frac{d(V)}{d(S)d(V-S)} = 0.343, \tag{3.45}$$

therefore, the right hand side of (3.37) based on an arbitrary cut set is

$$\frac{d(V)}{d(S)d(V-S)} \sum\nolimits_{e\in\partial(S)} w_e = 1.2 \tag{3.46}$$

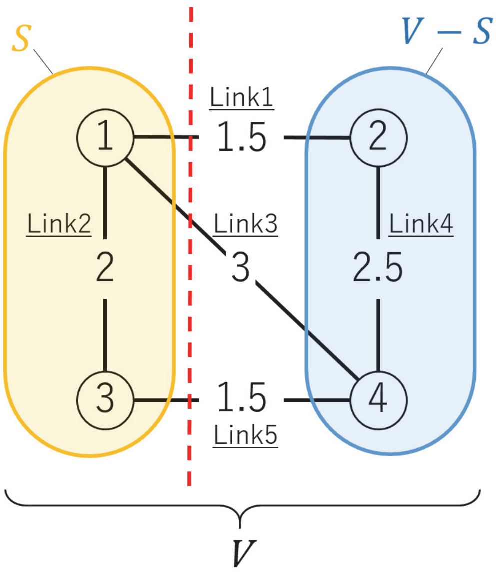

Figure 3.2 The cut set of test network

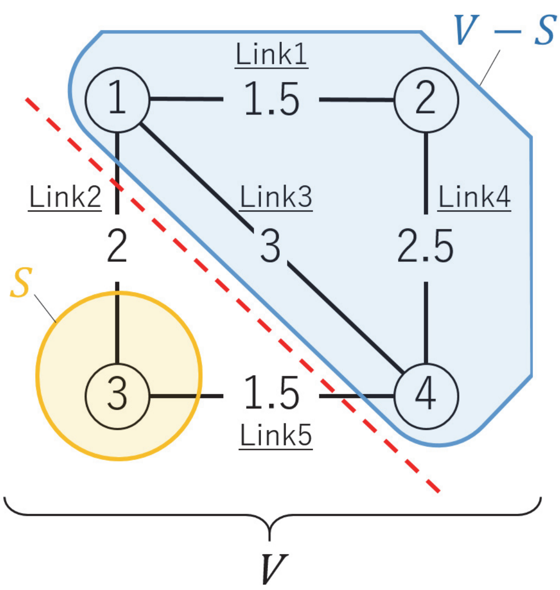

Figure 3.3 The cut set determined arbitrarily

The cut set shown in Figure 3.3 is not evenly divided even though the sum of weights is small. The right hand side of (3.43) is larger than that of the cut set obtained by minimising the second smallest eigenvalue of normalised Laplacian matrix. Although only one example of other cut set was introduced, the cut set obtained by minimising the second smallest eigenvalue realises equal division of the subnetwork and minimisation of the sum of weight by this details.

3.4. Eigenvector Centrality

As another network topological indicator, the eigenvector centrality is introduced in this section. The eigenvector centrality (EC) gives each node an evaluation value. A specific node is highly evaluated in cases where important nodes are connected to adjacent nodes of that specific node. Therefore, the parts of the network where important nodes connect one another becomes more important, while the part where less important nodes connect one another become less important. EC analysis in weighted networks evaluates the strength and weakness of connectivity based on the measures of traffic function set as weights.

3.4.1. Definition and derivation of eigenvector centrality

As mentioned in Chapter 2, EC defined by Bonacich (1972) is one of the network topology indicators. This method can be applied to directed network and does not require an adjacency matrix to be symmetric. Here the mathematical characteristics of EC will be described. Let,

$$\mathbf{Ax} = \lambda \mathbf{x}, \tag{3.47}$$

where $\mathbf{x}$ is an eigenvector, λ is an eigenvalue and $\mathbf{A}$ is a weighted adjacency matrix with elements,

$$a_{ij} = \begin{cases} \text{weight of the link from node } i \text{ to node } j \\ 0 \text{ otherwise} \end{cases}$$

The Rayleigh quotient is

$$\lambda = \frac{\mathbf{x}^T \mathbf{Ax}}{\mathbf{x}^T \mathbf{x}} = \frac{\sum_{i,j} x_i a_{ij} x_j}{\sum_i x_i^2}. \tag{3.48}$$

I am interested in the largest eigenvalue (demoted by*)

$$\lambda^* = \max_{\mathbf{x}} \frac{\mathbf{x}^T \mathbf{Ax}}{\mathbf{x}^T \mathbf{x}} = \frac{\mathbf{x}^{*T} \mathbf{Ax}^*}{\mathbf{x}^{*T} \mathbf{x}^*} = \frac{\sum_{i,j} x_i^* a_{ij} x_j^*}{\sum_i x_i^{*2}} = \frac{\cdots + x_i^* a_{ij} x_j^* + \cdots}{\cdots + x_i^{*2} + x_j^{*2} + \cdots}. \tag{3.49}$$

Proposition 1: The eigenvector corresponding to the largest eigenvalue is non-negative and the largest eigenvalue is positive.

Proof 1: $\mathbf{x}^* = \mathbf{0}$ would imply $\lambda^* = 0$. However, (3.49) shows that $x_i^* > 0$ and $x_j^* > 0$ would guarantee $\lambda^* > 0$ if $a_{ij} > 0$, so, since $a_{ij} > 0$ for some pair of nodes i and j, at least two elements of $\mathbf{x}^*$ must be positive and $\lambda^* > 0$. Hence $\lambda^* > 0$ and $\mathbf{x}^* \geq \mathbf{0}$. QED.

Proposition 2: If every node can be reached from any other node in the network, then $\mathbf{x}^* > \mathbf{0}$ (or alternatively $\mathbf{x}^* < \mathbf{0}$).

Proof 2: Suppose that $x_j^* > 0$ for some node j for which $a_{ij} > 0$ for some node i. It follows from (3.47) that $x_i^* = \frac{\sum_j a_{ij} x_j^*}{\lambda^*} > 0$. If it is possible to get from any node to node j and if it is possible to get from node j to any node then by induction $\mathbf{x}^* > \mathbf{0}$. Similarly, if $x_j^* < 0$ and the network is connected it follows that $\mathbf{x}^* < \mathbf{0}$. QED.

If Proposition 2 is true, then $\mathbf{A}$ is said to be irreducible. Note that neither Proposition 1 nor

Proposition 2 requires $\mathbf{A}$ to be symmetric. This is in contrast to spectral partitioning, which does require symmetry.

Note furthermore that a large value of a_{ij} leads to large values of x_i^* and x_j^* from (3.49), countered by the denominator, which grows faster than the numerator as x_i or x_j get larger. Since $x_i^* = \frac{\sum_j a_{ij} x_j^*}{\lambda^*}$, the eigenvector element for node i is a linear combination of the adjoining eigenvector elements, weighted by elements in both directions. Hence nodes i for which $x_i^* \gg 0$ constitute a strongly connected community and λ^* offers a measure of the strength of connection in the network as a whole.

3.4.2. Eigenvector centrality analysis on test network

The network shown in Figure 3.4 is used as an example of EC analysis. Since the EC analysis supports directed network, the test network was changed directed. An adjacency matrix of this directed test network is

$$\mathbf{A} = \begin{pmatrix} 0 & 1.5 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 \\ 2.5 & 2.5 & 1.5 & 0 \end{pmatrix}. \tag{3.50}$$

By using this adjacency matrix $\mathbf{A}$, the eigenvector corresponding the largest eigenvalue that satisfies (3.47) is the eigenvector centrality. Table 3.1 shows the largest eigenvalue and the corresponding eigenvectors on each node. Since there are scalar multiple eigenvectors corresponding to the largest eigenvalue satisfying (3.47) in scalar time, the values of EC are normalised so that the sum of EC in the network become 1. In small network, it is difficult to clarify the effects of propagation by adjacent nodes connect to critical nodes. Node 4 with the inflow of large weight link is highly evaluated. Conversely, the evaluation of node 3 is smaller than that of node 2 despite the inflow link from node 4. This may be because the weight of inflow link from node 4 is not so large. In this way, EC analyses the connectivity considering weights in directed network.

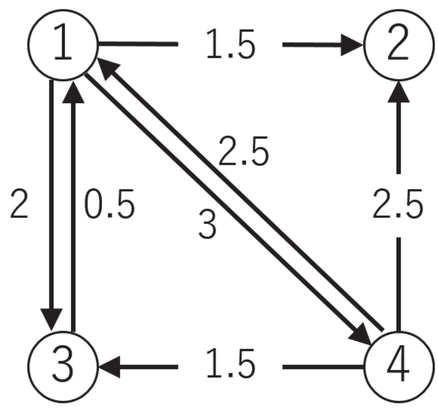

Figure 3.4 Directed test network

Table 3.1 Largest eigenvalue and eigenvector centrality

λ	Largest eigenvalue	4.691
x_1	EC on node1	0.277
x_2	EC on node2	0.256
x_3	EC on node3	0.067
x_4	EC on node4	0.400

3.5. Evaluation Objectives by Weighted Network

This research attempts to analyse the road network by network topological indicators using weighted network. Notation method for weighted network by matrix was introduced, and two network topological indicators analysed by weighted network, spectral partitioning method and eigenvector centrality method were described. This section shows the measured values of traffic function that are considered as weight. Moreover, the evaluation objective by weighted network analytics depending on each measured value is summarised.

Table 3.2 is organised for comprehensive weight settings. In this research, 10 types of traffic measured values are set as weight and that weighted networks are analysed. The challenges the weighted network to evaluate are roughly divided into four:

- The evaluation of road improvement
- Characterise the region on road network
- Understanding the traffic conditions which flow on the road network
- The evaluation of disaster impact on the road network

The road network is evaluated by the analytics that set the measured values of traffic function according to each challenge. Furthermore, the combine the evaluation results by analytics with each weight setting may make a deeper discussion about these challenges. Based on the type of traffic function, measured values such as road use situations, construction conditions and environmental conditions (like hazard risk) are set. Notes on these settings and data information to be used are mentioned in the parts where the analytics and evaluation are performed.

The main parts of this study on both methods are network topological analytics using capacity weighted network. Thus, the evaluation of capacity weighted network by both methods are explained. Capacity weighted spectral partitioning finds the vulnerable partition considering traffic capacity. The links in the cut set must have been composed of small capacity roads. This means that this cut consists of potential bottlenecks. Because it is likely to become a bottleneck regardless of demand if small capacity roads construct a cut set. Traditionally, the bottlenecks have been identified by traffic assignment based on OD traffic volume data. However, it is difficult to obtain accurate traffic demand data in the disaster or in the future. Also, the evaluation results may be different depending on the magnitude of the uncertain demand. As a proxy of the analysis based on demand data, this study proposes using capacity as a weight. With regard to EC analysis, unweighted EC shows the connectivity purely in terms of network topology. In many cases, however, the practitioners may also want to consider the "weakness" of infrastructure in identifying the vulnerable areas. In general, the probability of link disruption caused by various hazards should be considered, but it is very difficult to estimate it accurately since the occurrence of hazards may depend on geography, meteorological and social conditions. It is supposed here that capacity weighted EC shows the connectivity considering the difficulty of link disruption, because the link disruption may not easily happen on larger capacity roads. As a proxy of such probability, this study proposes to use link capacity as a weight. Even the same capacity weighted analysis, the interpretation of evaluation is different, the spectral partitioning which evaluates the likelihood to become bottlenecks and EC which considers the ease of links disruption.

In Chapter 6, long-term road networks with changes are analysed to evaluate the impact of road network improvement. The weight settings are adopted capacity-length as supply side weight and traffic volume as demand side weight. These weights are applied only to the EC analysis for the connectivity evaluation. The capacity-length is calculated as a multiplication of length and capacity of each link, and it represents the "magnitude of road areas". The capacity-length weighted EC evaluates the connectivity of road supply performance. By considering the length, it is possible to understand the relationship with road improvement costs. Traffic-volume sets the number of vehicles on each link which is one of the demand side characteristics as weights. The traffic volume weighted EC evaluates the level of traffic concentration based on the actual usage.

Basically, each weighted network is applied to the spectral partitioning analysis or EC analysis or both. The objectives and targets to evaluate by both methods are depending on the measured values as organised in this section. For the spectral partitioning analysis, reserve capacity and link disruption probability are used as weights in addition to capacity weighted. The reserve capacity represents the space of links by the difference between capacity and traffic volume. If the traffic volume exceeds the capacity, all of those values are 1. The analysis using reserve capacity weighted network attempts to identify the cut set which is likely to become bottlenecks by finding the parts with no remaining capacity. The parts where are likely to become bottlenecks due to the small available road network capacity can be said vulnerable. The analysis using link disruption probability weighted network attempts to identify the cut set which divides the network by link disruptions at the disaster. The identified cut set consists of links that have a high risk of being degraded at the same time.

For the EC analysis, speed, BPR function, travel time, distance, and congestion rate are used as weights in addition to capacity, road area and traffic volume. The measured values of speed use the speed limits of links. The EC analysis using speed weighted network represents the connectivity distribution of links with high and low speed limits. The speed limits of links should correlate with the road rank of links. The BPR function is the travel time considering the congestion. Therefore, the EC analysis using BPR function weighted network represents the connectivity distribution of links with short and long travel time considering congestion. On the travel time weighted network, the travel times as weights use travel times of links for the survey data. The EC analysis using distance weighted network represents the spatial density of network by the connectivity of length on each link. When roads with long distances are connected, the network must be sparse. Hence, in the parts where the connectivity of roads with long distances is high, even if there are detour options at the link disruption, it can be said that the detour rate is often high. The measured values of congestion rate are the traffic volume divided by the capacity. The congestion rate can also indicate the value that the traffic volume exceeds the capacity. The EC analysis using congestion rate weighted network represents the concentration and distribution of crowded roads. The contents of analysis according to each objective and target will be introduced in each chapter of this thesis. Analytics using the spectral partitioning method is described in Chapter 4, and analytics using EC is described in Chapter 5 and Chapter 6. The location mentioned in this paper in Table 3.2 shows the detailed section.

Table 3.2 Evaluation objectives based on weight settings

Classification of Challenges	Challenges	Weight	Equation	Supplementary explanation about the weight	Spectral Partitioning	Eigenvector Centrality	Location mentioned in this paper
		Unweigted	$w = 1$		The potential to significantly reduce network functionality based on network topology. The weakness of road network.	The strength and weakness of netowrk connection relationship.	
The evaluation of road improvement	How is the supply function improved as a network based on the viewpoint of "movement" which is the basic supply function of roads ? Where are insufficient improvement areas ?	Capacity	$w = C_e$		Vulnerable parts that are easy to become bottlenecks	The magnitude and strength of movement ability on road network. Connectivity considering the ease of link disruption based on the traffic capacity.	4.2, 4.3, 5.2, 5.3
	How is the impact of road construction on the entire network? Visualisation of the cost effectiveness of improvement on netowrk functions.	Road area (multiplication of capacity and length)	$w = L_e C_e$	Amount of road improvement	-	Contribution for the supply performance by the road improvements.	6.2, 6.4
Characterise the region on the road network	Classification of areas based on road features and functions. Descrimination of inter-city road areas where high-standard roads specialised for traffic functions and urban areas where roads specialised for access functions located.	Speed	$w = S_e$	Fixed speed limit	-	The distribution of road rank connectivity. Connectivity distribution of links with high and low speed limits.	5.4
		Capacity	$w = C_e$		Vulnerable parts that are easy to become bottlenecks	The magnitude and strength of movement ability on road network. Connectivity considering the ease of link disruption based on the traffic capacity.	4.2, 4.3, 5.2, 5.3
		BPR function	$w = t_{0i}\left(1 + \alpha P_e^{\beta}\right)$	Travel time (considering congestion)	-	The distribution of road rank connectivity. Connectivity distribution of links with short and long travel time considering congestion.	5.4
		Travel time	$w = t_e$		-	The distribution of road rank connectivity. Connectivity distribution of links with short and long travel time.	5.4
	Clarification of the spatial connection features of road networks. Identify the spatial density of the network.	Distance	$w = L_e$		-	The spatial density of network by the connectivity of length on each link.	5.4
Understanding the traffic conditions which flow on the road network	How is the available traffic capacity located ? Are there locations where are easy to become bottlenecks because of no traffic capacity to spare.	Reserve capacity	$w = C_e - V_e$	If the traffic volume exceeds the capacity, all of that values are 1.	Vulnerable parts that are easy to become bottlenecks because the available road network capacity is small.	-	4.4
	Where is the concentration of roads where most of the traffic capacity is used or used beyond the traffic capacity ? Where are the impact areas of those heavily used roads ?	Congestion rate	$w = \frac{V_e}{C_e}$	If the traffic volume exceeds the capacity, it is represented as exceeding 1.	-	Concentration and distribution of crowded roads.	5.4
	Identify locations where demand is significantly higher or lower. Understandf the geographical distribution of demand.	Traffic volume	$w = V_e$		-	Concentration and distribution of traffic volume.	5.4, 6.5
The evaluation of disaster impact on the road network	How are links susceptible to damage at the disaster distributed ? Identify parts that have the potential to give a significant impact for the whole of network at the disaster.	Link disruption probability	$w = -\ln p_e$		The links that have high risk of being degraded at the same time due to a disaster, and the disruption of their links divide the network.	-	4.4

C_e: Traffic capacity on link$e$ L_e: Length of link$e$ S_e: Speed limit of link$e$ V_e: Traffic volume on link$e$
t_{0e}: Free flow travel time on link$e$ t_e: Travel time on link$e$ p_e: Disruption probability of link$e$ in disaster

3.6. Concluding Remarks

This chapter showed the properties of weighted networks, their analysis methods and the evaluation objectives. The notation method using the matrix for weighted networks was presented. Then, I introduced various kinds of matrices used in graph theory for cases that consider the values as weights. As methods for analysing the characteristics of network using weighted networks, this chapter mentioned about the spectral partitioning method and eigenvector centrality method.

Spectral partitioning method is one of the graph partitioning method by using graph theory. This method identifies the cut set that divide the network evenly and are easy to cut into two subnetworks. Described the derivation process and definition, and the interpretation of the partition obtained by this method was explained using derived formulations. Further, the example network calculations made clearly the details of partition result obtained by spectral partitioning method. Eigenvector centrality method is another analysis method for weighted networks introduced in this chapter. EC method tends to get higher evaluation if they have adjacency nodes that connect to important nodes. Hence, the strength of connectivity based on the measured value of traffic function that are considered as weights. The definition and derivation are described and the directed example network was calculated. The evaluation result of directed example network indicated the characteristics of considers the weights.

This research attempt to evaluate the road network by using these network analytics indicators based on the weighted networks. Applicability to road networks, evaluation results, discussions and utilisations, etc. will be performed in later chapter for each network analytics indicator and weight settings. For the interpretation of those analytics, the challenges that are expected to be solved by weight settings and the objectives to be clarified by network evaluations using weighted networks were indicated. The contents summarised in this chapter will assist in the interpretation of subsequent analysis and evaluations.

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Chapter 4

Network Vulnerability Analytics by Topological Indicators

4.1. Introduction

Chapter 4 verifies the road network vulnerability analysis by network topology indicator. Identification of critical parts of the network where connectivity is significantly reduced is very important for the vulnerability of road networks. The improvement of critical parts helps to construct a robust road network efficiently. As shown in the Chapter 2, a lot of methodologies and literatures have been studied to evaluate the vulnerability of the road network. Most of them require estimates of demand and traffic assignment. Thus, they have some assumptions and restrictions of the target network such as perfect information (if deterministic user equilibrium is assumed), and perfect control (system optimal be all users following the administrator). The analysis based on the network topology indicator does not require to calculate with large computation loads, and it is valuable to be able to identify vulnerable parts easily and quickly.

In this chapter, the vulnerability of road network is evaluated by using the spectral partitioning method which is one of the graph partitioning method. The definition and details of derivation process were given in Chapter 3. The objective of this chapter is to validate that the usefulness of the evaluation method based on the network topology indicator and to demonstrate the tractability of the proposed method onto large-scaled practical road networks.

The method proposed by Bell et al. (2017) is used in Chapters 3 and 4. This is the first paper applying the spectral partitioning method to transport network while the method has been studied for many kinds of other networks (ex; von Luxburg, 2007; Bandeira, 2015; Spielman, 2015).

4.2. Comparison with Conventional Methods

To test its usefulness, spectral partitioning method described in 3.3 is applied to the Sioux Falls road network. Moreover, the validity of the method will be confirmed by comparing with a maximum flow problem, which is one of the conventional road network evaluation methods.

Here, following to Bell et al. (2017), a traffic capacity is used as weight. Because the cut set obtained by the second smallest eigenvalue of capacity weighted Laplacian has the least capacity and therefore could constitute a network bottleneck. From the comparison results, if the capacity weighted spectral analysis can identify the critical bottleneck in a network, it means that spectral analysis is useful measure of transport network vulnerability.

4.2.1. Maximum flow in capacitated networks

As a conventional road network evaluation method, a maximum flow problem is adopted. This method uses the theory of capacitated user equilibrium assignment with right-angle function (Bell and Iida, 1997). The maximum link traffic volume is obtained by solving a total travel time minimisation problem. For the links that may overflow, a delay time is added as a shadow price to express the principle of equal travel time.

The formulation is as follows.

$$\max_{\mathbf{x}} Z \tag{4.1}$$

subject to

$$\sum_{a \in in(i)} x_{ad} - \sum_{a \in out(i)} x_{ad} = \begin{cases} -Z g_{id} & \text{if } i \in \mathbf{O} \\ \sum_{j \in \mathbf{R}-d} Z g_{ji} & \text{if } i = d \\ 0 & \text{otherwise} \end{cases}, \forall d \in \mathbf{D} \tag{4.2}$$

$$\sum_{d \in \mathbf{D}} x_{ad} \leq s_a \quad \forall a \in \mathbf{E} \tag{4.3}$$

$$x_{ad} \geq 0 \quad \forall a \in \mathbf{E}, d \in \mathbf{D} \tag{4.4}$$

where,

Z	:	Total trip production (unknown variable, network capacity)
x_{ad}	:	Destination-specific link traffic volume (unknown)
g_{od}	:	A probability that a traffic is originated from o and destined to d($\sum_{od} g_{od} = 1$, known as OD probability)
s_a	:	A capacity of link a
$In(i)$	:	A set of links flowing into node i
$Out(i)$	:	A set of links flowing out from node i
$\mathbf{O}$	:	A set of origins
$\mathbf{D}$	:	A set of destinations
$\mathbf{E}$	:	A set of links.

Shadow prices of link capacity constraints should be positive if a link overflows. Therefore, the cut set that causes overflow is obtained by connecting the links that have positive shadow prices. It assumes here that OD pattern is given and unchangeable.

4.2.2. Sioux Falls Network

The Sioux Falls road network (Bar-Gera, Transportation Networks) shown in Figure 4.1 has 24 nodes and 76 directed links. Traffic capacity indicated by the values on links in Figure 4.1 is same in both directions. Figure 4.2 shows the distribution of OD traffic volume, and the maximum and minimum OD traffic volume are 4,400 and 100 (vehicles), respectively. The partition produced by capacity weighted spectral analysis is compared with the partition produced by the active link capacity constraints of the maximum flow problem.

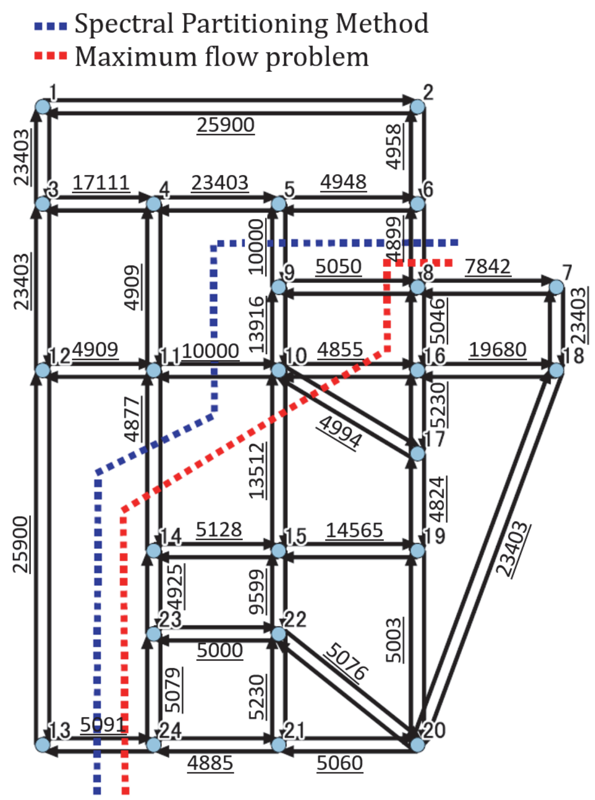


Figure 4.1 The cut set on the Sioux Falls network

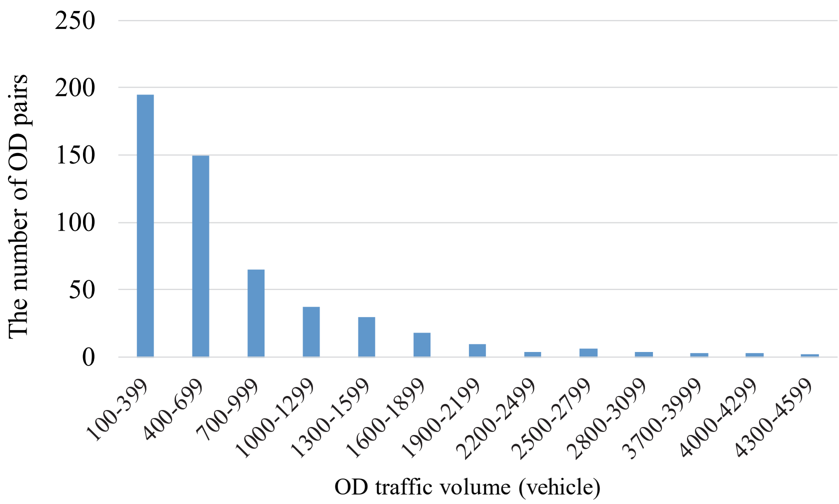
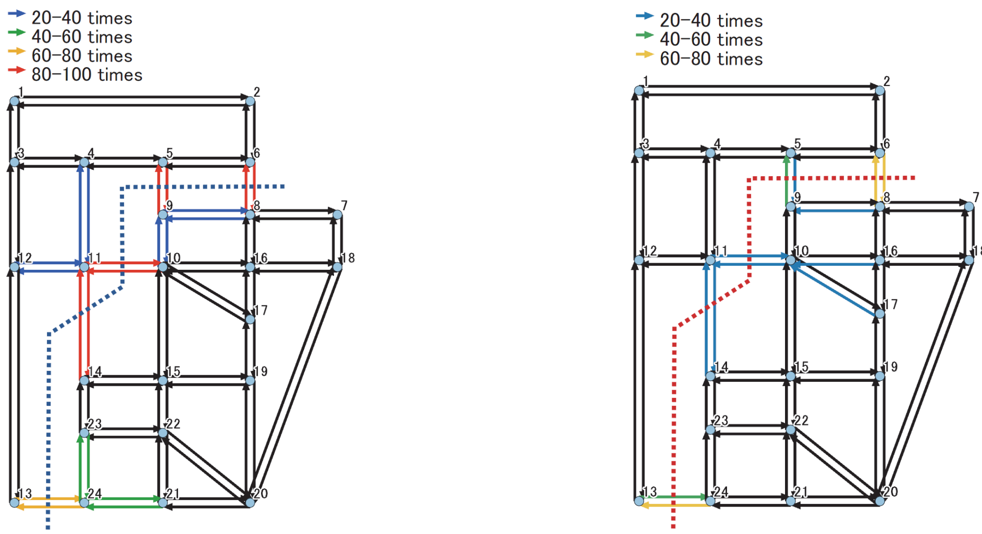


Figure 4.2 The distribution of OD traffic volume

4.2.3. The partition results by both methods

The dotted lines in Figure 4.1 show the partition produced by capacity weighted spectral analysis and maximum flow problem. They are similar but not totally the same. The reason is that the partition produced by the maximum flow problem depends on the OD pattern traffic volume, whereas the partition produced by capacity weighted spectral analysis is derived only from the network topology and traffic capacity. Then, traffic capacity sensitivity is analysed by iterative calculations with fixed OD traffic volume and fluctuated traffic capacity.

In this case, a uniform random number, $\gamma = [0,1]$, is created for each link and traffic capacity is changed to $(0.5 + \gamma) \times s_a$. This means that the traffic capacity fluctuates uniformly from 50% to 150%. 99 sets of fluctuated traffic capacity are prepared, and 100 calculation results including original case are obtained. By connecting links that are included many times in the cut set, it is possible to find the cut set which is likely to occur regardless of the fluctuations of traffic capacity. Figure 4.3 shows the partition connecting the links that are included many times within the 100 instances. The result indicates that the most likely cut sets by the fluctuation of traffic capacity are same regardless of the methods. Moreover, in the Spectral Partitioning method, there are many links that are frequently selected as the cut. This means that the cut set produced by the Spectral Partitioning method is more stable than the cut set produced by the maximum flow problem. This comparison result of both methods shows that the Spectral Partitioning method can identify the potential bottleneck.



(a) Spectral Partitioning method (b) Maximum flow problem

Figure 4.3 The cut set by two methods

4.2.4. Effect of the weight

In this section, the critical bottlenecks of road network are identified by capacity weighted spectral partitioning analysis. To confirm the effect of the weight in this analysis, the partition produced by the Spectral Partitioning method with unweighted $(w_e = 1)$ is shown in Figure 4.4. The partition result that depends only on the network topology divide almost evenly up and down. Because the trend is different from the partition considering the traffic capacity, it is confirmed that a weight setting provides a large impact on the extraction of critical parts of the network.

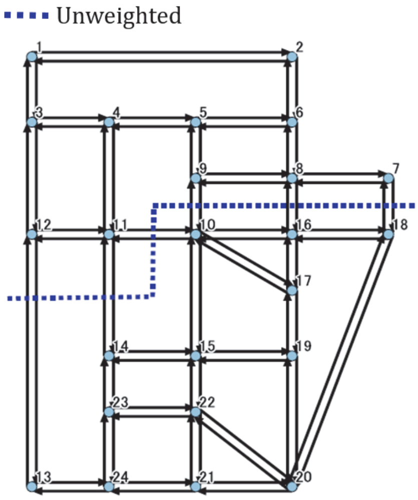


Figure 4.4 The partition by unweighted spectral analysis

4.3. Application to Practical Road Network

To test a usefulness of the spectral partitioning analysis for a practical road network, the Gifu Prefecture road network shown in Figure 4.5 is analysed. This network, which includes intercity expressways, national highways and the prefectural roads, contains 1,468 nodes and 2,348 links. The normalised Laplacian is used for the identification of the critical cut. The cut corresponding to the second smallest eigenvalue indicates where network capacity poses a potential vulnerability.

The cut set consists of red links shown in Figure 4.5. The result indicates that the cut set connecting urban area including Gifu City (the western area of Gifu Prefecture) where the network is dense and the eastern area where the network is rather sparse and has the weakest connectivity. Since the cut set lies on a 'constricted area' within the Prefecture, it makes sense. As mentioned above, there are nodes with positive values of eigenvector on one side of the boundary and nodes with negative values of eigenvector on the other side. In this case, the number of nodes with non-negative eigenvectors is 767(52%), and the number of nodes with negative values is 701(48%). The boundary divides the network into two almost evenly by a cut set with a small total traffic capacity. Application to the Gifu Prefecture road network reveals that the spectral partitioning analysis can identify a set of vulnerable links and critical parts even in practical road networks.

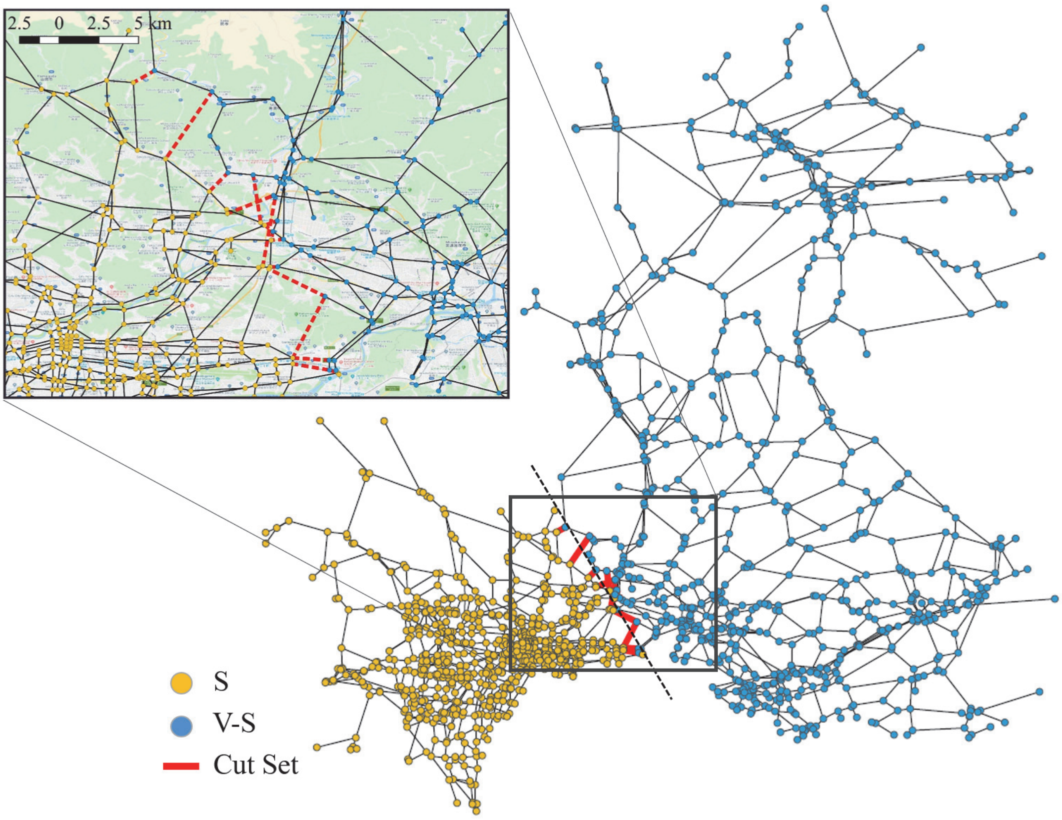


Figure 4.5 The cut set by traffic capacity weighted

4.4. Impact of Weight Setting

One of the advantages of spectral partitioning is that the link weights can be selected according to the research objective. Capacity weighted spectral analysis can be used to identify potential bottlenecks in the network. Other weight definitions would lead to different network partitions. This section tests the changes in the characteristics of the cut set according to different weight definitions by the spectral partitioning analysis on the Gifu Prefecture road network.

4.4.1. Reserve capacity

According to Wong and Yang (1997), reserve capacity is the difference between link capacity and actual traffic volume. In this case, the spectral partitioning method are applied by considering the weight of links by their reserve capacity. The traffic volume survey data is used to determine the reserve capacity (National road traffic census survey, 2005). As the spectral partitioning method does not allow negative weights, we set the link weight to 1 in the cases where the link traffic volume exceeds its capacity.

Figure 4.6 shows the resulting cut set. This method partitioned a very small area. This may be because the cut set includes only links with traffic volumes exceeding their link capacity. There are only 22 partitioned nodes in the sub-network. It may be needed to consider how much the traffic exceeds the link capacity.

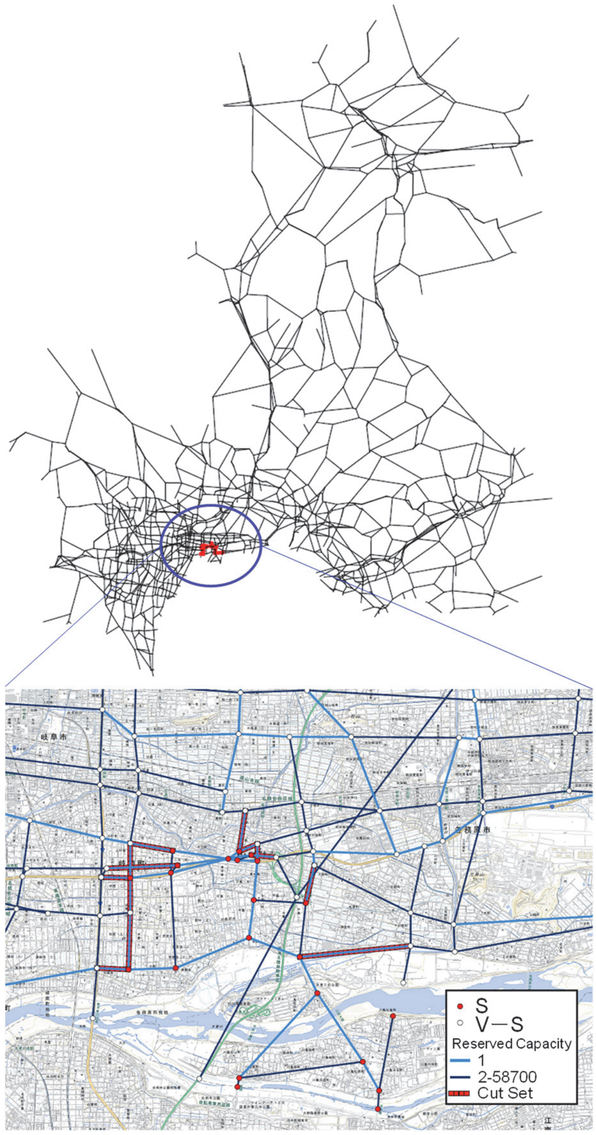


Figure 4.6 The cut set when links are weighted according to reserve capacity

Therefore, an exponential transformation of the reserve capacity is applied with sensitivity parameter k, as is shown in (4.5).

$$w_e = \exp(kr_e), \tag{4.5}$$

where r_e is the reserve capacity of link e and k is the sensitivity parameter.
If $k = 0$, the weight is 1 regardless of the reserve capacity. As k increases, the weighting varies more as the reserve capacity varies. The effect of using different values of k on the cut set is shown in Figure 4.7. As k increases, the cut set gradually moves to the west. The network is denser and more congested in the western part of Gifu Prefecture, including Gifu City and Ogaki City. This may be why the cut shifts to west. When we further increase k, the result became unstable, so we could not obtain a proper cut set. This may be because as k increases, as the weights of the over-capacitated links tend towards 0, the network are divided into several sub-networks. Further work is required to determine the use of reserve capacity

indicator to the spectral partitioning method.

Figure 4.7 Effects of different values of $\boldsymbol{k}$ on cut set

4.4.2. Link disruption probability

To determine the reliability of the network, it is useful to consider how easily links can become disconnected. We can do this by weighing links by link disruption probabilities. When the link disruption probability, p_e of a specific link is independent from other links, a probability that links will be disrupted simultaneously can be calculated by multiplying these probabilities. We then take a logarithm of these and write a function for the cut set with the maximum probability of disruption as

$$\max_{\partial(\mathbf{S})} \ln\left(\prod\nolimits_{e\in\partial(\mathbf{S})} p_e\right) = \max_{\partial(\mathbf{S})} \sum\nolimits_{e\in\partial(\mathbf{S})} \ln p_e = \min_{\partial(\mathbf{S})} \left(\sum\nolimits_{e\in\partial(\mathbf{S})} - \ln p_e\right). \tag{4.6}$$

Therefore, the cut set with the maximum probability of disruption can be extracted by setting w_e to $-\ln p_e$.

The Gifu Prefecture has many slopes with a high risk of landslides and rock falls when heavy rain occurs. Honjo et al.(2011) and Koita et al.(2010) estimated a probability of landslides or rock falls within Hida that is located in the northern area of Gifu. To evaluate the part of the network where there is a risk of disconnection due to landslides and rock falls, we set link weights on the link disruption probabilities estimated by Honjo et al. (2011). Note that we have the link disruption probabilities of 103 links only within the Hida area, the link disruption probabilities in other areas were set to 0.

Figure 4.8 shows the resulting cut set together with links with non-zero disruption probabilities. Even if the disruption probability is considered, the result that critical partition is composed of links with no probability. This result depends on the network topology rather than the disruption probability of links. The reason why the link disruption probabilities do not have large effects is considered to be the characteristics of the data. Links with non-zero disruption probability in Figure 4.8 are located only north part in Gifu prefecture, and more links have no probability. If the weight of each link in the network were set, the results may differ significantly. However, the cut set is different from both cases of capacity weighted (Figure 4.5) and unweighted (Figure 4.7, $\boldsymbol{k} = 0$). The result of this case study showed that the analysis with the probability values as weights is possible.

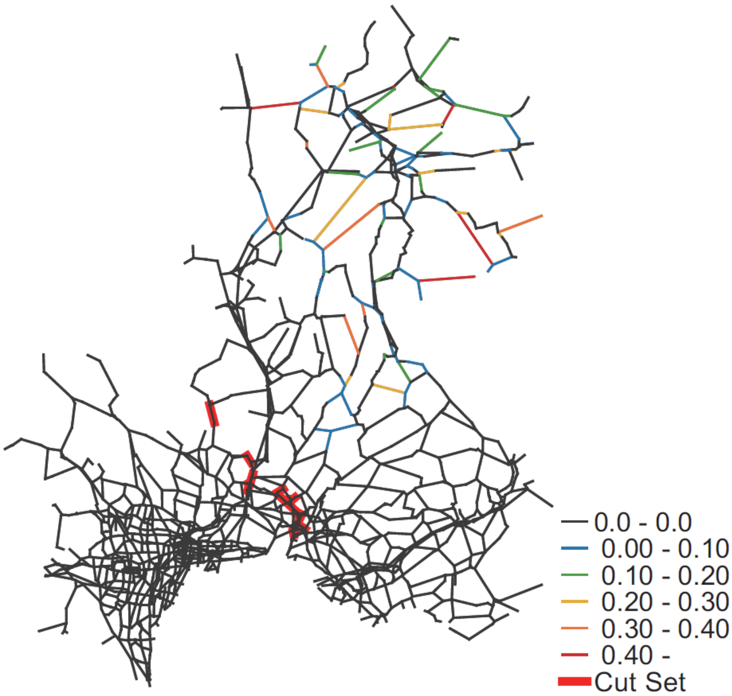


Figure 4.8 The probability and the cut set

These three examples (including the case of traffic capacity) show that different weight scenarios result in different cut sets. Therefore, when using the spectral partitioning method, it is important to select proper weights that reflect the objectives of the analysis. Nonetheless, the Gifu network tends to be cut around Mino area, which is the constricted area in the middle of the network. This finding suggests that connections to this area should be strengthened.

4.4.3. Spectral partitioning method by three weighted networks

Table 4.1 is extracted from Table 3.2 that was analysed by the spectral partitioning method in this chapter. Three kinds of weighted network with different weight types were applied. The evaluation by spectral partitioning using capacity weighted network showed consistency with the maximum flow problem. Hence, it is effective to identify the part that could easily become bottlenecks. The bottleneck on the road network was identified by finding a cut set with extremely small capacity. The knowledges from the application of Gifu prefecture road network are described.

- High potential bottleneck part is at the connection between the western area where urban cities are located and the eastern area where there are many mountain areas.
- Improvement of west-east connection in Gifu prefecture road network can be expected by the construction of Tokai Ring Road in the future.
- In the topology of Gifu prefecture, the cut set is located in the places where it is difficult to improve the connection with multiple large capacity roads.

The analysis using reserve capacity weighted network attempted to identify the cut set which is likely to become bottlenecks by finding the parts with no remaining capacity. From the result of sensitivity analysis, as the effect of reserve capacity became larger, the cut set moved to urban areas. Compared with the cut set by capacity weighted network without traffic volume data, the cut set by reserve capacity weighted is located in heavy traffic area such as Ogaki city and Gifu city. The high potential bottleneck parts move to the area where congestion is likely to occur by considering the traffic volume. Although, the cut set connects the west and east sides of the prefecture.

The analysis using link disruption probability weighted network attempted to identify the cut set which divides the network by link disruptions at the disaster. However, all links included in the cut set of Gifu prefecture do not have link disruption probability data. Hence, it was not possible to identify the cut set in which the network is divided by the links that easily disruption in practical road network. Since the spectral partitioning method using the probability values as weights was able to analyse, it is considered that the problem of this result is insufficient data of the link disruption probability. As future tasks, the analysis that considers the data acquisition situation and target area will be required.

Table 4.1 Evaluation objectives of spectral partitioning method based on weight settings

Classification of Challenges	Challenges	Weight type	Weight	Spectral Partitioning
The evaluation of road improvement Characterised the region on the road network	How is the supply function improved as a network based on the viewpoint of "movement" which is the basic supply function of roads ? Where are insufficient improvement areas ?	Supply	Capacity	Vulnerable parts that are easy to become bottlenecks
The usage situation of road network	How is the available traffic capacity located ? Are there locations where are easy to become bottlenecks because of no traffic capacity to spare.	Demand	Reserve capacity	Vulnerable parts that are easy to become bottlenecks because the available road network capacity is small.
The evaluation of disaster impact	How are links susceptible to damage at the disaster distributed ? Identify parts that have the potential to give a significant impact for the whole of network at the disaster.	Disaster	Link disruption probability	The links that have high risk of being degradated at the same time due to a disaster, and the disruption of their links divide the network.

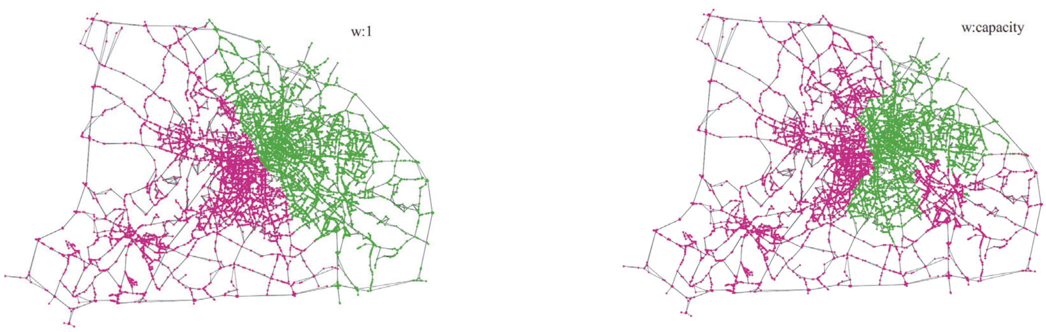
4.5. Test for Larger Road Networks

As is explained, the Spectral Partitioning method makes it possible to identify critical links without traffic assignment or route enumeration. This section therefore shows the usefulness of spectral partitioning analysis using large-scale road networks where any conventional vulnerability methods such that requires traffic assignment cannot be applied because of high computational load.

Table 4.2 shows a summary of large-scale road networks in 6 regions around the world (Bar-Gera. H, Transportation Networks) that we analysed. The largest network is Sydney network and the smallest network is Gold Coast network. Here, the Spectral Partitioning method is applied to two cases: unweighted and capacity weighted. Figure 4.9 to Figure 4.14 shows the partitioning results. The cut set produced by unweighted spectral analysis shows the critical part which is identified only from the viewpoint of the network topology. On the other hand, the cut set produced by capacity weighted spectral partitioning analysis extracts vulnerable parts that are critical in terms of capacity for each city. Some cities, such as Gold Coast and Chicago, have almost same partitioning results for weighted and unweighted cases, while other cities such as Berlin and Sydney, have significant changes depending on the weight setting. These are significant related to the construction status of large-capacity roads and geographical conditions such as whether the regions are inland or seaside. For example, if there is a large difference in the results between capacity weighted and unweighted, the cut set may be a part where small capacity links are significantly connected though that is not vulnerability in term of network topology. Detailed discussion requires consideration of the link capacity distribution and geographical features in each region however it is clear that such differences in characteristics affect the critical cut set results by the capacity weighted spectral analysis in large-scale road networks from the example calculations. These results demonstrate that the spectral partitioning method works well for large-scale road networks. Moreover, the calculation times (PC: Intel Xeon 3.50Ghz *2, 32GB, OS: Windows10, 64bit) are also shown in Table 4.2. It was possible to obtain the cut set for all cases and the calculation time is not very long even for the largest network, Sydney (201.85 min).

Table 4.2 Networks

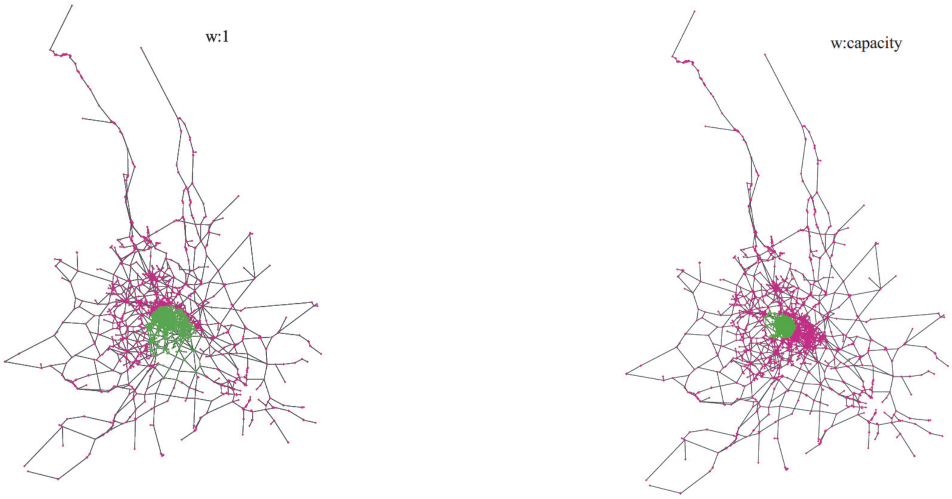
City	Country	Number of links	Number of nodes	Calculation time (min)
Berlin	Germany	28,449	12,981	12.25
Birmingham	UK	33,867	14,578	17.31
Philadelphia	USA	40,003	13,389	13.03
Gold Coast	Australia	11,140	4,779	0.6
Sydney	Australia	75,379	33,113	201.85
Chicago	USA	39,018	12,979	12.49



Unweighted

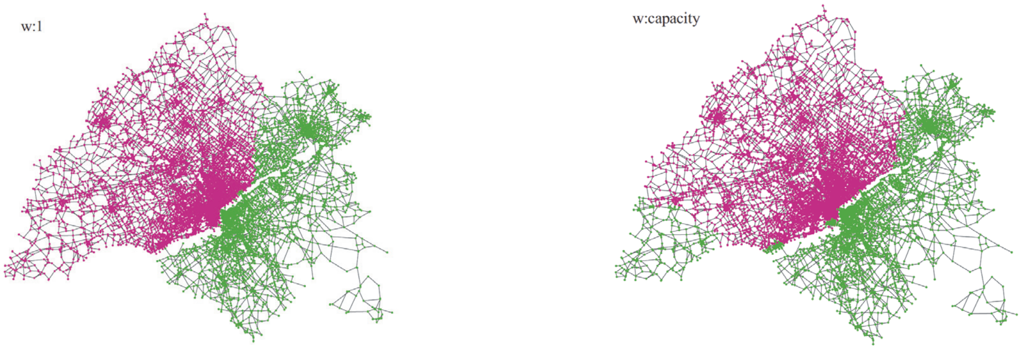
Capacity Weighted

Figure 4.9 The partition in Berlin



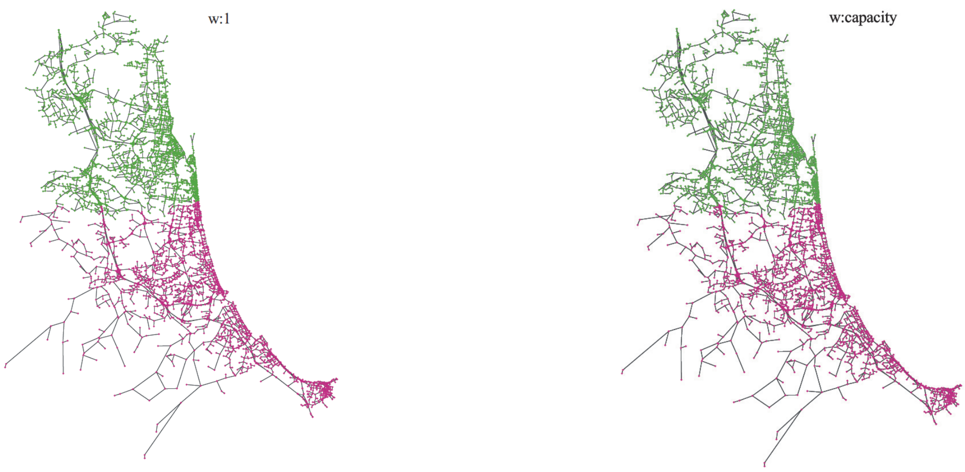
Unweighted Capacity Weighted

Figure 4.10 The partition in Birmingham



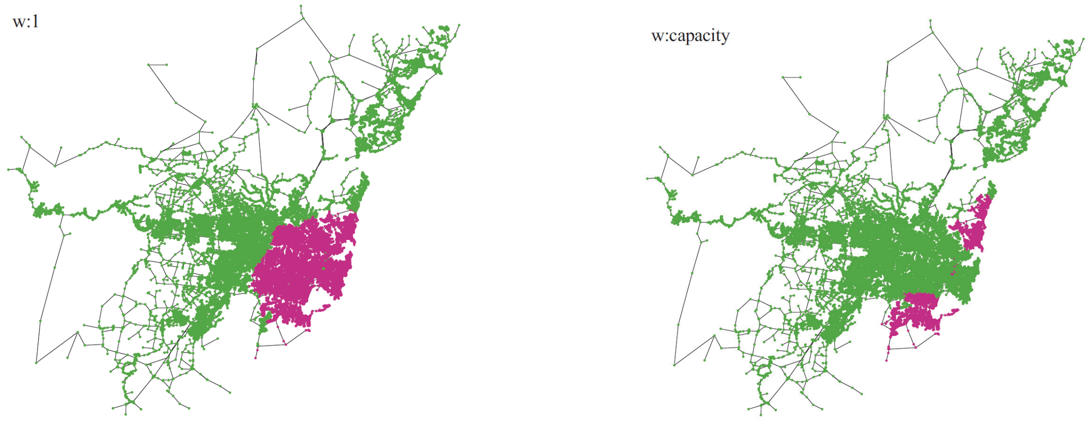
Unweighted Capacity Weighted

Figure 4.11 The partition in Philadelphia



Unweighted Capacity Weighted

Figure 4.12 The partition on Gold Coast



Unweighted Capacity Weighted

Figure 4.13 The partition on Sydney

Unweighted Capacity Weighted

Figure 4.14 The partition on Chicago

4.6. Concluding Remarks

This chapter used the spectral partitioning method with the traffic capacity weighted to identify the parts of the network that are significantly weak, i.e. vulnerable parts of road network. The vulnerable parts of a capacity weighted network are likely to become bottlenecks where the capacity is significantly small. The knowledges obtained from the analysis results are summarised below.

First, the comparison with the conventional road network evaluation method and capacity weighted spectral partitioning method clarified the advantage of spectral partitioning method for the road network evaluation. The capacitated links obtained from the maximum flow problem and the links within the cut set produced by the Spectral Partitioning method are almost same. This comparison result showed that the Spectral Partitioning method which does not require OD pattern traffic volume can identify the potential bottleneck easily.

Next, to test the usefulness of the spectral partitioning method for a practical road network, the

Gifu Prefecture road network is analysed. The proposed method identified the vulnerable part of the network and the result are reasonable considering the characteristics of the Gifu Prefecture. Also, other weight cases were examined. The examples of weight settings are link disruption probability and link reserve capacity. By the analysis of link disruption probability, it was confirmed that probability values can be used for the weights. Since different partition results were obtained depending on the weights, further discussion is needed how to apply various link features as weights.

Finally, the proposed method is applied to a large-scale network, which is the main advantage of this method. The results of applying the proposed method to the practical large-scale road networks in 6 cities revealed that the Spectral Partitioning method works well even for such large network. The characteristics of vulnerable parts may vary from city to city, and different result may be obtained by changing the different weight settings. The result of partitioning may also be affected significantly by geographical conditions and social factors such as land use and population. Relationship between network features and other indicators should be investigated.

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Chapter 5

Network Connectivity Analytics by Topological Indicators

5.1. Introduction

Chapter 5 verifies road network connectivity analytics by topological indicators. As I mentioned in Chapter 2, methods to evaluate connectivity of road network have been well studied. In case of traffic accidents and disasters, disruption of some links may affect extensively to road network. In order to evaluate the affected magnitude, it is considered that the parts where the connectivity is weak in term of network topology is easily affected by the disruption at the disaster. On the other hand, parts with strong connectivity of network are not easily affected by such failures. Therefore, it is very important to understand areas with weakly and strongly connected. In Chapter 4, the spectral partitioning method with capacity weighted was introduced to identify the critical bottlenecks as network vulnerability analytics by topology indicators. This chapter attempts to identify weak areas where its connectivity is easily affected by bottlenecks and strong areas where its connectivity is not affected as network connectivity analytics by topology indicators.

Recently detailed road network can be used freely, and such detailed network data may provide different insights on network connectivity analysis. Computationally tractable methods are thus required to analyse of detailed networks including the small road. This research uses the eigenvector centrality method shown in 3.4 as indicator to analyse the network connectivity. The eigenvector centrality (EC) is one of the evaluation methods based on network topology with a small computational load. Another approach EC is that it shows the strength of the connection of a node to its neighbours, taking into account the strength of the connection.

The objective of this chapter is to validate that the usefulness of the EC evaluation method. Moreover, it is important to understand the characteristics of the EC evaluation on the road networks. The computational tractability, which is one the great advantages of the proposed method, is also tested.

5.2. Comparison with Other Methods

5.2.1. Eigenvector centrality with other centrality measures

To understand similarities and differences among them in the application of the road network, centrality measures other than EC are verified here. Degree Centrality (DC) defined by Proctor and Loomis (1951), Closeness Centrality (CC) defined by Beauchamp (1965), Betweenness Centrality (BC) defined by Freeman (1977) and Eigenvector Centrality (EC) are compared. Weights of CC and BC are set as link length, and as for EC both unweighted and capacity-weighted cases are compared.

A small-scale road network shown in Figure 5.1 is used to compare some centrality measures. This Gifu City network is a part of Gifu Prefecture network that was used to test the usefulness of Spectral Partitioning method in the previous chapter. The network only contains major roads including national highways and prefectural roads, and there are no expressways in this area. The number of nodes and directed links are 177 and 554, respectively. At first, let us explain the geographical situation of Gifu City. The thickness of links in Figure 5.1 represents link capacity. The urban area in this city is located around Gifu Station indicated by a yellow star in Figure 5.1. Also, there are bypass roads of national highway R21 with larger capacity in the southern area (shown in red circle). Conversely, density of roads becomes sparse to the north. There are mountains in the northern part and road network is limited around there. It is therefore expected that the network in the middle is strongly connected whereas the connectivity is low in the northern area.

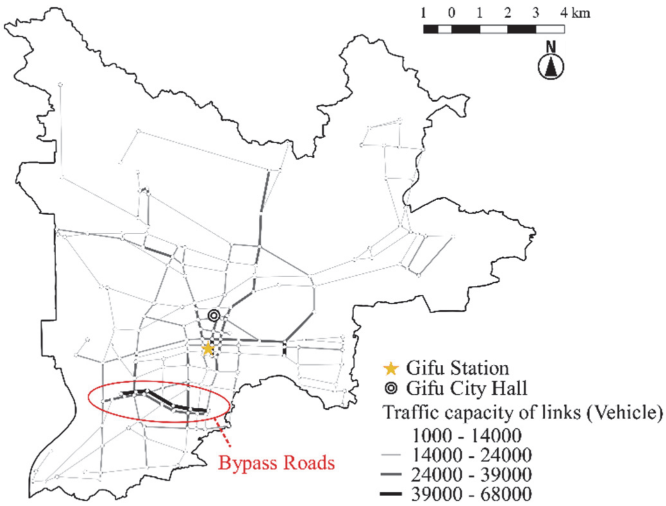
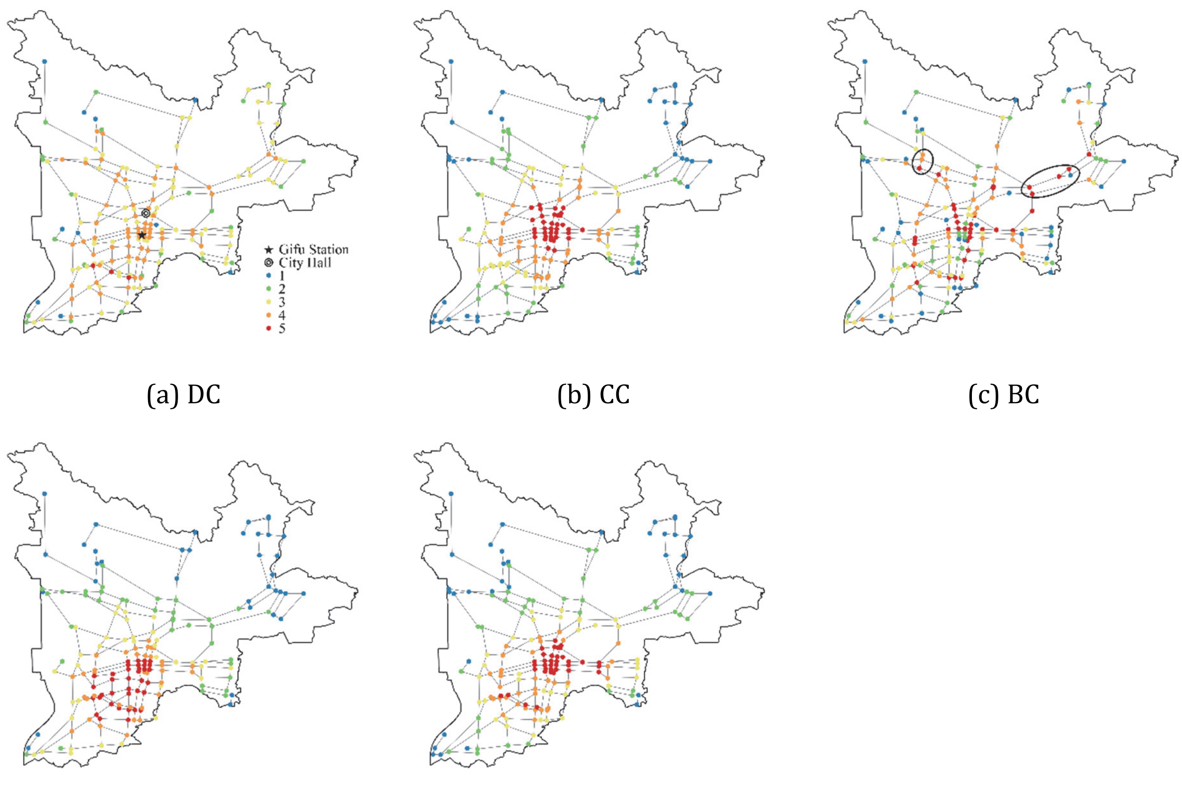


Figure 5.1 Gifu City road network

Figure 5.2 shows the results of different centrality calculations. Nodes in each centrality measure are classified into five levels with equal number of nodes, except for the DC which has only five values from 1 to 5. Table 5.1 summarises the statistical correlation among the five measures. Although the CC and capacity-weight EC use different weights, they have similar tendencies. Both measures evaluate the nodes in the city centre as high, and the nodes in the northern mountain area as low. Spearman's rank correlation among them is also very high. This means that the EC without using the index of "shortest path length" can get the similar tendency to CC that requires the calculation of the shortest distance between all nodes. Also, unweighted EC has similar tendencies and high rank correlation to CC. As for the DC, nodes connected with only one or two degrees are located at the edge of network. Conversely, there are a few nodes connected to five links in the middle of network. Because of the feature of road network that the maximum degree may be around five or six at most, the degree centrality cannot be used to highlight the connectivity of the road network.

BC counts the number of node pairs whose shortest path traverses the node. Parts with high centrality other than central urban areas seem to be located where the network can be easily divided. This means that the important links determined by the BC are evaluated as the links whose centrality decrease in other centrality measures, and by the disruption of such links can be 'critical' based on the definition of Taylor et al. (2006). In particular, links in the black circle in Figure 5.2 (c) may divide the network is they are disconnected. These nodes have high BC values whereas the values of CC and EC are low. From this observation, the CC and EC can identify the vulnerable nodes whose connectivity may reduce by the disruption of any critical links.

EC has an advantage of applicability to large-scale networks than the CC and BC that require shortest path search among all node pairs. As conclusion compared with other centrality measures on the road network in Gifu City, EC can show similar tendencies to CC. Moreover, the different tendencies against BC suggest that the evaluation results by EC can identify vulnerable nodes, as shown in the previous paragraph. These results lead to the conclusion that the contribution of road network evaluation using EC is large.



(a) DC (b) CC (c) BC

(d) Unweighted EC (e) Capacity-weighted EC

Figure 5.2 Centrality measures on road network in Gifu City

Table 5.1 Correlation coefficient

Spearman's rank correlation	Degree Centrality	Closeness Centrality	Betweenness Centrality	Eigenvector Centrality (Unweighted)	Eigenvector Centrality (Capacity weighted)
Degree Centrality	1				
Closeness Centrality	0.5004	1			
Betweenness Centrality	0.5554	0.5132	1		
Eigenvector Centrality (Unweighted)	0.5717	0.7679	0.3546	1	
Eigenvector Centrality (Capacity weighted)	0.4974	0.9263	0.4646	0.8405	1

5.2.2. Comparison of eigenvector centrality and the number of non-overlapping routes

To test the suitability of the eigenvector centrality for road network connectivity evaluation, the method is compared with the conventional evaluation results by using the same road network. As a conventional method, the number of non-overlapping routes proposed by Kurauchi et al. (2009), which is one of the connectivity vulnerability evaluation methods, is adopted. The number of non-overlapping routes counts independent routes that do not share a link between origin and destination. If a node pair has Nth non-overlapping routes, connectivity between a node pair is secured even if the $N-1$ links are disrupted as the worst case. To compare this method with EC, the average number of non-overlapping routes from node n_i to all nodes is used as the connectivity vulnerability evaluation of node n_i. The non-overlapping route needs to count the number of distinct routes between any node pairs, and if the EC can give a similar result with the non-overlapping route, connectivity evaluation by EC will be greatly supported.

This problem is formulated as an optimisation problem to maximise the number of non-overlapping routes as follows;

$$\max_{N_{ij}} N_{ij} \tag{5.1}$$

subject to

$$\sum\nolimits_{a\in out(i)} x_a = N_{ij}, \sum\nolimits_{a\in in(i)} x_a = 0 \tag{5.2}$$

$$\sum\nolimits_{a\in out(j)} x_a = 0, \sum\nolimits_{a\in in(j)} x_a = N_{ij} \tag{5.3}$$

$$\sum\nolimits_{a\in out(k)} x_a = 0 - \sum\nolimits_{a\in in(k)} x_a = 0 \quad \forall k \in \{k \in \mathbf{V}, k \neq i, j\} \tag{5.4}$$

$$x_a = \{0,1\}, k \in \mathbf{V}, k \neq i, j, \tag{5.5}$$

where,

N_{ij} : The number of non-overlapping routes between node i and node j

$\mathbf{V}$	:	A set of nodes
x_a	:	A control variable 0-1 (1: in non-overlapping route 1, if not 0)
k	:	A node that components the non-overlapping routes
$in(k)$	:	A set of links flowing into node k
$out(k)$	:	A set of links flowing out from node k
N	:	The number of nodes.

By solving this optimisation problem, the number of routes that do not share links between all node pairs is obtained. For example, in the network shown in Figure 5.3, the number of non-overlapping routes between the pairs of nodes A and B is 2. The average between all node pairs is adopted to evaluate the whole of network. The average number of non-overlapping routes, R_i, between a node and all other nodes evaluates connectivity in the network. Nodes that have many destination nodes without overlap have higher connectivity. R_i is obtained for each node as follows,

$$R_i = \frac{\sum_{j \neq i \in \mathbf{V}} N_{ij}}{N-1}. \tag{5.6}$$

Figure 5.4 shows evaluations by the average of non-overlapping routes and unweighted EC. Nodes are divided into four levels with equal number of nodes for each level. In the middle area of the network where many links are connected to one another, both evaluations are high. The connectivity evaluations by both methods are high in the middle area of network where Gifu Station and Gifu City Hall are located, as shown in Figure 5.1. This area including Bypass Road has many links are highly connected to one another. In the EC evaluation, the top-level nodes are gathered in these areas and the connectivity gradually decreases toward the outside area. Conversely, in the average number of non-overlapping routes, upper than 50% level nodes are rather mixed, and the size of area where they are located is larger than EC. Also, nodes in the northern area in Gifu City and at the edge of network are evaluated as low connectivity in both evaluations. Next paragraph analyses the relationship between these evaluation results.

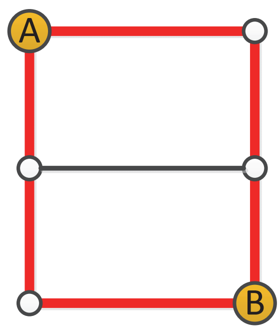
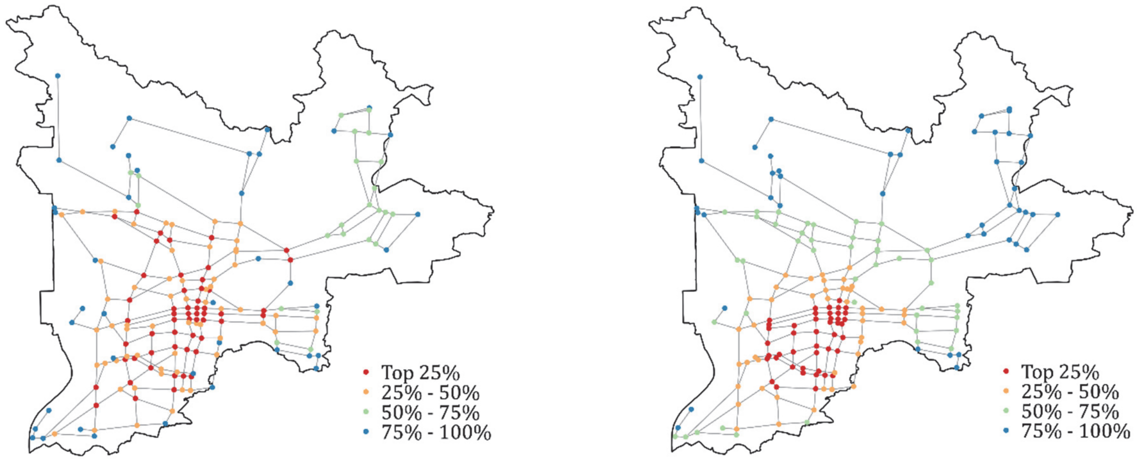


Figure 5.3 Non-overlapping routes between node A and B



(a) The average of non-overlapping routes (b) EC

Figure 5.4 Comparison of evaluation results

Figure 5.5 shows the scatter plot of EC and the average number of non-overlapping routes. There are nodes with low connectivity on EC evaluation and high connectivity on the average number of non-overlapping routes. However, there are no nodes with high connectivity on EC evaluation and low connectivity on the average number of overlapping routes. At least there seems to be a relationship between both evaluations.

Table 5.2 shows the result of no correlated test about Speaman's rank correlation. Since it is difficult to compare by values, the rank of value is compared. The p-value is less than 1%. There is a statistically significant correlation between EC and the average number of non-overlapping routes evaluations. Also, the rank correlation coefficient is 0.741. Thus, it is concluded that EC is useful for evaluating connectivity of road networks.

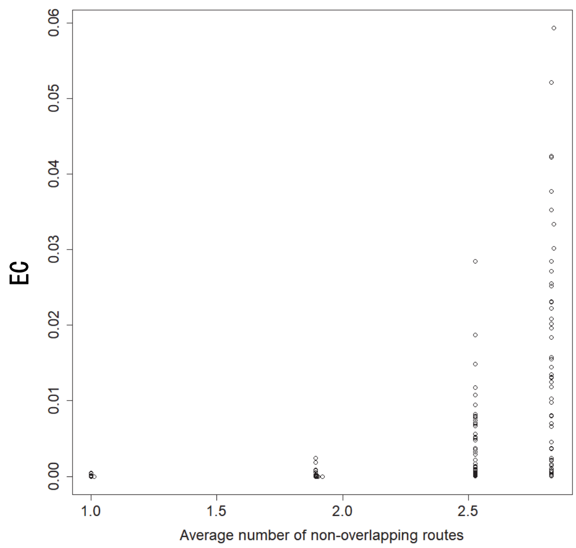


Figure 5.5 The scatter plot of EC and average number of non-overlapping routes

Table 5.2 The result of no correlated test (rank correlation)

t-value	14.582
df	175
p-value	2.2e-16
Correlation	0.741

5.3. Application to Practical Road Network

In the previous section, it is revealed that EC can evaluate connectivity of road networks. This section attempts to calculate the values of EC to the practical road networks. In applying to practical road networks, capacity weighted EC is used to consider road serviceability in addition to connectivity evaluation by unweighted EC. Compared with the unweighted EC, which evaluates connectivity based on only the network topology, capacity weighted EC evaluates connectivity considering the supply ability of roads.

5.3.1. Capacity weighted eigenvector centrality

In EC, the evaluation result depends largely on the weight for each link. The weights can be set as any non-negative values. The links on road networks have many features, such as distance, travel time, traffic volume and so on. These indices can be selected as a weight depending on what you want to evaluate. This research seeks for strongly and weakly connected part as a service provided by the road network. Therefore, this research proposes to use link capacity as a weight. Nodes connected with motorways or national highways with high capacity can be highly evaluated by the proposed method.

It was revealed that unweighted EC can evaluate the impact of link disruption based on the relationship with the average number of non-overlapping routes. In addition, I believe that capacity weighted EC can consider the ease of link disruption, because links that have large capacity are generally difficult to disrupt. EC can indicate the diffusion effect of such easiness of disconnections.

5.3.2. Gifu prefecture road network

To test the usefulness of capacity-weighted EC measure, the Gifu Prefecture road network shown in Figure 5.7 is analysed. The Gifu Prefecture road network, which includes intercity expressways, national highways and prefectural roads contains 1,460 nodes and 4,578 directed links. The elements of the adjacency matrix are directional capacities as mentioned earlier. In this network, the minimum and maximum link capacities given by National Road Traffic Census (2005) are 1,000 and 80,000 vehicles per day, respectively. Figure 5.6 shows the distribution of traffic capacities. Most of links have traffic capacity less than 20,000 vehicles per day. The reason for this is that Gifu Prefecture has large mountain area with narrow roads. Also, there are many narrow roads even in the city area, like collecting/distributing roads.

A few links with large capacities are located on expressways or national highways. For the calculation, this paper uses MATLAB ver. R2017b using eigs function. The eigenvector corresponding to the largest eigenvalue by MATLAB is normalised by the L2-norm, that is defined as $\|x\|_2 = \sqrt{\sum_i x_i^2}$. The sum of squared EC value is thus equal to 1 on every network. The example of practical city road networks shown in 4.6 also used the squared eigenvector values.

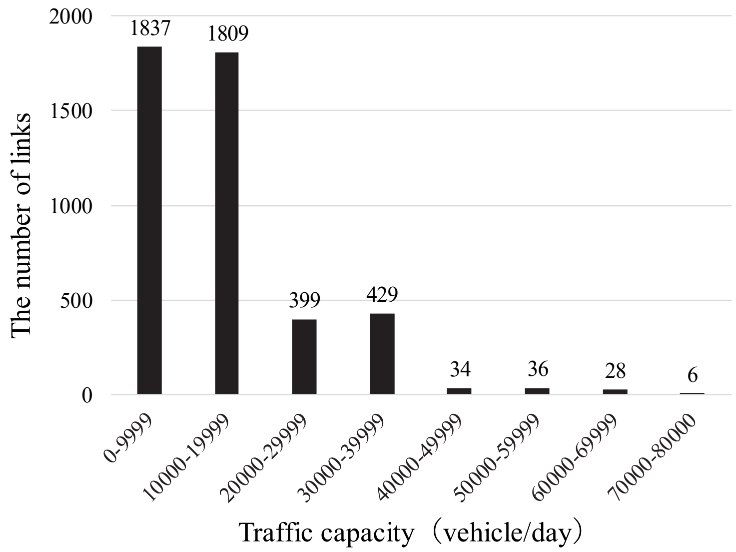


Figure 5.6 The distribution of traffic capacity

5.3.3. Strongly and weakly connected parts

Figure 5.7 shows the distribution of the EC. EC is an evaluation by relative values in the network, and there is no definition of how much to consider strongly connected nodes and how much to consider weakly connected nodes. Hence, it is necessary to set an arbitrary threshold. The characteristic of EC distribution depends on the network, for example, there are a few nodes with extremely large EC. Figure 5.8 shows the logarithm EC values (in descending order) in Gifu Prefecture road network. This study decides the threshold of EC level for five intervals so that the range of log-scaled EC values divided equally. Each level on the distribution and the percentage of included nodes are shown in Figure 5.8. Based on this threshold rule, nodes included in Level 1 have extremely strong connectivity and nodes included in Level 2 have strong connectivity. Conversely, nodes included in Level 4 have weak connectivity and nodes included in Level 5 have extremely weak connectivity. Figure 5.7 shows the geographical distribution of nodes classified into each level in Gifu Prefecture road network.

First, the strongly connected part of the network is discussed. The number of nodes included in the level with EC larger than the one-fifth of the value range (Level 1) is 220, which is 15% of total nodes (shown in red). From Figure 5.8, most of these nodes are located in the centre (urbanised area) of the Gifu Prefecture around Gifu City. Moreover, nodes in Levels 1 and 2 spread around the urbanised area or along expressways shown by bold black lines in Figure 5.7. From the definition of capacity-weighted EC, it is understandable that higher values can be obtained along roads with higher capacities, but it is interesting to say that such effect does not spread out if the adjacent links do not have enough capacity. For example, the node representing Hida-Kiyomi Interchange (IC) on Tokai Hokuriku Expressway has Level 2 EC value and its effect has spread to the east via Chubu-Jukan Expressway, but this effect may not spread beyond Takayama IC. Moreover, the node representing Shirakawago IC is ranked as Level 3 although it is along the Expressway and some of nodes around it are often ranked as Level 4 (less connected). Most of sections on Tokai Hokuriku Expressway have two lanes for each direction, whereas there is only 1 lane for each direction in Tokai Hokuriku Expressway between Hida-Kiyomi IC and Shirakawago IC as well as Chubu-Jukan Expressway from Hida-Kiyomi IC to Takayama IC. This is a reason why the effect of high capacity on Tokai Hokuriku Expressway may spread up to Hida-Kiyomi IC, but its effect does not spread beyond them. Capacity-weighted EC thus can evaluate the nodes connected with higher capacity links, but its effect may not spread over when nodes are connected with lower capacity links.

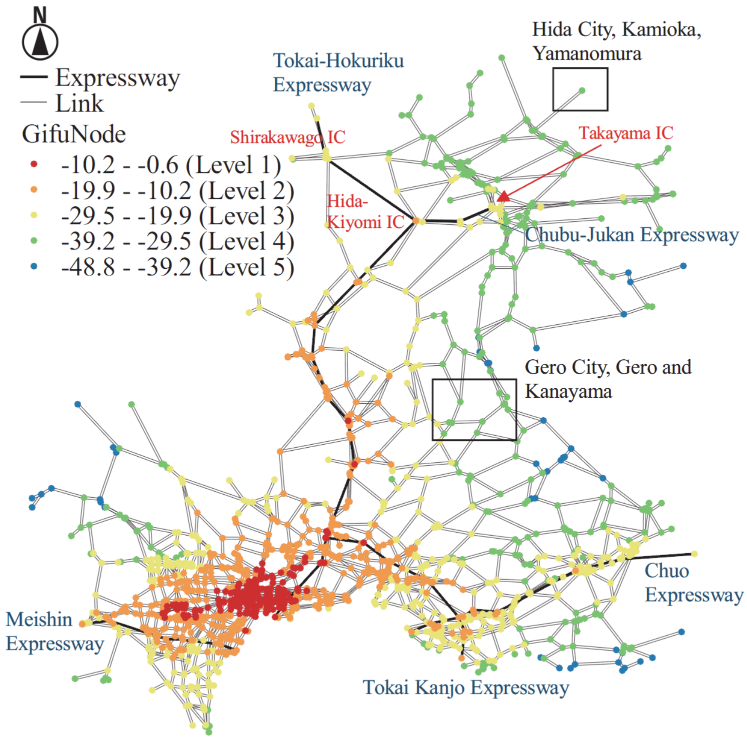


Figure 5.7 The capacity-weighted EC in Gifu Prefecture road network

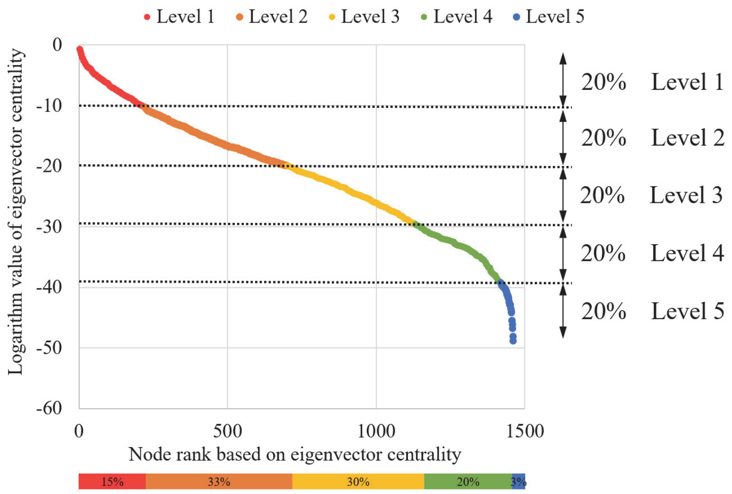


Figure 5.8 The rank of eigenvector centrality (Logarithm)

Next, the weakly connected part is discussed. The number of nodes in the lowest connectivity level (Level 5, shown in blue) are 43 (3% of the total nodes) and are mainly located at the edge of the network. Note that the connectivity of edge nodes should be low since a network is cut off from the surrounding network. We will check the effect of the boundary in the next section. Figure 5.7 also shows that the values in the north-eastern part are small (Level 4) even though the corresponding nodes are not near the boundary. The degree of the nodes in this area is enough, but the influence of important roads with large capacity does not spread out, and the connectivity of these nodes may be limited by the narrow roads. As a concrete example, in western Japan torrential rain occurred in July 2018, the residents in Hida and Gero City shown by the rectangular shapes in Figure 5.7 were isolated due to road disruptions (Gifu Pref., 2018). Several road disruptions may result in isolation in areas when the connectivity is poor. Gifu Prefectural Government specially identifies such districts where such isolation is expected and has been investing roads to improve the connectivity (Gifu Pref., 2019). Thus, capacity-weighted EC analysis can identify the threatened areas.

5.3.4. Boundary effect

To check the effect of the boundary, an extended network is analysed so that the boundary nodes become inner nodes. Figure 5.9 shows the extended network containing the original Gifu Prefecture network. The colour of nodes in Figure 5.9 shows the rank of EC values by using this extended network. Only nodes within original Gifu Prefecture network are analysed to compare with the result obtained from the original network. The value of ECs is normalised only by nodes in Gifu, and Figure 5.11 shows the result. As shown in Figure 5.10, the connectivity around Hida and Gero areas remains low, like as the result of the original network. The distribution of strongly and weakly connected areas on larger network is very similar to the original one. This may be because high mountains lie between Gifu and Nagano Prefectures, the roads connecting between them are limited and their capacity is low. Hence, the proposed method can identify such weakly connected nodes even they are not located around the network boundary.

Further, the rank of EC values for both networks are compared. The difference between the rank of EC values for the extended network (Figure 5.10) and its original network (Figure 5.7) shows the impact of the boundary. Only overlapping nodes in both networks are extracted, normalised and ranked. The number of nodes to compare is 1,460. Figure 5.11 shows the relationship of the rank of EC values between both networks in descending order. If the rank of node in the extended network is similar to the original one, the corresponding plot should lie roughly along the 45-degree line. However, several nodes are far different from the line. Nodes with particularly significant rank differences are picked up, they are named as 'specific nodes' and are shown by stars in Figure 5.10. The colour of the star represents the level of connectivity. As Figure 5.11 shows, all specific nodes have relatively low ranks in the original Gifu Prefecture network but are ranked high in the extended network. All specific nodes are located at the western edge of the original network. This is because there is 4-lane expressway to the west of them, Meishin Expressway. This means that large rank differences may occur if there are high capacity roads in the outside areas that are close from the boundary of the network. If there are, such links should be included in the evaluation.

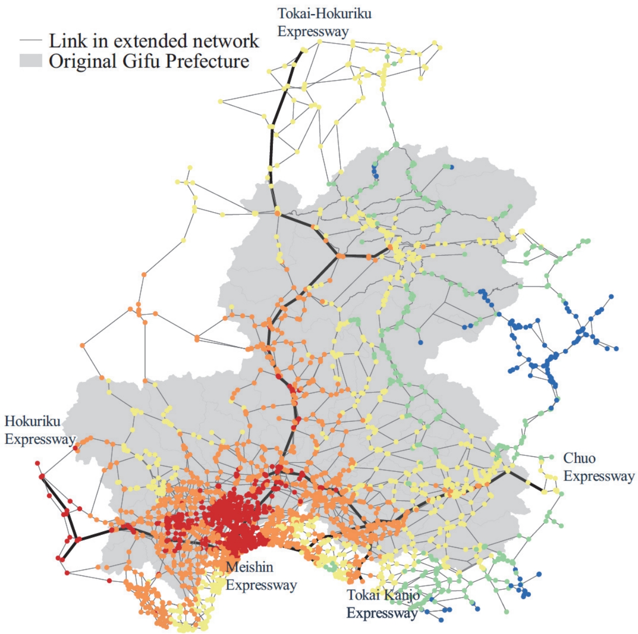


Figure 5.9 Extended network

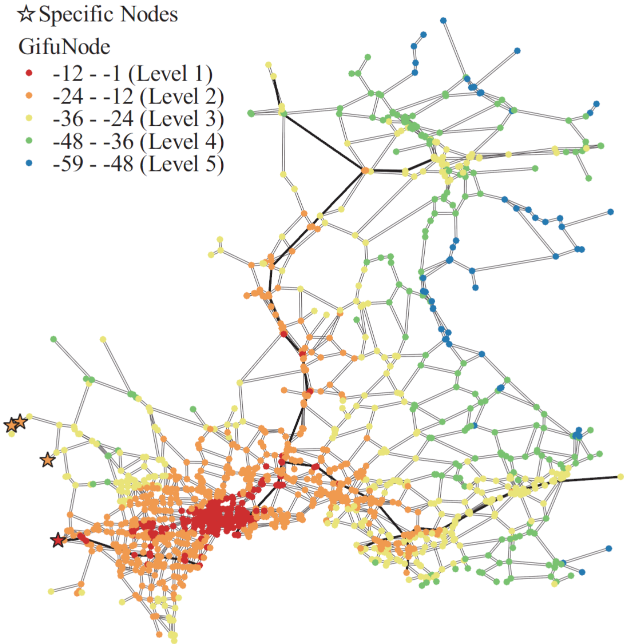


Figure 5.10 The normalised eigenvector centrality on the extended network

By the application to Gifu Prefecture road network, it was confirmed that the proposed method can identify the weakly connected part of network and the result coincides with the recently isolated areas. It was also confirmed the effect of boundary and the boundary should be selected so as to avoid the biased estimation of outer links. Besides, the calculation only requires 0.239 seconds (PC: Intel Xeon, 32GB, OS: Windows10, 64bit) and the computational advantage is quite large. To confirm this advantage, the proposed model will be applied to larger networks in a later section.

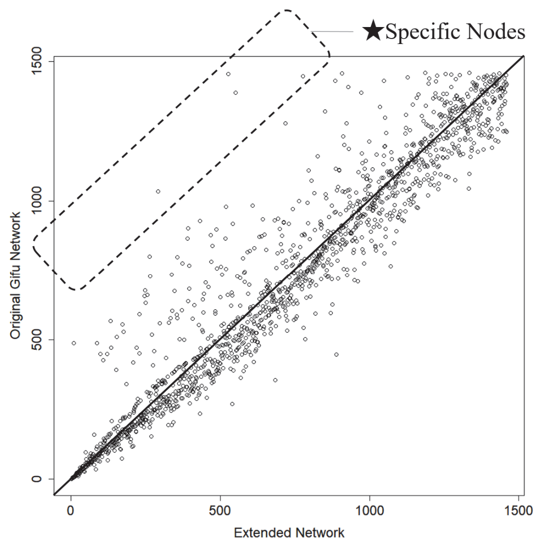


Figure 5.11 The rank of eigenvector centrality

5.4. Verification of Weight Settings

In the previous section, it was confirmed that capacity weighted EC can identify the strongly and weakly connected area of a practical road network. As mentioned above, the weights can be set any nonnegative values. This section attempts to verify the impact of weight settings by applying several indices as weights.

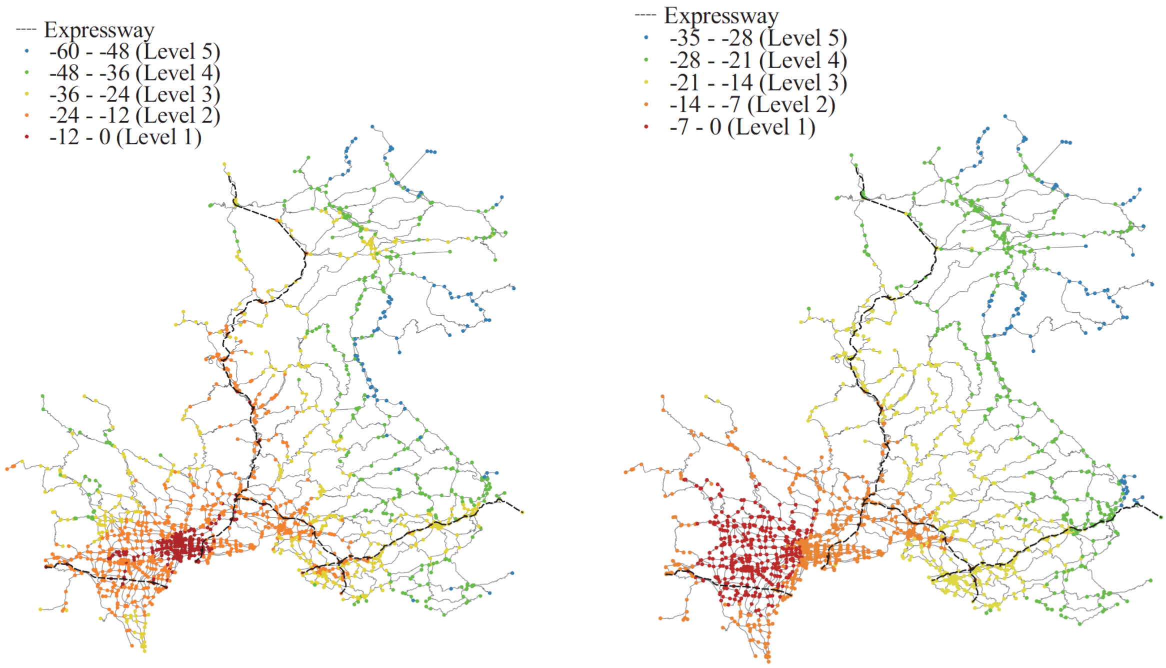
5.4.1. Comparison of unweighted and capacity weighted eigenvector centrality

At first, to test the impact of weight setting, unweighted and capacity weighted EC are compared. This road network does not correspond completely with the network applied in the previous section. To avoid the effect of network boundary, it is calculated by using the extended network, and the results in Gifu Prefecture are picked up and normalised again. In Gifu Prefecture road network, the number of nodes and links are 1,783 and 4,780, respectively. The traffic capacity data is based on National Road Traffic Census (2005) as with analysis in 4.4.

Figure 5.12 shows the distribution of capacity weighted and unweighted log-scaled EC. The range of the EC values after taking logarithm transformation is categorised into five intervals divided equally by the range of the values. This is a same rule as 4.4. The pure topology of the road network can be evaluated by the unweighted case. From Figure 4.11 (b), Level 1 nodes are located in the western part of Gifu Prefecture. From there, the connectivity is gradually becoming weaker towards the east. On the other hand, the result of the capacity weighted network (Figure 4.11(a)) shows that Level 1 nodes are located in further eastern side than the unweighted case, and the nodes with better connectivity spread along the expressways that has larger link capacities. This difference is the impact of considering road capacity on eigenvector centrality.

Figure 5.13 shows the number of nodes in each level. This is similar in both networks. The level with the greatest number of nodes is Level 2, and one with the lowest number of nodes is Level 5. However, Figure 5.14 shows the existence of nodes with different levels in both networks. There are several nodes with a high level in unweighted network and low level in capacity weighted network, and vice versa. Where are the nodes with the large level difference located? Figure 5.15 shows the nodes with large level difference between two networks. At first, nodes with lower levels in unweighted network than in capacity weighted network are obviously located along the expressway. This means that the capacity weighted eigenvector centrality clearly shows the influence of the large capacity road like expressways. Moreover, nodes which have large level difference are not only lying on the expressways but also spread to adjacent nodes. From this result, the capacity weighted network can take into account the ripple effect of roads with larger capacities (expressways). On the other hand, nodes with a higher level in unweighted network than the level in capacity weighted network, especially nodes with a difference of 2 levels or more are located mainly in the western mountainous area. Gifu Prefecture, however, also have large mountain areas in the northern part. Therefore, the reasons why these areas are picked up is not only because of this. It can be said that although these areas have high connectivity from the topological point of view, they are insufficient from the viewpoint of capacity. If the road network improvement is considered, the capacity expansion may be more effective than adding a new link.

By the comparison result of two weight settings, it was found that the level of difference between capacity weighted and unweighted network mainly occurs along the expressway in the case of Gifu Prefecture. Since this result makes sense for the real road situation, the capacity weighted EC can evaluate effect of connectivity considering traffic capacity. Also, it was confirmed that weight setting has a large impact on EC evaluation by verification using the practical road network.



(a) Capacity weighted (b) Unweighted

Figure 5.12 Capacity weighted and Unweighted EC on log-scale

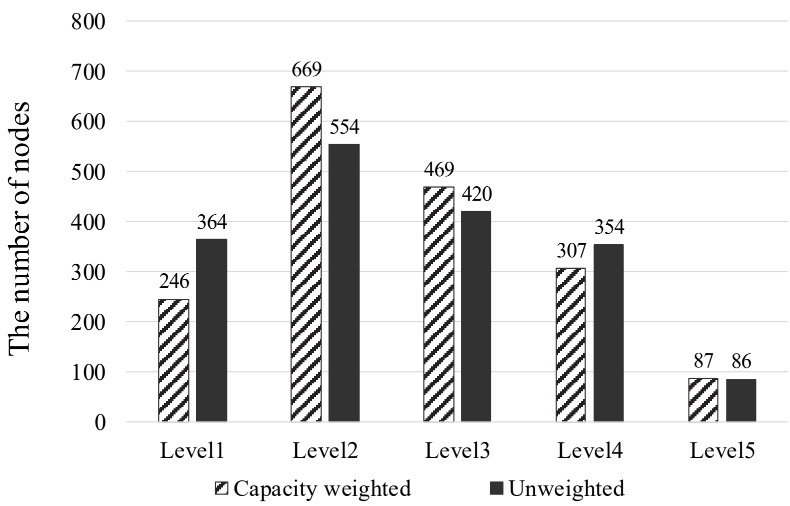


Figure 5.13 The number of nodes in each level

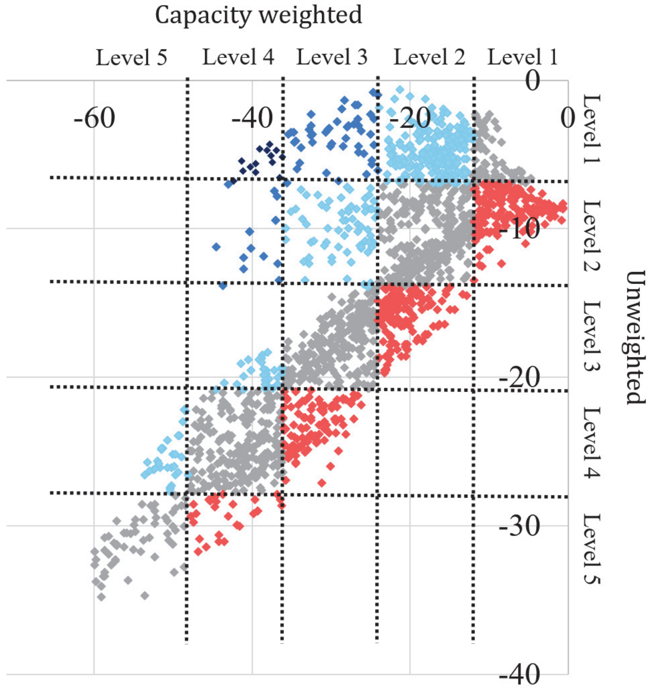


Figure 5.14 Logarithm values of EC on both networks

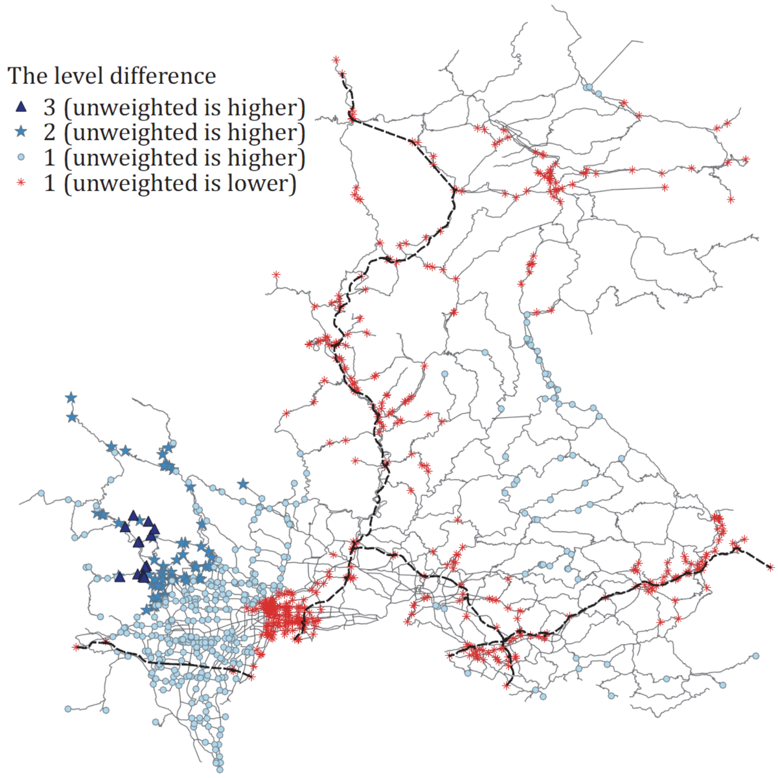


Figure 5.15 The difference of level between both networks

5.4.2. Traffic indicators

A large impact of weight settings to EC evaluation was confirmed by comparing unweighted and capacity weighted network. As a next step, seven traffic indices are used as weights to understand the characteristics of the road networks from the difference in evaluations by weight settings.

Seven traffic indices obtained by traffic survey data are used. They are traffic volume (the number of vehicles passing a road section), traffic capacity, congestion rate (traffic volume/traffic capacity), speed (km/h), distance (km), travel time (min) and travel time calculated by the Bureau of Public Roads (BPR) function. The congestion rate and the BPR function are represented by (5.7) and (5.8). The coefficients for BPR function are set as α=0.15 and β=4.

$$BPR_e = t_{0i}\left\{1 + \alpha P_e^{\beta}\right\} \tag{5.7}$$

$$P_e = \frac{V_e}{C_e} \tag{5.8}$$

where,

P_e	:	Congestion rate on link e
V_e	:	Traffic volume on link e
C_e	:	Traffic capacity on link e
t_{0e}	:	Free flow travel time on link e (min)
α, β	:	Coefficients.

These data were obtained from the Road Traffic Census in Japan, which is a nationwide statistical survey collecting basic data on road planning, construction, management and so on. Data from the 2005 Census were used in this study. Traffic volume is the total number of vehicles passing an observed cross-section within a 24-hour period. ‘Speed’ is set as a speed limit on each link. The weights as traffic volume can represent the area where the traffic demand gathers, and the weights as capacity represent how roads are well connected. The weights as a congestion rate take into account not only demand but also congestion. This indicates whether crowded road is concentrated. The weights as speed represent how roads with a higher speed limit are connected. The speed limit has some relationship with the rank of roads. The weights as distance represent road density since when roads with long distances are connected, the network must be sparse. The travel time is the required time for each road. Basically, the longer the distance, the longer travel time takes. However, some of roads with short distance but long time (such as narrow roads in mountain area), and some with long-distance but short time (such as expressways) exist. This has a relationship with distance and speed limit. The BPR function is the travel time considering the congestion as shown in the above equations. The BPR function is used since this value represents the congestion level well by the relationship of travel speed, capacity and traffic volume.

5.4.3. Road network

EC with seven weights are applied to the Gifu Prefecture road network shown in Figure 5.16. This road network is same as the network indicated in 5.5.1. Traffic indices described in the previous section are set as link weights. Table 5.3 summarises the features on the network. The most unique characteristic of Gifu Prefecture is the combination of both urban and mountainous areas. The five cities shown in Figure 5.16 are the highly populated cities in the region. Because these five cities are located in the south, and the southern part of the prefecture is urbanised with a larger population and economy with richer transport services. Although the northern part of the prefecture is rather mountainous with a sparse population, it still needs to maintain urban functions and the connectivity of the road network should also be guaranteed. The dotted lines in Figure 5.16 represent expressways.

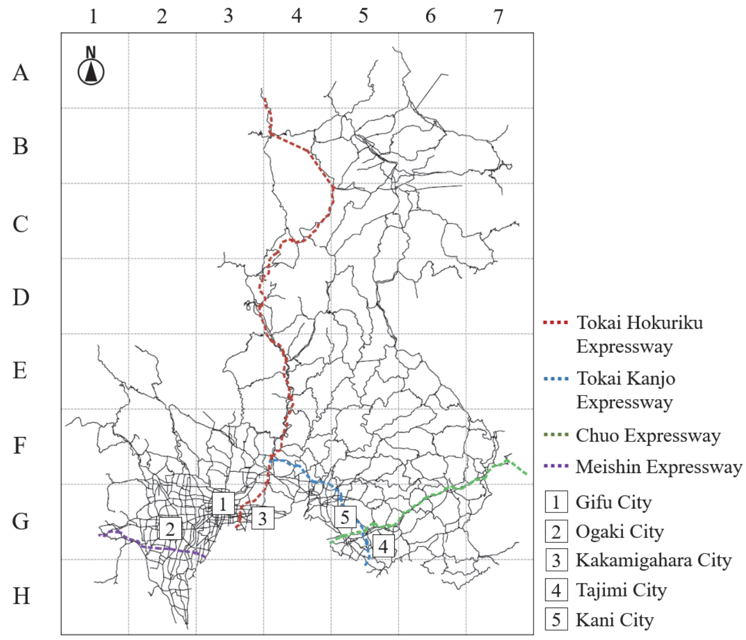


Figure 5.16 Gifu Prefecture road network

Table 5.3 Features of the Gifu Prefecture road network

	Unit	Min	Max	Median
Traffic Volume	Vehicle	39	76,247	7138
Capacity	Vehicle	1000	80,000	10,000
Congestion Rate	%	0.0056	9.46	0.71
Speed	km/h	20	100	35
Distance	Km	0.038	10	0.83
Travel Time	Minute	0.016	13.33	0.42
BPR Function	Minute	0.076	7923.53	3.02

5.4.4. EC evaluation by various traffic indices

The results must be interpreted carefully because the network is evaluated as better in some indices when the value of the eigenvector centrality is larger, but not in the other indices. For example, the network is evaluated as better when travel time is low, but not when speed is low. Figure 5.17 presents the results of eigenvector centrality analysis by each traffic index. The comparative evaluation is carried out based on the rank of eigenvector centrality because the number of nodes in each case is identical. The nodes are equally divided into five classes with 20% (ranked from 1st to 5th from the highest). Hence, the number of nodes in each class is equal in all classes and in all cases. The findings are summarised as follows:

Traffic Volume

The regions with a high centrality of traffic volume (meshes G2 and G3 in Figure 5.17) are only located in urban areas (Gifu and Ogaki City in Figure 5.16). From this result, the impact of traffic volume is concentrated in the urban area. There are no areas ranked in the first level other than Gifu and Ogaki City.

Traffic Capacity
The evaluation by traffic capacity is greatly affected by the expressways. The nodes ranked in the 1st level are located both in urban areas and around the cross-point of the Tokai Kanjo and Tokai Hokuriku Expressways. Furthermore, the highly ranked nodes spread to the north along the Tokai Hokuriku Expressway.

Congestion Rate
A high centrality value means a concentration of excess demand links. In addition to areas identified by above indices, meshes F2 and F3 have higher centrality values. Nodes in these places are included in the 2nd rank (20–40%) by traffic volume evaluation but are ranked in the first level since the capacity is not significantly high.

Speed
The evaluation result by speed is easy to interpret. Basically, the maximum speed limit of the surface road is 60 km/h and that of the expressway is 100 km/h in this area. Therefore, the centrality value along the expressway becomes inevitably larger. The result shows that the areas where the expressway routes cross have a particularly high value. Interestingly, since we don't have expressways within Gifu City, the value of this weight is rather low.

Distance
The centrality by distance is highest in the north. It decreases towards the south and the values are low in most of the urban areas. A large centrality by distance means that the road network density is low. These results show that the network in the north is rather sparse and that the urban area has a dense road network.

Travel Time
There is a positive correlation between distance and travel time of links. The distribution of the nodes however differs somewhat. Some of the nodes included in the rank of top 20% by distance are located in meshes D3, D4, E3 and E4, whereas nodes in the evaluation by travel time are located in meshes E5, E6, F5 and F6. Travel time of the links in the latter area is rather long, although the distance is not very long. This is a feature of a steep mountain route in the mountainous area. A combined evaluation of distance and travel time is therefore effective.

BPR Function
The evaluation by the BPR function gives an interesting insight: the higher centrality areas are scattered to each city. Areas with large centrality, like meshes C5, C6, G2, G3 and G5 are urbanised area. Unlike other

indices, the large centrality area does not form a giant cluster. The appearance of several small clusters with large centrality values is a characteristic of the BPR function evaluation and this may happen because the congestion rate is nonlinearly evaluated by the power of β.

From these results, the eigenvector centrality gives different results based on the different weights. Strong and weak areas of the network can be identified from various aspects. There are some indices whose evaluations have similar trends, and combination with other indices is sometimes useful. In the next chapter, the relationship of eigenvector centrality by different weights is verified.

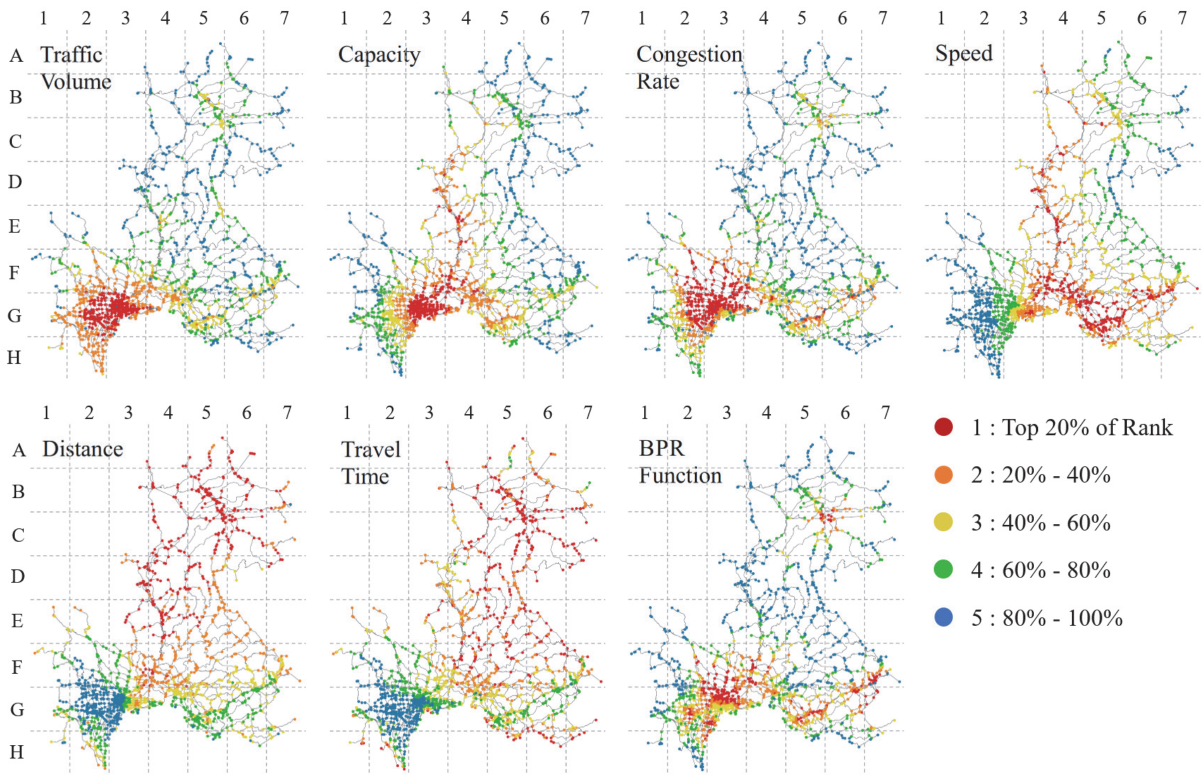


Figure 5.17 Eigenvector Centrality by traffic indices

5.4.5. Characterisation of the road network by a factor analysis

Figure 5.17 presented seven case studies of eigenvector centrality analysis. These all evaluate the same Gifu Prefecture road network. However, the previous chapter shows that the evaluation results vary depending on the indices. This chapter attempts to find common factors within the evaluation results. Factor analysis can be used to find hidden factors leading to these results. The logarithm of the eigenvector centrality is used for the factor analysis, as it is suitable to highlight the small changes on this scale. The logarithm values are always negative because all components of eigenvectors are less than 1 by normalising the size of the eigenvector as 1.

Factor analysis is a statistical method that estimates unobserved variables ('factors') that describe the variability of experimental or observed data. It can be used to identify common factors, and factor loadings are used to evaluate the influence of common factors on each observed variable. These relationships are represented by the following formula(5.9). A unique factor is a factor that affects only on one observed variable.

$$\mathbf{Y} = \mathbf{\Lambda f} + \boldsymbol{\varepsilon} \tag{5.9}$$

where,

$\mathbf{Y}$	:	A set of observed variables
$\mathbf{\Lambda}$	:	A set of factor loadings of each variable and each factor
$\mathbf{f}$	:	A set of common factors
$\boldsymbol{\varepsilon}$	:	A set of unique factors.

In this analysis, we used the maximum likelihood estimation method for factor extraction, as well as a varimax rotation. At first, the number of factors is determined by scree plot criteria. A scree plot is used to plot the eigenvalues of the correlation matrix in descending order, and the rank of eigenvalues apart from the trend line determines the number of factors. Here, the number of factors is set at 3 by a scree plot based on the correlation matrix shown in Table 5.4.

Figure 5.18 represents the factor loadings of three common factors. In the first factor, congestion rate, traffic volume and BPR function have large positive loadings. Conversely, travel time and distance have negative large loadings. The large positive loading factors (congestion rate, traffic volume, BPR function) relate to the traffic demand. In contrast, the indices whose magnitude of the loadings are less than 0.5 do not have this relationship. The first factor can therefore be interpreted as the "traffic demand factor." The second factor has particularly large positive loadings of both travel time and distance. These indices are correlated (cc=0.794) as shown in Table 5.4. Nodes with large centrality of these indices lie in areas with a sparse road network. Additionally, traffic volume, capacity, congestion rate and the BPR function have large negative loadings. These indices tend to have small values on dense road network areas. Therefore, the second factor can be interpreted as the "road sparsity factor." The third factor has three effective indices; other than these, the magnitude of factor loadings is small. The three indices have larger values on capacity, speed and BPR function. The values for these three indicators may increase for trunk roads such as expressways and national highways. Hence, the third factor is interpreted as the "road rank factor." If the value is large, the road should be highly ranked such as expressways and highways, whereas the road may be lowly ranked such as narrow prefectural roads when the value is small.

Thus, the evaluation of seven eigenvector centrality measures is summarised into three common factors. The cumulative contribution rates of the common factors from the first to third factors are 0.305, 0.567, and 0.805, respectively. The findings reveal that more than 80% can be represented by these three common factors. It is possible to clarify the three main network evaluation elements by using the data only from the National Road Traffic Survey.

Table 5.4 The Correlation Matrix of Indices

	Traffic volume	Capacity	Congestion rate	Speed	Distance	Travel time	BPR
Traffic Volume	1.000						
Capacity	0.389	1.000					
Congestion Rate	0.900	0.407	1.000				
Speed	-0.178	0.585	-0.029	1.000			
Distance	-0.516	-0.151	-0.457	0.374	1.000		
Travel Time	-0.516	-0.304	-0.414	0.266	0.794	1.000	
BPR	0.463	0.621	0.621	0.317	-0.284	-0.247	1.000

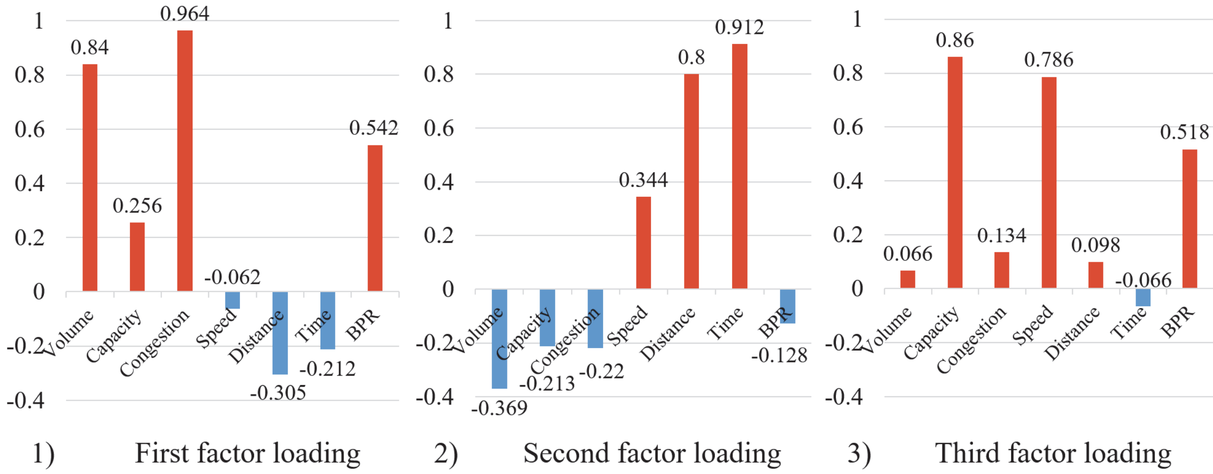


Figure 5.18 Factor loadings of three factors

5.4.6. Hierarchal clustering by the common factors

Next, the nodes in the network can be classified using the three common factors. This analysis is carried out to characterise the geographical distributions of nodes by common network features. Characterising the current road network may help in identifying the required levels and functions of road networks. The factor score for each node is used for clustering. This clustering analysis adopts a hierarchical method based on the furthest neighbour method in Euclidean distance. The number of clusters must be determined in advance in the case of the hierarchical clustering. In this study, the nodes are classified into five clusters.

Table 5.5 lists the number of nodes with its percentage included in each cluster. The share of the largest cluster (Cluster 4) is 38%, so there is no giant cluster. Although Clusters 3 (8%) and 5 (7%) are slightly smaller, they still contain more than 100 nodes. Table 5.5 also lists the average factor scores in each cluster, which reveal the characteristics of the cluster. Positive scores in the “traffic demand,” “road sparsity,” and “road rank” factors mean high demand, sparse roads, and higher rank roads, respectively. Additionally, Figure 5.19 shows the geographical distribution of clusters. The characteristic of each cluster is summarised as follows:

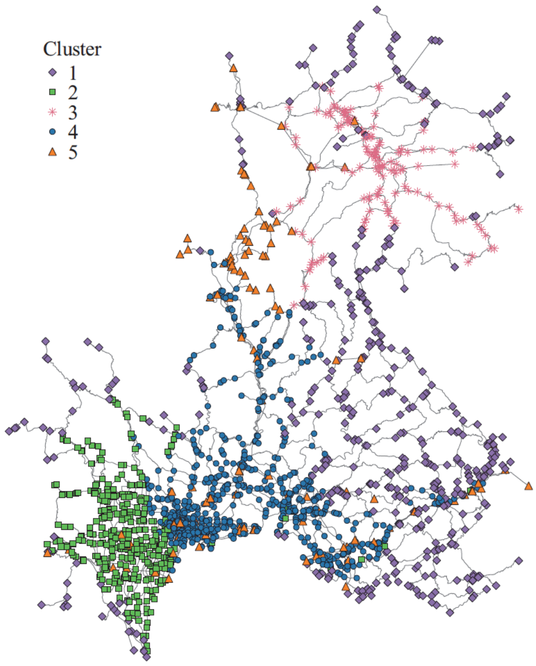


Figure 5.19 Node clustering in Gifu prefecture

Table 5.5 Summary of clustering

Cluster	The number of nodes	Percentage	Average factor score 1 "Traffic demand"	Average factor score 2 "Road sparsity"	Average factor score 3 "Road rank"
1	472	26%	-0.376	0.348	-0.830
2	372	21%	0.378	-1.267	-0.729
3	140	8%	0.247	2.252	-0.541
4	678	38%	0.479	0.060	0.861
5	121	7%	-2.665	-0.402	1.281

Cluster 1

Nodes in Cluster 1 are located in sparse areas and the edge of the network. This cluster can be called as the "rural part cluster." This set of nodes has low demand and low road rank. Also, the roads in this cluster area is sparse. Therefore, the mountainous areas with insufficient road improvement are indicated by this Cluster.

Cluster 2

Nodes in Cluster 2 are located in the southwestern part of Gifu Prefecture around Ogaki City. This cluster can be called as the "western urban part cluster." From the average factor score, this set of nodes has a large demand, a low sparsity, and an insufficient road rank. Therefore, the western urban part needs to reinforce roads with higher ranks such as expressways.

Cluster 3

Nodes in Cluster 3 are located in the areas around Takayama City in the north. Takayama City is the largest city in the north of Gifu Prefecture. This cluster can be called as the "northern urban part cluster." The average factor scores also represent the characteristics of urban areas in the north of Gifu Prefecture; the road network is rather sparse consisting of lower rank roads. Since the road density is sparse the new high-rank road investment may be needed.

Cluster 4

Nodes in Cluster 4 are mainly located around Gifu City, and also in a region that can be moved from Gifu City within a short time by using expressways and national highways. This cluster can be called as the "central urban part cluster." The values of three loading factors are well balanced and this cluster represents the 'average' feature of this network.

Cluster 5

Nodes in Cluster 5 are mainly located along the expressways. This cluster can be called as the "expressway cluster." Although this set of nodes has roads with a high road rank, the average factor score of traffic demand is strongly negative. Because Cluster 5 has the considerable remaining capacity, the policies for promoting the use of such roads are effective.

Nodes in Clusters 2, 3, and 4 are located around large cities such as Ogaki, Takayama and Gifu City. In contrast, Clusters 1 and 5 have nodes that are similar in terms of geographical conditions and environments. The node clustering method adopted in this study does not consider the geographical locations. Nonetheless, except for some nodes within Clusters 1 and 5, the method classifies the nodes mostly by geographical locations, like Clusters 2, 3 and 4.

Seven traffic indices obtained by traffic survey data were set as the weights. The evaluation results using seven indices identify the areas with strong and weak impact from the viewpoint of each index. Also, this section applied factor analysis to find the common factors among seven indices. These common factors help explaining the characteristics of the road network. Moreover, by using the results of the factor analysis, the nodes were categorised into five clusters. The distribution of nodes revealed that the common factor represents characteristics of the nodes by regional situations and functions. EC analysis with several weights can characterise and classify the road networks, and such categorisation can further be used to decide future road investment policy.

5.5. Test for Larger Road Networks

To test the usefulness of the proposed method for larger road networks, road networks of six regions around the world are analysed. These networks are from Bar-Gera. H, Transportation networks that is the same data used in the Chapter 3. A summary of large-scale road networks is shown in Table 3.1. The traditional connectivity evaluation methods reviewed earlier are difficult to apply to such large networks as the computational time and maximum memory make computation prohibitive. Using these practical larger road network data, this paper seeks for the strongly and weakly connected areas using the capacity-weighted EC method.

Figure 5.20 shows the distribution of the squared EC values on log scale in the six networks. As with Gifu Prefecture road network, the nodes are classified into five levels by equal range of the log EC values. Figure 5.21 illustrates the share of the nodes for five levels. Specific characteristics of each network are described below.

Berlin, Germany

Figure 5.21 shows that lots of nodes are classified in Level 1. These nodes spread over the whole network (Figure 5.20(a)). Moreover, about 94.0% of nodes belong to Level 1 and Level 2. This result shows that the connectivity is uniformly strong across the network. On the other hand, nodes with connectivity at Level 3 are located in the east of the city and largely surrounded by more strongly connected areas. The area of poor connectivity is most likely a relic of the old East Berlin.

Birmingham, United Kingdom

As is shown in Figure 5.21, both shares of Level 1 and Level 5 nodes are small. In the central area of the Birmingham located in the middle of the map, there are many Level 1 nodes, and so as the city centre of Wolverhampton and Coventry. The difference shows the scale and shape of the city; the connectivity of central Birmingham area remains high. Weakly connected nodes are located rather in residential areas and parks. Birmingham City has a green belt surrounding the urban area to control urban growth (Birmingham Development Plan 2031 as of October 21, 2018, listed on Birmingham city council website). As shown in Figure 5.20 (b) few nodes and roads are located in the green belt. The effect of high connectivity in the central area cannot spread due to the green belt. In Birmingham, urban and residential area are clearly segregated, and residential areas and parks are evaluated rather low with respect to connectivity.

Philadelphia, United States

Delaware River crosses the middle part of the Philadelphia. Level 1 node is located only in the south. This area is covered with a forest and the road network is rather sparse, but it was located between U.S. National Interstate Highway 42 and U.S. Highway 30. Surrounded by large-capacity roads can be a reason why this area is evaluated as strongly connected. Also, the urban central area located to the north of Delaware River shows strong connectivity (Level 4). This effect spreads beyond the river to the south and the contribution of bridges for spreading the connectivity is substantial. Weakly connected areas are only located on the periphery of the network.

Gold Coast, Australia

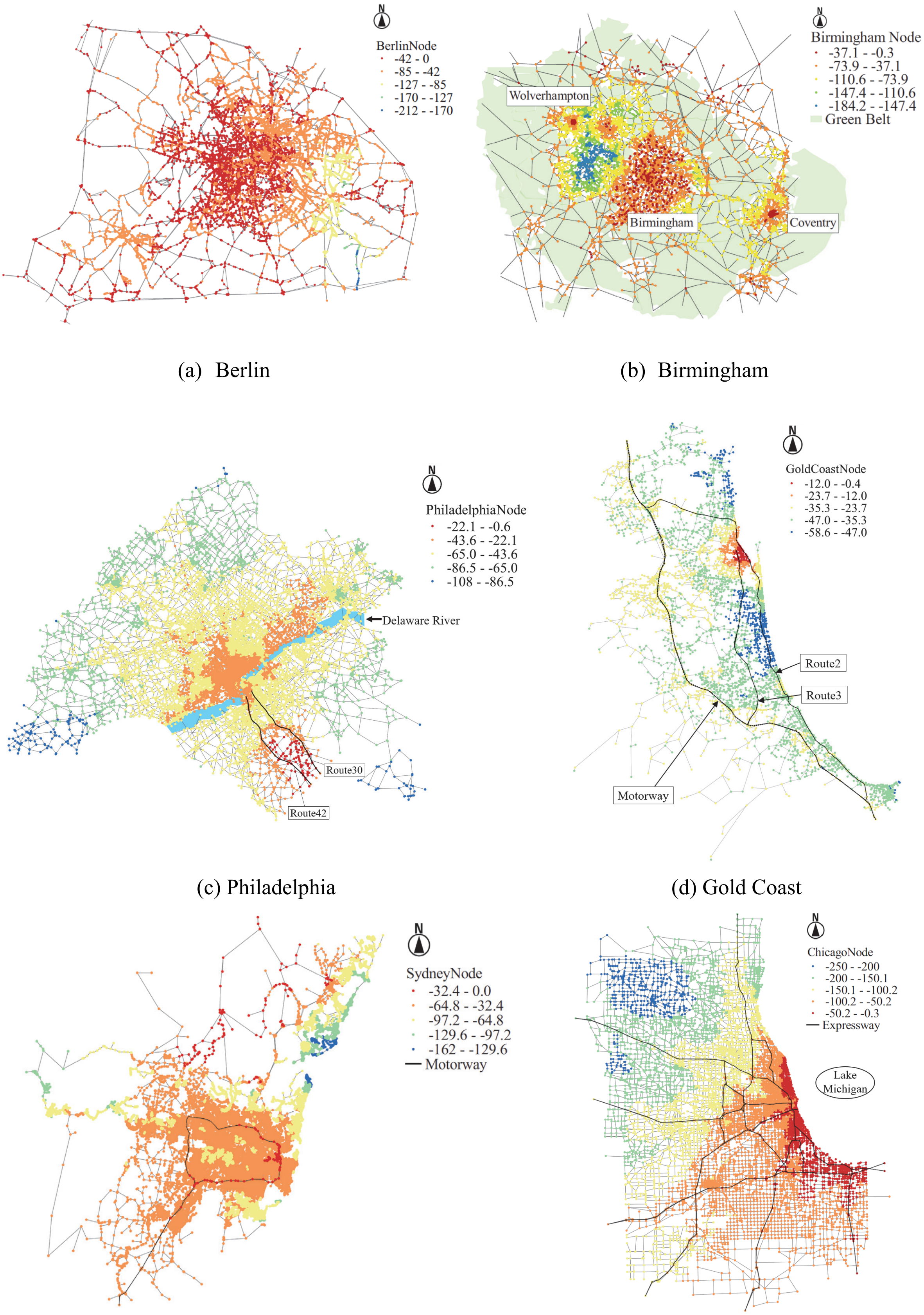
In the Gold Coast network, less than 4% of nodes are classified as Level 1, which are concentrated in one region. This area is the most prosperous part of the Gold Coast. Conversely, Level 5 nodes are located in largely residential areas. Level 1 and 5 nodes are located close to each other, and they are connected by national roads Route 2 and Route 3, which have large capacities. Level 5 nodes area is close to the Level 1 nodes area and they are connected by high-capacity national highway 2 and 3. We found that the area where nodes are evaluated as Level 5 is, although it is very close from the Level 1 area, there are many canals that all houses are accessible by boat, and only the limited number of roads are available.

Sydney, Australia

In the Sydney network, the nodes in Level 1 exist only in a limited central area and along with parts of the Motorway network and the links to the north. We found that links in the northern part are aggregated and thus have large capacity. The influence of the strongly connected nodes does not spread out as is seen in other cities. On the other hand, most nodes in the middle of the network belong to Level 2. Because of the influence of aggregated links, only limited number of nodes with huge capacities are ranked as Level 1 and most of the other nodes are ranked as Level 2. This could be a reason why we cannot identify strongly connected areas. Nodes belonging to Level 5 are located at the end of a peninsula or on Central Coast, where connectivity is known to be an issue for geographical reasons.

Chicago, United States

Chicago is located next to Lake Michigan. More than one-fifth nodes are classified in Level 1, and most of them are located along Lake Michigan. It seems that connectivity gradually spreads from the lake shore to the inland area. Even though some U.S. National Interstate Highways pass through both north-south and east-west direction, the influence on connectivity is strong to the south but not strong to the north. Weakly connected areas with Level 5 are only located in the north western part. There are lots of farms in these areas with lower capacity roads. The most strongly connected areas are located in the CBD, and the distribution in which the EC gradually decreases toward the north matches the characteristics of the Chicago city identified from Zoning and Land use map (The Chicago Department of Planning & Economic Development) and an aerial photograph.



(a) Berlin (b) Birmingham

(c) Philadelphia (d) Gold Coast

(e) Sydney (f) Chicago

Figure 5.20 EC in log scale for each network

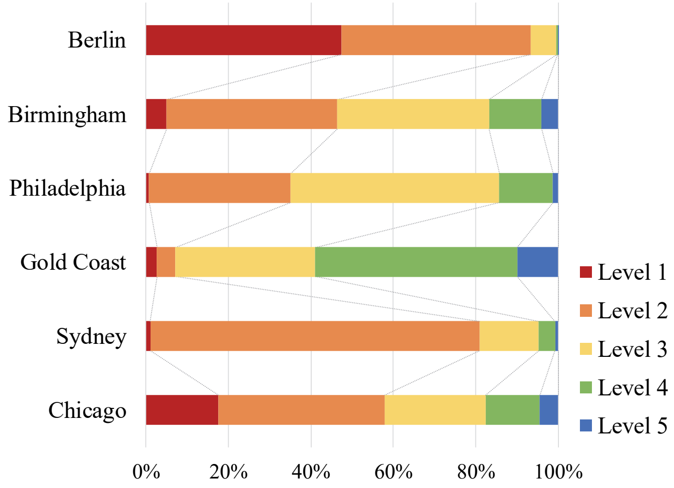


Figure 5.21 Node share in the six city networks

The results of six networks are compared. Figure 5.21 shows that the fraction of nodes in Level 1 in Berlin is the largest and it was followed by Chicago and Birmingham. However, the distributions of nodes in Level 1 on the map are all different. Berlin has uniformly strongly connected across the whole network, whereas Chicago exhibits a gradual transition from the weakly connected northern suburbs to more connected southern suburbs. In Birmingham, the strongly and the weakly connected areas are clearly segregated by the green belt. In Sydney, the fraction of nodes in Level 1 is small but the fraction of nodes in Level 2 is very large. This classification is similar to that in Berlin as the whole network has high connectivity. For cities like Berlin or Sydney, high connectivity seems to be maintained across the whole network, with small pockets of weak connectivity. In the Gold Coast and Philadelphia, the fraction of nodes in the strongly and the weakly connected areas can be small. It was found that all six networks describe very different characteristics, and the natural, geographic, political and social conditions may lead such differences. The computational time for Sydney (the largest size among six cities) is 1.828 seconds, and the proposed method is still computationally tractable.

5.6. Application to Detailed Network

The main advantage of EC analysis is the ability to evaluate large-scaled networks to which the traditional methods with high computational loads cannot be applied. The large-scaled network includes not only the vast network but also highly detailed network which has a lot of nodes and links. This section demonstrates the significance of detailed network analysis by using the evaluation results in a road network including small city roads.

The detailed network of the Gifu Prefecture to analyse has 138,871 nodes and 399,438 directed links. This network is named as “detailed network” in this section. The capacity weighted EC distribution is shown in Figure 5.22. The nodes are equally divided into five classes with 20%. Detailed network includes all kinds of roads, therefore the difference of the evaluation results with and without omitting small narrow roads can be discussed. The calculation of capacity weighted EC on the detailed network only requires 57.98 seconds (PC:Intel Xeon, CPU:E5-1620v4@3.50GHz, memory:32GB, OS: Windows10, 64bit). It is very easy to handle even such detailed networks.

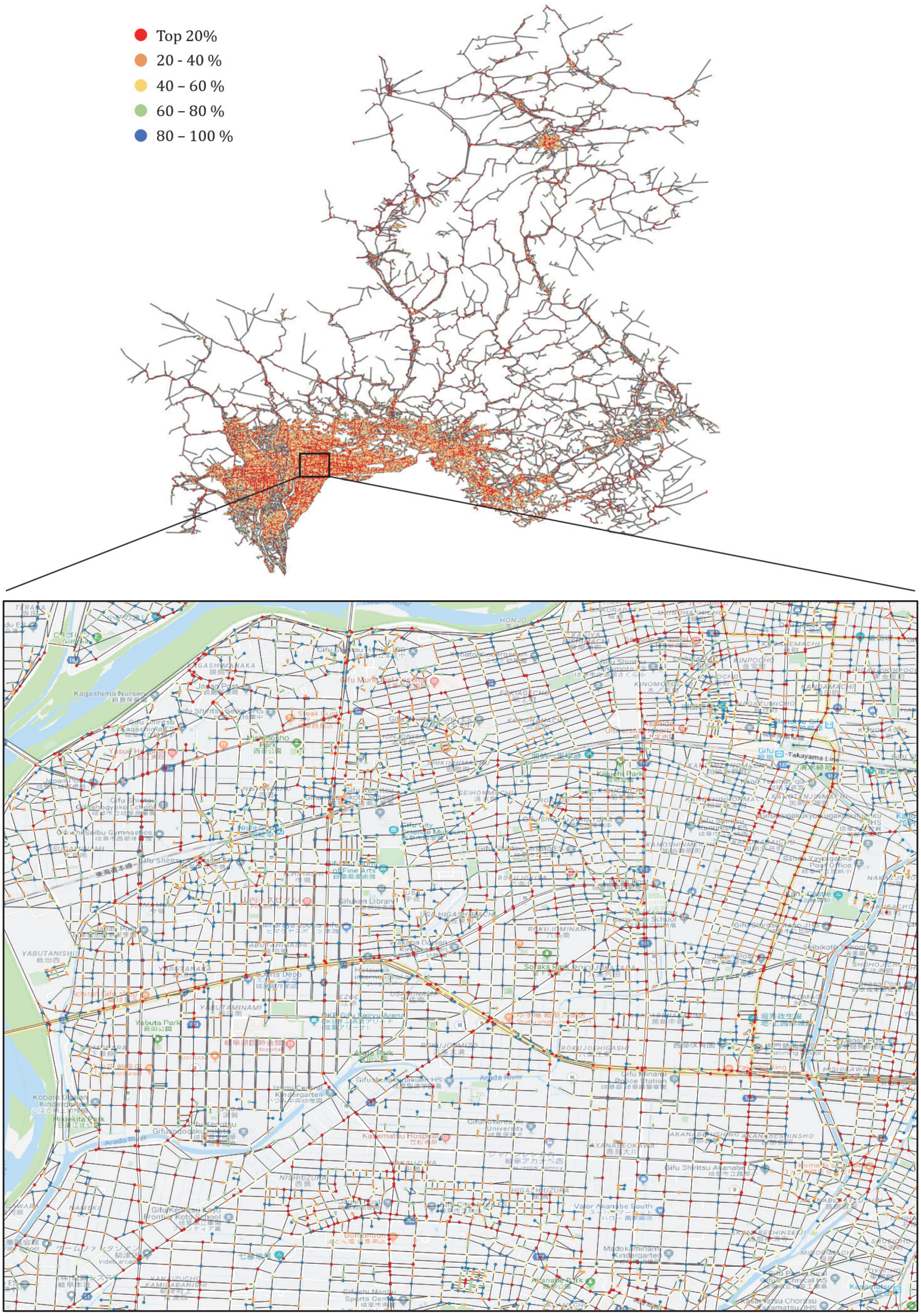


Figure 5.22 Capacity weighted EC distribution in detailed road network

Since the level of detail is too high and the results of analysis shown in Figure 4.21 are not clear on the whole Gifu prefecture, several cities are focused to discuss. The analysis of each city uses EC normalised in the Gifu prefecture network. However, the EC values are very small and to make the characteristics of distribution in each city more clearly, it is evaluated by the relative rate compared with the minimum value of each city. Therefore, all values of nodes in each city are more than 1. The following analyses in this section use this rule.

As an example, three cities (Gero city, Gujo city and Gifu city) are discussed with the comparison of EC between simple network and detailed network. The location of these cities is shown in Figure 5.23. Gero city and Gujo city are located in a mountainous area, and Gifu city is located in an urban area. The simple version of Gifu prefecture road network is same as the one discussed in 4.4. The size of network is 1,460 nodes and 4,578 directed links. This network is called "simple network" in this section.

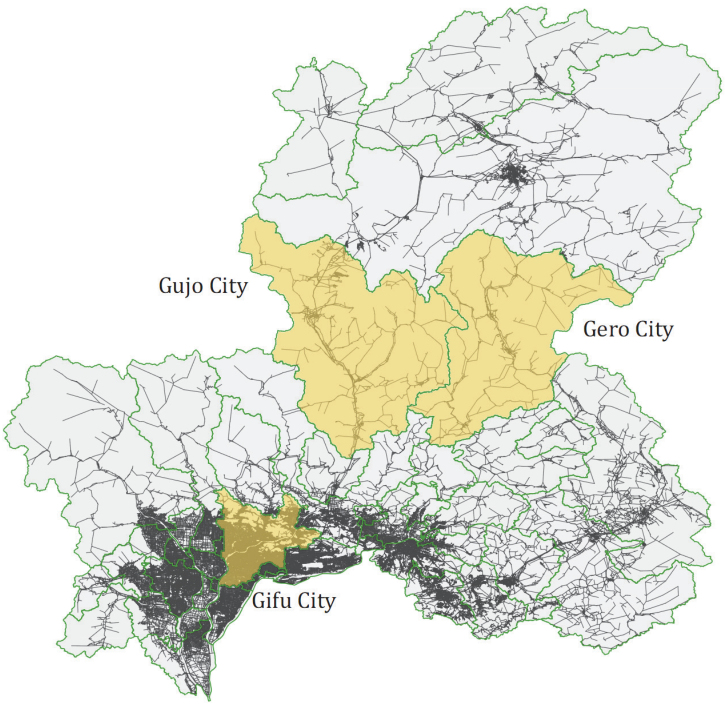


Figure 5.23 The location of three cities (Gero, Gujo and Gifu city)

Gero city

Figure 5.24 shows the relative rate of EC based on minimum value in Gero city. Gero city has no motorways, and a national highway (Route 256) is connected from Tokai Hokuriku Expressway which passes the west side of the Gero city. EC evaluation of simple network shown by orange triangle is high in the western and southern part. This may be an effect of Tokai Hokuriku Expressway with sufficient capacity. On the other hand, the evaluation around city centre where the city hall (green star in Figure 4.23) is located is not high. In the simple network, an impact of roads with large capacity is strong, but it seems that it is difficult to identify the strongly and weakly connected part at the small city level.

EC evaluation of the detailed network is shown in blue circles in Figure 5.24. There are high connectivity areas just north and south of the city hall. Also, it is indicated that even the western and southern parts of the city with uniform high connectivity in the simple network have both strongly and weakly connected areas. EC values in the detailed network are sparsely distributed across the city. This may mean that the strength of connectivity in the city by small or narrow roads is represented. Figure 5.25 shows the population in Gero on 500m square mesh. This population data is obtained from the national population census at same year as the road network used. As you can see the population distribution, the detailed network almost covers the place where population exists. Moreover, EC in the detailed network evaluates that the connectivity in the north-eastern area of the city with almost no population is very low. On the other hand, there are areas where the connectivity is evaluated strongly although the population is not large (e.g. the edge of city in the south-eastern area). These areas may be influenced by neighbouring cities. Because the area mentioned as example has a road connecting with Nakatsugawa City located in the south, the connectivity is evaluated high. In this way, it shows that it is possible to consider the evaluations of external connectivity even though this is an evaluation of Gero city in this section.

Heat maps based on the EC evaluation of the detailed and simple network are shown in Figure 5.26 and Figure 5.27. Compared to heat map of simple network where most areas with nodes except for the north-eastern area of the city are represented uniformly, heat map of the detailed network has strongly and weakly parts in the areas with nodes and extracts some villages by places where high connectivity is concentrated. At the city centre of Gero, the EC evaluation by the detailed network that includes small city roads is more remarkable than the EC evaluation by the simple network. This means that the city centre is rather isolated because of insufficient connectivity. In simple network, areas that are not affected by large-capacity roads tend to be not much different and have low connectivity. However, it was confirmed that the detailed network can indicate the strongly and weakly connected parts even in areas where the impact of large-capacity roads is not sufficient. As the network resolution level increase like a detailed network, the connectivity of small narrow roads can be evaluated. Moreover, their connectivity evaluations are the result considering the impact from outside of the city.

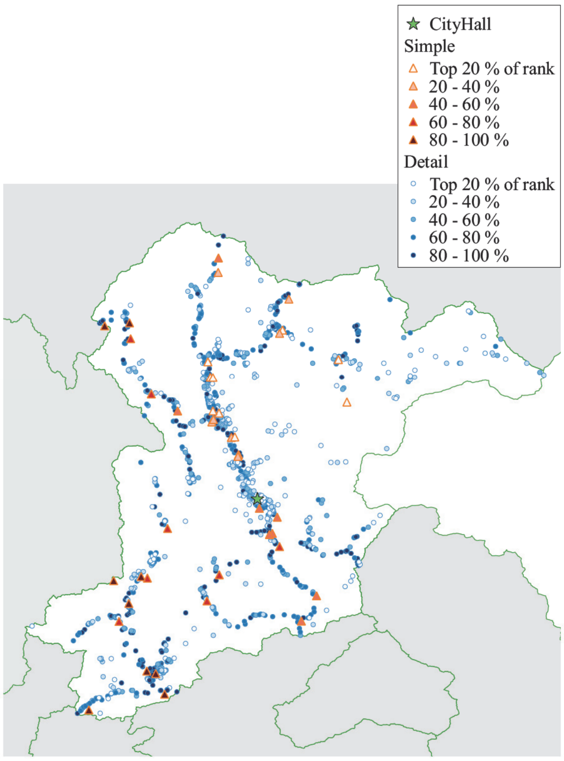


Figure 5.24 The relative rate of EC (Gero)

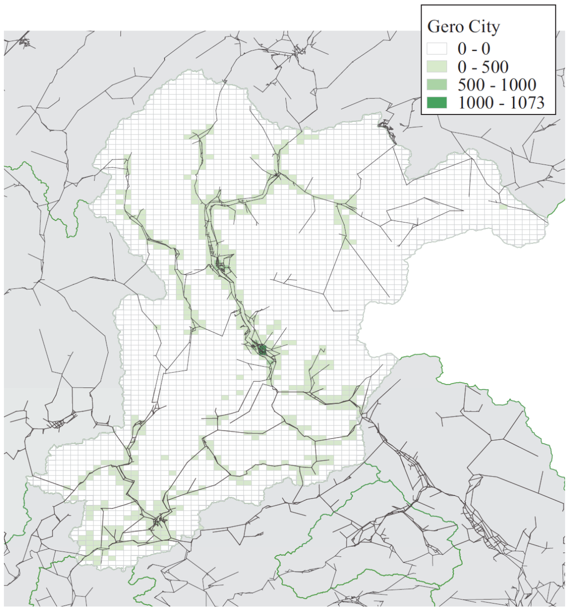


Figure 5.25 The population in Gero city

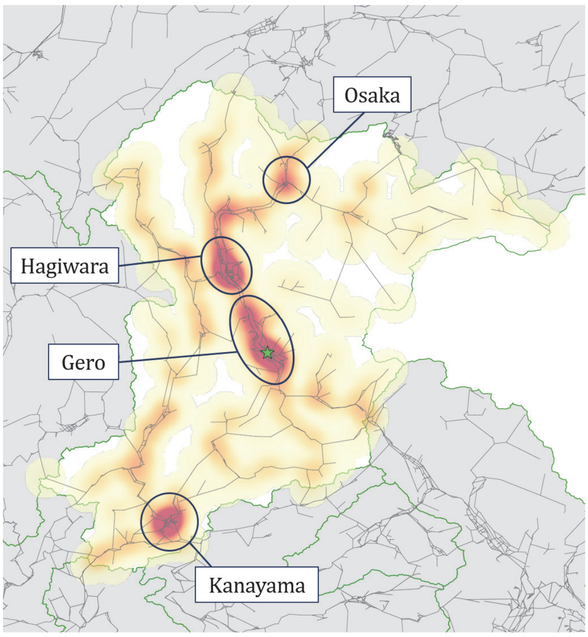


Figure 5.26 Heat map of EC in detailed network

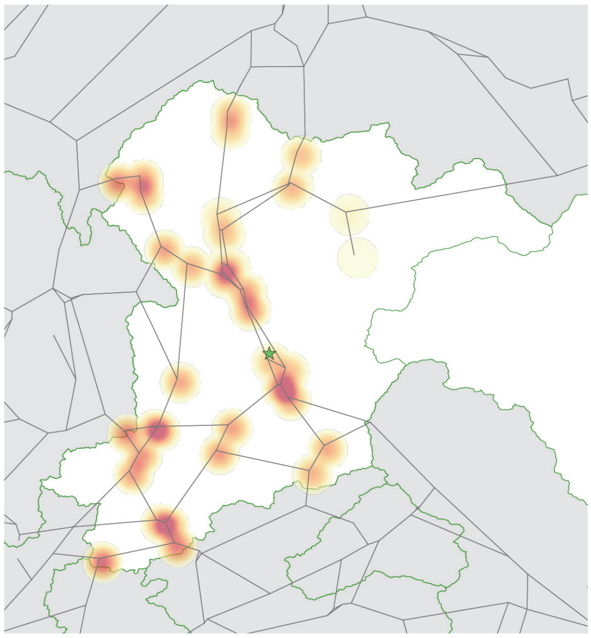


Figure 5.27 Heat map of EC in simple network

Gujo City

Figure 5.28 shows the relative rate of EC. Tokai-Hokuriku Expressway lies in Gujo city. Therefore, EC evaluation by both networks shows strong connectivity along the expressway. However, some nodes along the motorway surrounded by the black dotted line in the detailed network have rather weak connectivity. This may be the result of an area that is close to large capacity roads but has many low capacity roads. Such an evaluation result obtained in the detailed network is not shown in the simple network because the simple network omits such small roads. The use of the detailed network indicates that not all areas along the motorway have strong connectivity.

Let us look at the mountainous area where the road network is sparse. From the distribution of EC in Figure 5.28 and heat maps of EC evaluation in Figure 5.30 and Figure 5.31, some villages (Meiho and Wara) are picked up as an example in the mountainous area. In the simple network heat map shown in Figure 5.31, the connectivity is high in the urban area around the city hall and along with large-capacity roads and the connectivity is low in the mountainous with small population. In the detailed network, the connectivity in similar places is high, but the connectivity in mountainous areas has also differences. Small villages such as Meiho and Wara has shown in the detailed network heat map of Figure 5.30 are evaluated as parts with high connectivity in the mountainous areas. Wara is relatively more strongly connected than Meiho. This may be because National Highway 256 connecting Gujo with Gero lies along Wara. The locations of villages in the detailed network are consistent with the populated data shown in Figure 5.29. In this way, the detailed network analysis can evaluate connectivity in mountainous areas that do not include large-capacity roads.

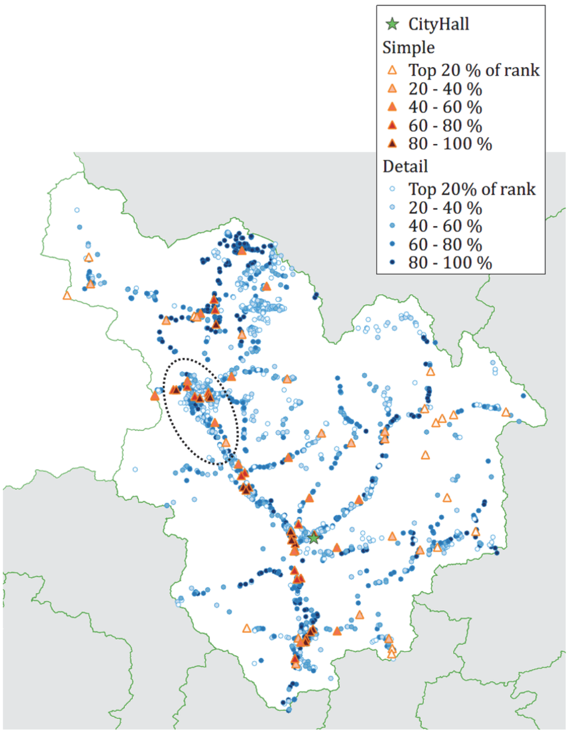


Figure 5.28 The relative rate of EC (Gujo)

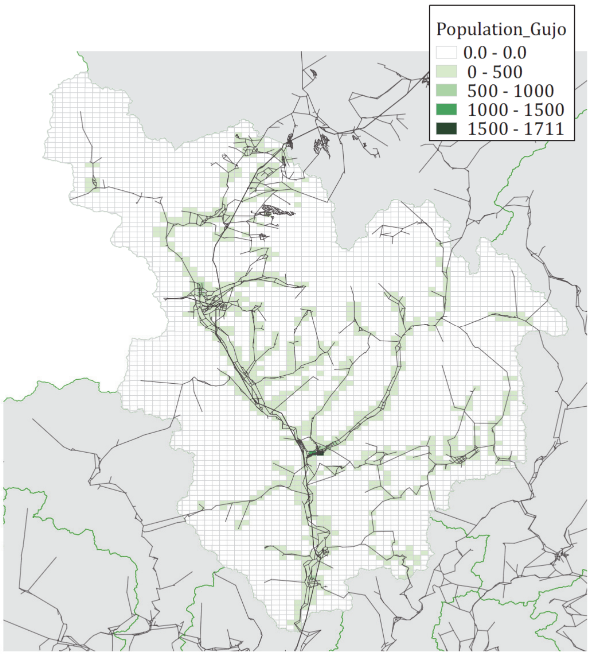


Figure 5.29 The population in Gujo city

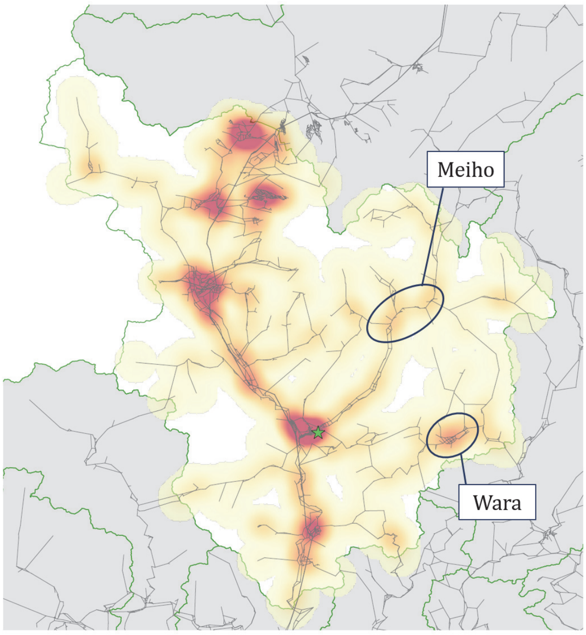


Figure 5.30 Heat map of EC in detailed network

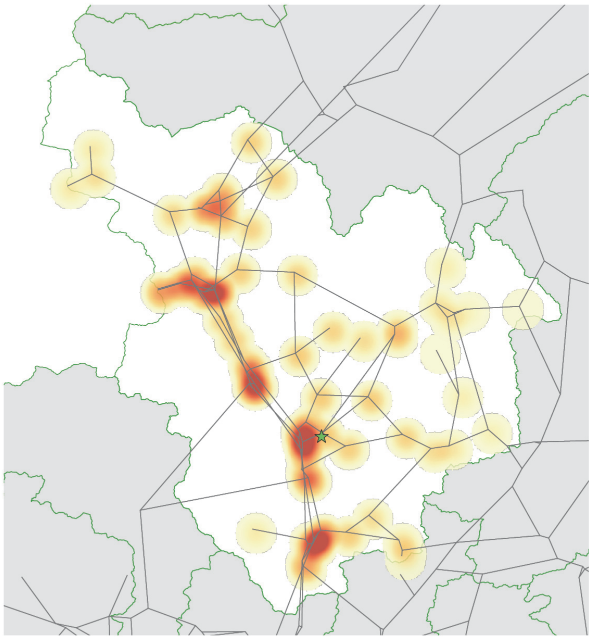


Figure 5.31 Heat map of EC in simple network

Gifu City

The purpose of discussion by using an example of Gifu city is to focus on connectivity in areas where roads with small capacity are dense. In the case of Gujo city, there was an area where connectivity is weak by many low capacity roads although they are close from large capacity roads. It is verified whether such evaluation result occurs in Gifu city as well. Gifu city has few mountainous areas. In the simple network, the eastern areas of the city are strongly connected. This is an effect of the large capacity roads by Tokai-Hokuriku and Tokai-Kanjo Expressway which are located in the east of city. Conversely, the north-western areas of the city are weakly connected. In the detailed network, the characteristics in the simple network did not appear and nodes with strongly and weakly connected are sparsely distributed. For example, the most north-east areas (black flame in Figure 5.32) are selected to understand the difference of evaluations between the simple and detailed network. All nodes in the simple network are evaluated as strongly connected. On the other hands, the strongly and weakly connected nodes scatters in the detailed network. By the population data shown in Figure 5.34, some people live in this area. However, detailed network has nodes that are evaluated very weakly connected. As a result of confirmation by using aerial photograph shown in Figure 5.33, this area contains farmland and roads with low capacity. On the heat map of EC in simple network shown in Figure 5.36, the EC evaluations in black flame are relatively high. However, on the heat map of EC in detailed network shown in Figure 5.35, the EC evaluations in black flame are not too high. In this example, there are many roads that are closely connected but their capacity is small, and the connectivity may be evaluated as weak. By using the detailed network, it is possible to identify small areas where roads are dense but not affected by the surrounding large capacity roads. In addition, in the simple network, high EC evaluations are concentrated in the centre and east of the city, while in the detailed network, high EC evaluations are also located in the west and south sides of the city. These differences show the areas that are evaluated only by the detailed network.

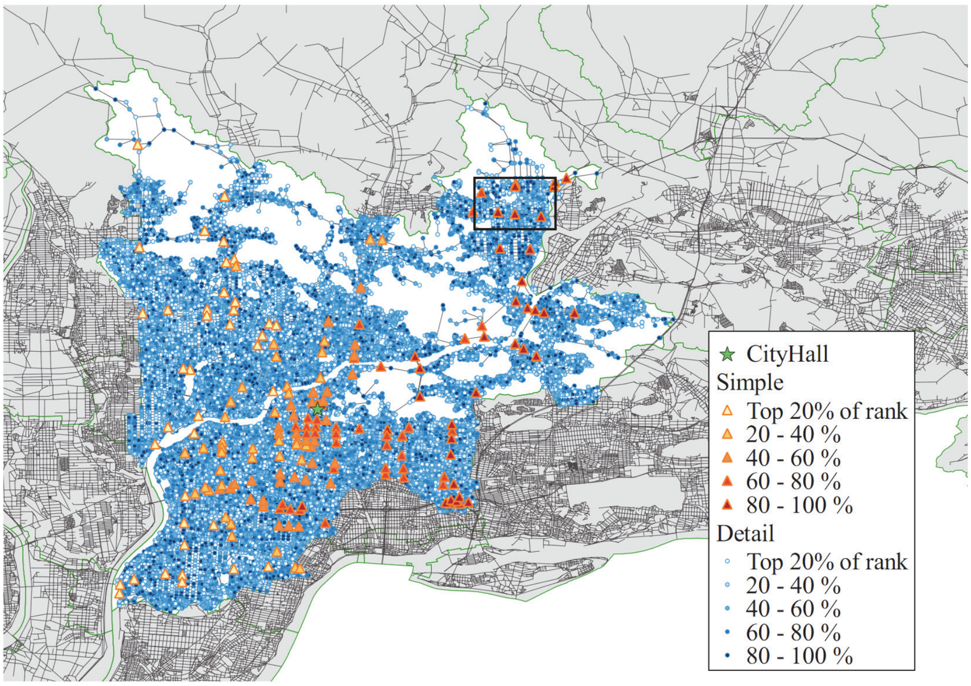


Figure 5.32 The change ration of EC based on minimum value

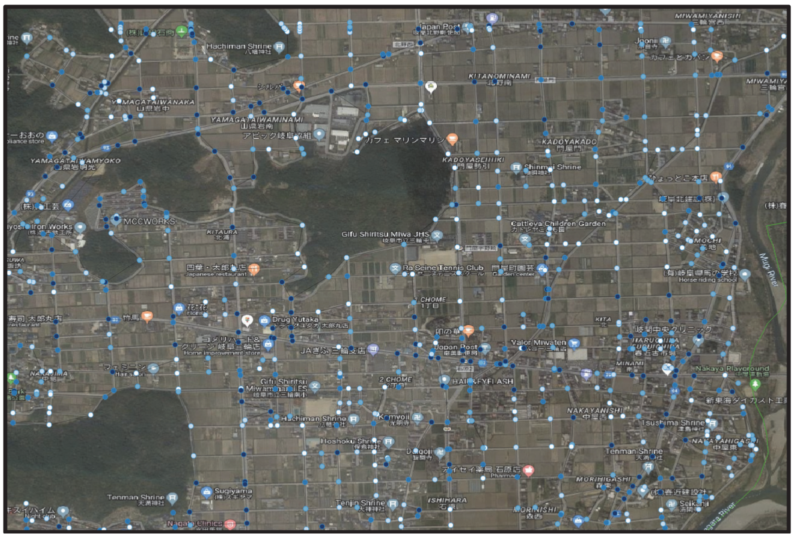


Figure 5.33 The enlarged view inside black square in Figure 5.32

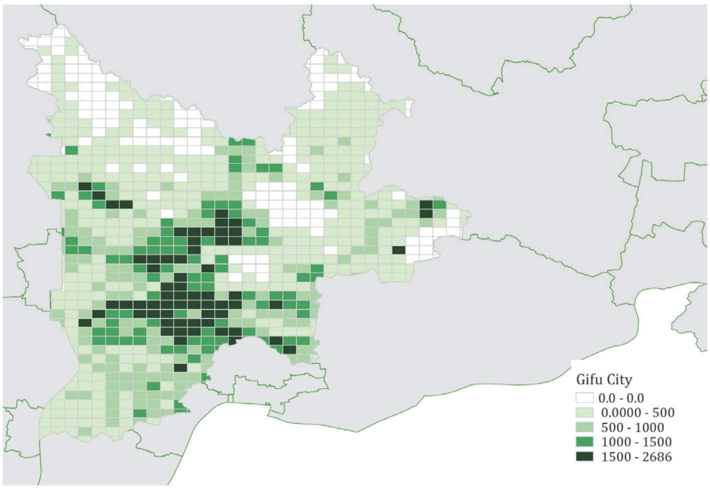


Figure 5.34 The population in Gifu city

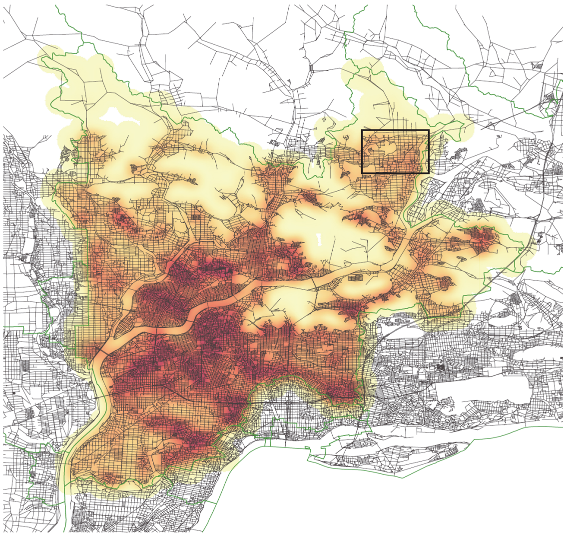


Figure 5.35 Heat map of EC in detailed network

Figure 5.36 Heat map of EC in simple network

When the connectivity of road network in a small region is evaluated, it is necessary to consider both the effects of expressways and national highways connecting cities and the effects of small roads in the city. The analysis of the prefecture-wide scale and highly detailed road network revealed that there are a lot of information that cannot be found without using this detail level of network. For example,

- identification of areas where connectivity is weakened by many small roads even around the large capacity roads,
- identification of places where connectivity become strong by the effect of villages in areas where connectivity is uniformly low in the road network limited to larger than prefectural roads,
- identification of areas where connectivity is evaluated to be weak in a road network limited to prefectural roads compared to the detailed network including city roads. If a city facility such as a city hall is located, the connectivity of that facility to the outside of the city is insufficient.

Hence, the ability to apply large-scaled networks is a huge advantage of the proposed EC analysis.

5.7. Concluding Remarks

This chapter used the EC analysis which is one of the centrality measures as a method to identify the strongly and weakly connected part of networks. At first in this chapter, the definition and derivation of EC were described. Furthermore, characteristics of EC were explained by analysing the relationship with other centrality measures. EC analysis can handle directed graphs, and all road networks to analyse in this chapter have directed links. The knowledges obtained from the analysis results are summarised as follows.

The proposed capacity weighted EC analysis was confirmed to be a suitable method for road network connectivity evaluation. The indicator of non-overlapping routes defined by Kurauchi et al. (2009) is one of the conventional connectivity vulnerability evaluation methods for road network. The comparison with EC evaluation and the number of non-overlapping route by using practical road network revealed that they have a significant correlation.

Capacity weighted EC can also consider the ease of link disruption. The application results to the Gifu prefecture network showed that the capacity weighted EC can identify the areas where have large effect of higher capacity roads. Moreover, it was shown that the capacity weighted EC can identify high-risk parts by the link disruption at the disaster. These results were discussed based on the actual situation in Gifu prefecture. Furthermore, the effect of network boundary suggests that if there are roads with large capacity nearby, the study network should be expanded to consider their effect.

The weight setting has a large impact for the EC evaluation. The analysis by using different weights identified the common factors of road network, and factors help explaining the characteristics of the road network. In particular, the factor analysis is carried out using the EC results. Nodes classified by the results of the factor analysis revealed the characteristics of regional situations and functions.

The EC analysis is very useful for large-scaled networks, as same as the spectral partitioning method discussed in the Chapter 3. The computational time for Sydney (the largest size among example cities) is only 1.828 seconds. This computational time shows that EC is really trackable for large-scaled network. Some examples of application to large-scaled network showed that EC method can identify the strongly and weakly connected areas. The distribution of EC results is totally different depending on the networks. These differences were discussed according to the characteristics of each city. However, the road characteristics may depend on other factors such as land use, geographical structures and so on. As a future study, understanding such differences of characteristics may include valuable information related to land use and urban policies, that may also contribute to evaluate social sustainability.

The analysis of detailed networks has the large potential to find important information of road networks. Applicability to large scaled networks is a huge advantage of EC. Because the analysis in detailed network can reveal the characteristics which cannot be found on approximated networks. By comparing the detailed network and simple network (approximated network) in Gifu prefecture, there were characteristics that are identified only by the analysis of the detailed network. Specifically, there is a weakly connected area near large capacity roads. It was shown that analysis independent of network resolution is possible. Moreover, the analysis of the detailed network used the especially large road network (138,871 nodes and 399,438 directed links). The computation time is 57.98 seconds. This shows that the EC evaluations in detailed network can be obtained very easy and quickly.

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Chapter 6

Evaluation of Long-term Road Improvements by Connectivity Analytics

6.1. Introduction

The impacts of natural disasters have been increasing recently because of depopulation and climate such as global warming. In the case of emergency, the road network actually becomes more important than other transportation modes such as rail (IATSS, 2000) because of the extensive road coverage and robustness in maintaining the connectivity of the systems. Even in ordinary periods, the road networks play an important role in ensuring accessibility, and the road improvement is one of the urgent issues. In japan, the 4th national development plan in 1987 aimed to spread high-speed transportation services nationwide (4th national development plan, 1987). The objective of the plan was to construct 14,000 km motorways at the beginning of 2000s. However, the achievement rate is still around 70% in 2011. It is still necessary to improve road networks (Transition of high-standard road network planning).

However, the road construction is very expensive, and it takes a long time. For efficient road investment, it is essential to understand how the impact of road improvement is reflected in road services and usage. This chapter evaluates the long-term change of road network where many roads have been improved. The impact of about 30 years' road improvements is verified by using connectivity analytics.

6.2. Eigenvector Centrality and Weight Setting

As the network connectivity analytics by topological indicator, eigenvector centrality (EC) which was confirmed to be useful for road network evaluation in the previous chapter is used. To verify the impact of on transportation ability for vehicle based on ease of connection, the traffic capacity of each link is considered on the weights. Moreover, to distinguish between short links and long links with equivalent traffic capacity, the multiplication of traffic capacity and link length is set as weights. The setting of weight w_e is

$$w_e = L_e C_e \tag{6.1}$$

where,

L_e : A length of link e (km)

C_e : A traffic capacity of link e (vehicle/day).

In this chapter, the weight based on the multiplication of length and capacity of links is representing the “magnitude of road areas” and used as an indicator of road improvement. The eigenvector centrality using weighted adjacency matrix with these weight settings evaluates the connectivity of road supply performance.

6.3. Long-Term Network Changes in Gifu Prefecture

In this study, a road network in Gifu Prefecture with roads ranked higher than prefectural roads are considered, and study period is from 1985 to 2024. In Gifu Prefecture, as is the same as other areas of Japan, based on the 14,000km high-standard road network plan decided in 1987 by the Japanese Government, expressways have been vigorously constructed. Especially, Tokai-Hokuriku Expressway, Tokai-Kanjo Expressway and Chubu-Jukan Expressway have been constructed and extended. Gifu prefecture named these three expressways as “New three expressways” and they are regarded as effective roads for activating tourism and economy, reducing the traffic congestion and securing emergency transport roads at disasters. Figure 6.1 shows the locations of “New three expressways”. Table 6.1 summarises the construction histories and plans of the Tokai-Hokuriku Expressway and the Tokai-Kanjo Expressway that have been actively developed. These expressways have been or will be extended to neighbour prefectures such as Aichi, Mie, Toyama, Ishikawa and Nagano.

The Tokai Hokuriku Expressway, which was fully connected in 2008, has been continuously developed since 1996. However, most of sections are provisionally constructed as two-lane roads, and it has been gradually expanded to 4 lanes. On the other hand, the Tokai-Kanjo Expressway has opened the eastern section from Toyota East JCT to Minoseki JCT at first in 2005. After that, a construction plan has been established for the western section and is currently under construction. The Tokai-Kanjo Expressway has a lot of provisional 2-lane roads, so it will be required to expand to 4 lanes in the future. In this way, road improvement projects have been carried out actively in Gifu Prefecture in the past 30 years, and quickly and effective road improvements are also required in the future.

This chapter attempts to evaluate the effect of road network improvements by analysing network connectivity. The improvement transition of the road network from 1985 to 2024 in Gifu prefecture is used to analyse.

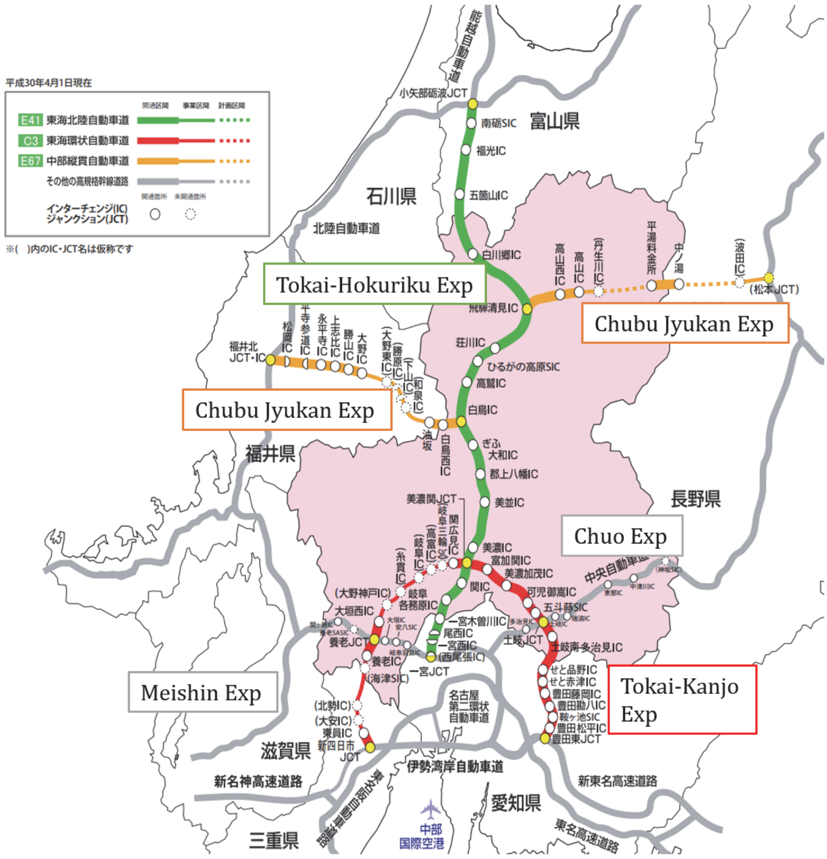


Figure 6.1 The location of "New three motorways"

(Gifu Pref. official HP, English captions are added by the author)

Table 6.1 The construction history of Tokai-Hokuriku and Tokai-Kanjo Expressway
(Gifu Pref. official HP, translated by the author)

	Tokai-Hokuriku Motorways	Tokai-Kanjo Motorways
1986	Opened Gifu-KakamigaharaIC～MinoIC L=19.1km（4Lanes）	
1987		
1988		
1989		
1990		
1991		
1992	Opened FukumitsuIC～Oyabe-TonamiJCTL=11.1km（2Lanes）	
1993		
1994	Opened MinoIC～MinamiIC L=17.2km（2Lanes）	
1995		
1996	Opened MinamiIC～Gujo-HachimanIC L=10.2km（2Lanes）	
1997	Opened IChinomiya-KisogawaIC～Gifu-KakamigaharaIC L=5.6km（4Lanes） Opened Gujo-HachimanIC～ShirotoriIC L=16.6km（2Lanes）	
1998	Opened BiSAiIC～IChinomiya-KisogawaIC L=3.8km（4Lanes） Opened IChinomiyaJCT～BiSAiIC L=3.9km（4Lanes）	
1999	Opened ShirotoriIC～ShokawaIC L=21.9km（2Lanes）	
2000	Opened GokayamaIC～FukumitsuIC L=16.3km（2Lanes） Opened ShokawaIC～Hida-KiyomiIC L=18.9km（2Lanes）	
2001		
2002	Opened ShirakawagoIC～GokayamaIC L=15.2km（2Lanes）	
2003		
2004	4 Lanes compLeted MinoIC～Fukubegatake PA L=18.5km South from ShirotoriIC L=2.1km	
2005		Opened Toyota-Higashi JCT～Mino-Seki JCT（L=73.0km）
2006		
2007	Hida tunneL opened	
2008	Opened Hira-Kiyomi IC～Shirakawago IC L=25.0km（2Lanes） 【ALL Lanes opened】 4 Lanes compLeted Fukubegatake PA～Gujo-HachimanIC L=8.9km	
2009	4 Lanes compLeted Gifu-YamatoIC～ShirotoriIC L=10.4km 4 Lanes compLeted Gujo-Hachiman IC～Gifu-YamatoIC L=6.2km	Opened Mino-Seki JCT～Seki-Hiromi IC（L=2.9km）
2010		
2011		
2012		Opened Ogaki-Nishi IC～Yoro JCT（L=5.7km）
2013		
2014		
2015		
2016		Opened ToinIC～Shin-YokkaIChi JCT（L=1.4km）
2017		Opened Yoro JCT～Yoro IC（L=3.1km）
2018	4 Lanes compLeted Shirotori IC～Takasu IC L=8km 4 Lanes compLeted Hirugano-Kougen SA～Hida-Kiyomi IC L=26km	
2019	4 Lanes compLeted Takasu IC～Hirugano-Kogen SA L=7km	Opened DaianIC～ToinIC （L=6.4km） Plan : Ohno-GodoIC～OogakinishiIC Plan : Seki-HiromiIC～TakatomiIC
2024		Plan : TakatomiIC～Ohno-GodoIC Plan : HokuseiIC～DaianIC

The target road networks are 8 year points in 1985, 1990, 1999, 2005, 2010, 2017 and 2024. This chapter uses the approximated network whose road ranks are higher than prefectural level including expressways and national highways. The network in 2024 is created based on the future road construction plan. Table 6.2 shows the number of nodes and directed links on the network for each year. All network data is obtained from the national road traffic census survey. As an example, Figure 6.2 and Figure 6.3 show the distribution of the length and capacity in 2010, the latest year of the road network that has real data. The distances of most of the links are 5 km or less, but there are links with a length of 15 km or more.

Most of them are links representing expressways. As for traffic capacity, most of the links have capacities less than 20000 (vehicle/day), and some of them are even smaller than 10000. This is because substantial number of links are narrow that are located in urban and mountainous areas. However, it can be seen that there are some links with large capacity exceeding 40,000 (vehicle/day), representing expressways.

Table 6.2 Networks of each year

Year	1985	1990	1994	1999	2005	2010	2017	2024
Node	1716	1727	1745	1770	1791	1793	1796	1802
Link（General）	4470	4494	4514	4618	4717	4723	4728	4745
Link（Motorway）	31	35	40	52	69	73	78	89

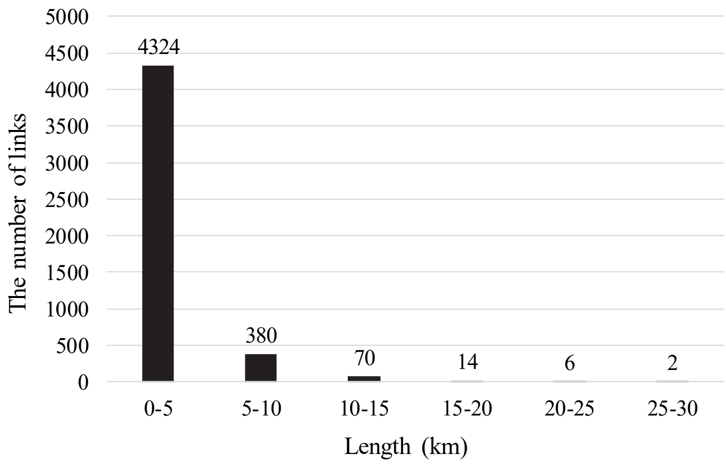


Figure 6.2 The distribution of length in 2010

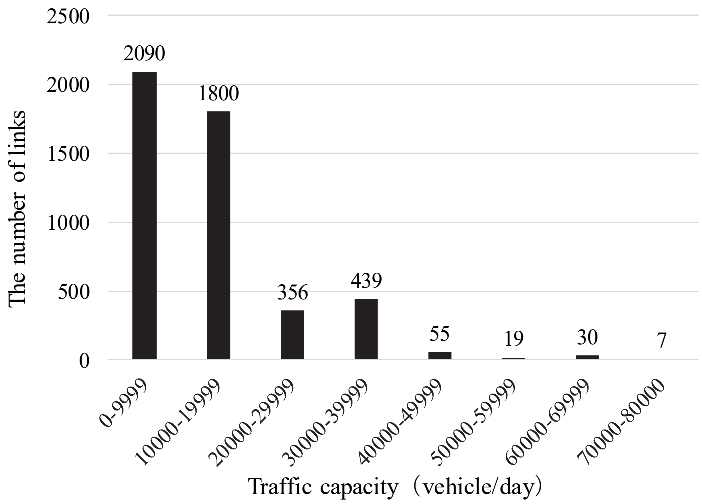


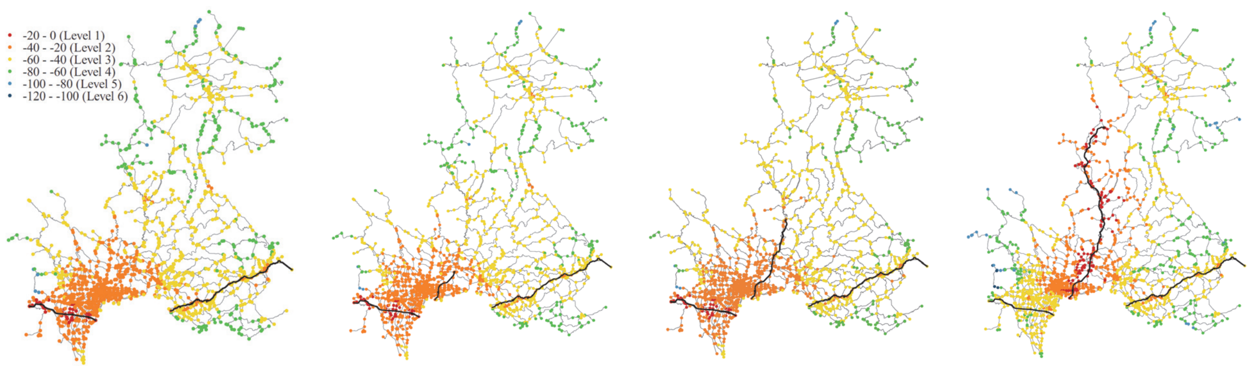
Figure 6.3 The distribution of capacity in 2010

6.4. Connectivity Analytics in Long-term Changes of Road Networks

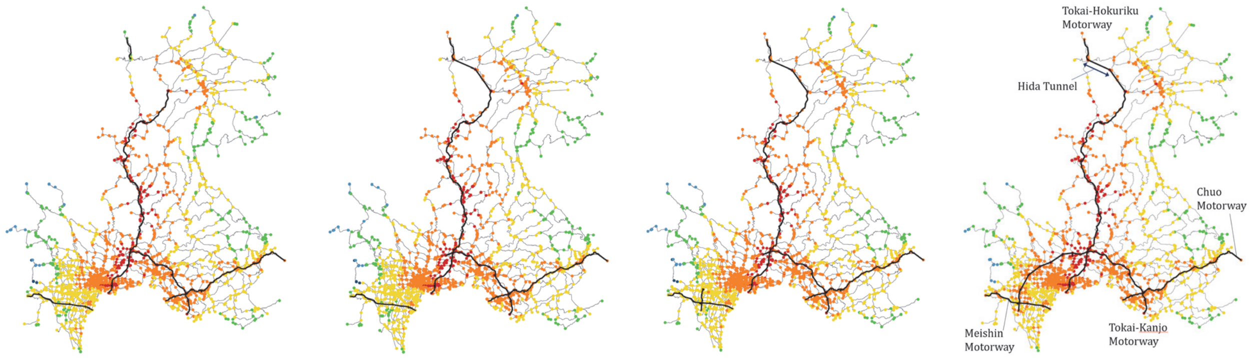
6.4.1. Application to multi-year road network

Figure 6.4 shows the distribution of the EC at eight-year points. The values of EC are normalised so that the size of the eigenvector becomes 1 for each network, and the EC takes a value between [0,1]. Since EC value often takes very small value, Figure 6.4 uses the logarithm value of the EC to distinguish the micro differences. The value of node i in Figure 6.4 is $\ln x_i^*$, which is always negative. Also, the nodes in every network are classified into six levels according to the same thresholds of the value of $\ln x_i^*$, as is shown in left upper part of the Figure 6.4. The bold black roads in the network indicate expressways.

The distribution of EC on the road network, nodes included in Level 1 are concentrated near the Meishin Expressway, and the nodes included in Level 2 are is mainly located around Gifu city. After that, the centrality of the nodes located in the north gradually increased according to the extension of Tokai-Hokuriku Expressway. In 1999, the centrality along the Tokai-Hokuriku Expressway becomes high, and most of the nodes included in Level 2 are located near the Tokai Hokuriku Expressway. Accordingly, the centrality of the nodes located in the western part of Gifu Prefecture, which has shown high centrality so far, is relatively decreased. Since 2005, the Tokai-Kanjo Expressway has been opened and extended. The effects of the Tokai-Hokuriku Expressway have spread to the east, and the centrality of nodes located around the existing Chuo Expressway increases as well as the nodes along the newly constructed roads. In 2024, the westward and eastward roads of the Tokai-Kanjo Expressways are scheduled to be opened. The range of nodes included in Level 2 is greatly expanded compared with the ones before all sections are opened, and most of nodes except for ones located at the edge are included in Level 3. From these discussions, road network improvements have a large effect on the connectivity in terms of road supply performance. The evaluation that considers capacity and length as weights makes it possible to distinguish the effects of high cost large road improvements with long-distance section. Hence, the spread of influence is large when the expressways with long distance and large capacity are opened.



(a) 1985 (b) 1990 (c) 1994 (d) 1999



(e) 2005 (f) 2010 (g) 2017 (h) 2024

Figure 6.4 The distribution of EC by the road networks in each year

Figure 6.5 shows the rate of log-scaled EC and the total weights in each year. The total weights are calculated by summing up the multiplication of length and capacity of links in each year. From the figure, from 1999 to 2005, the rate of EC changes drastically along with the major changes in total weight. The extension of Tokai-Hokuriku Expressway in 1999 increased the rate of nodes included in Level 1. Moreover, by the impact of starting operation of the Tokai-Kanjo Expressway in 2005, the rate of nodes with high centrality is maintained, and the rate of nodes included in Level 4, 5 and 6 decreases gradually. The major road improvement from 2005 to 2010 is the opening of the Hida Tunnel located in the north edge of Tokai-Hokuriku Expressway. Although the opening of Hida tunnel has a very small change in the total road capacity, its effect must be significant in terms of connectivity of roads. However, the evaluation based on the EC did not show much effects. This may be because we only evaluated the road network within Gifu Prefecture. Since the Tokai Hokuriku Expressway is connected to the Hokuriku Expressway in Toyama Prefecture, the opening of Hida Tunnel has a particularly large impact in the Hokuriku region. The opening of the all sections of Tokai-Kanjo Expressway from 2017 to 2024 affects the rate of EC. Although the weight has not increased so much, the rate of nodes included in Level 1 and 2 has increased. This expressway must contribute much to the improvement of road supply performance connectivity on the whole Gifu Prefecture.

Because EC values are normalised in the network of each year, increasing the value of EC over year may not represent the connectivity improvement. It can be seen that the dispersion of EC increases in the EC distribution shown in Figure 6.5 by the extension of the Tokai-Hokuriku motorway in 1999. A rate of nodes included in Level 1 increases the rate of nodes included in Level 3, 4 and 5 also increases. After that, the dispersion is resolved again as the construction of the Tokai-Kanjo Expressway was extended. The EC distribution is roughly uniform when the connectivity of road network is equal regardless of its level of connectivity. However, a multi-year analysis of actual road improvement based on the EC weighted by the distance and capacity can identify the magnitude of improvement impacts by new road constructions. The evaluation reveals increased network connectivity of road supply performance.

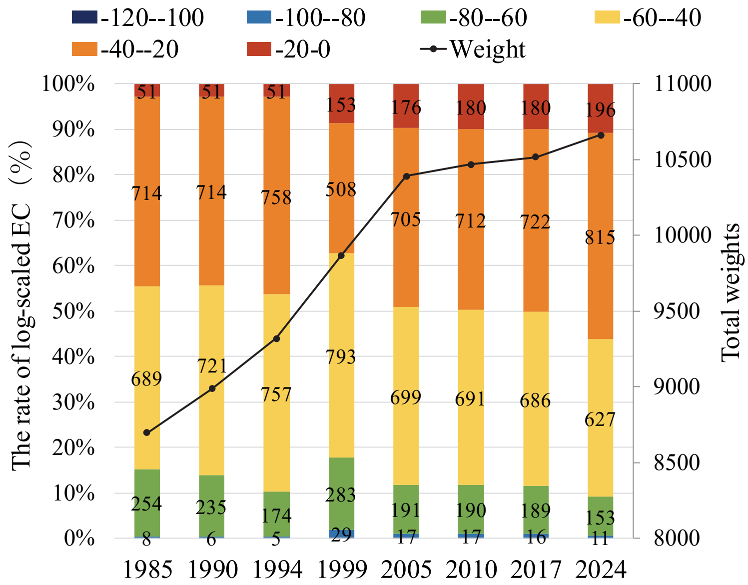


Figure 6.5 The transition of EC rate and weigh values

6.4.2. Effect of expansion to 4-lanes on Expressways

There still exist provisional 2-lane sections on the Tokai-Hokuriku and the Tokai-Kanjo Expressways. Figure 6.6 shows the current situation and plan for 4 lane expansions of expressways in Gifu prefecture. Here, the effect in the EC evaluation by the improvement of such provisional 2 lanes to 4 lanes are confirmed. To test the impact of link expansion, 5 improvement cases based on the road network in 2024 are prepared. Table 6.3 shows the calculation cases and some evaluation results. The expectation is the expected value of log-scaled EC on the distribution which has six class and class frequency 20 shown in Figure 6.7. The variance is based on the log-scaled EC of each case after improvement. Figure 6.7 illustrates the number of nodes included in each level which is divided into 6 based on log-scaled EC. In all cases, the number of nodes with high centrality increases and the number of nodes with low centrality decreases compared with the state of no link expansion. Since there are no nodes included in Level 5 and 6 in any cases, it can be said that the effect by expansion to 4 lanes in the entire road network is significant. Specific comparison examples are shown below.

Case 1 and Case 2 (Tokai-Kanjo Expressway, eastward and westward)

To verify which expansion of Tokai-Kanjo Expressway, eastward or westward, is more effective, results of Cases 1 and 2 are compared. Besides, the construction of the provisional 2 lanes road was started form the eastward. From the comparison with both cases, there is no significant difference between two cases, even though the total weights (Table 6.3) is larger in Case 2 (westward expansion). However, Case1 has slightly less nodes included in Level 3, and Case1 has more nodes included in Level 1. Moreover, the expectation of Case 1 is slightly larger than ones of Case 2. Therefore, it is more effective to expand the eastward of Case1. However, the variance of both cases are almost same. Although the preference improvement of Case 1 is more effective than Case 2, the difference is not significant. One of the reasons for this result seems to be that the eastward sections already have roads with 4 lanes. The effects of expansion to 4 lanes may increase by connecting to the already 4 lanes used section even if the expansion section to 4 lanes is short. Moreover, the effects of eastward improvements are expected to be greater than the evaluation in Gifu prefecture since the eastward section connects the Tokai-Kanjo expressway in Aichi prefecture which has 4 lanes completed.

Case 3 and Case 4 (Tokai-Hokuriku Expressway)

Two cases are compared for Tokai-Hokuriku Expressway. Case 3 represents an expansion from Shirotori IC to Hida-Kiyomi IC, which has finished from March 2019. Case 4 assumes that all sections on Tokai-Hokuriku Expressway has expanded to 4 lanes. Figure 6.7 shows that there is almost no difference between Cases 3 and 4, although they have large difference in the change of total road area. Also, the expectation and variance in Table 6.3 on both cases are almost same. It can be said that it is very effective to expand to 4 lanes up to Hida-Kiyomi IC, which was finished by March 2019. From this comparison results, the current improvement plan is a reasonable measure.

Case 5 (all expanded)

Finally, Case 5 expands all sections of Tokai-Hokuriku and Tokai-Kanjo Expressways to 4 lanes. The EC evaluation result of Case 5 has a tendency that there are many nodes with high centrality and few nodes with low centrality compared with the result in 2024. Compared to the other cases, the expectation increases and the variance decreases. It is clear that an example of Case 5 is effective for improving the supply performance of road network.

The verification by using 5 cases of 4-lane expansion did not include the construction of new roads. It was revealed that the EC weighted by the traffic capacity and length is useful even in the case that the network topology does not change. Also, the calculation of the partial capacity expansion showed the difference of impact magnitude and range. This is helpful for prioritising the improvement plans of road sections.

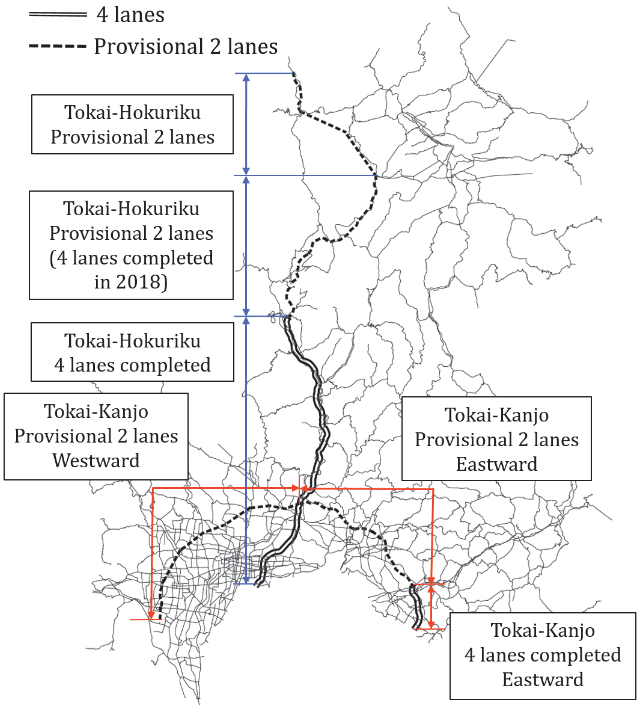


Figure 6.6 The situation and plan for 4 lanes

Table 6.3 Calculation cases for 4 lanes and evaluation results

	Target area for 4 lanes	The total weight after improvement	Expectation after improvement	Variance after improvement
Case1	Tokai-Kanjo motorway in eastward	109,318,056	-27.558	195.582
Case2	Tokai-Kanjo motorway in westward	109,399,286	-28.413	195.082
Case3	Tokai-Hokuriku motorway (Scheduled to 4 lanes in 2018)	109,100,682	-28.901	186.409
Case4	All Tokai-Hokuriku motorway	110,945,426	-28.846	186.039
Case5	All Tokai-Kanjo motorway All Tokai-Hokuriku motorway	116,003,250	-25.216	166.500

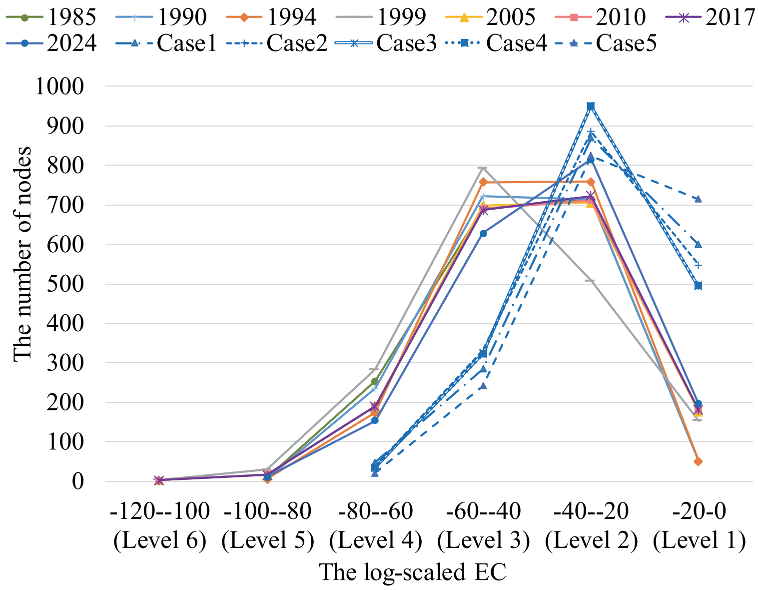


Figure 6.7 The distribution of log-scaled EC

6.5. The Impact of Road Improvements Based on Demand and Supply

Previous section verified the impact of road network improvement on the connectivity. The calculation result over years can show changes of connectivity by the road network improvement. The changes in supply performance by road improvements can be assessed immediately after the opening of the new road section. However, the effect of road investment may not emerge immediately because it takes a time for people to adjust their activity to new infrastructure. This section attempts to analyse the impact of road improvements on demand-side as well as supply-side. Focusing on the long-term changes of traffic demand and supply by EC measures, how traffic demand follows the supply of the network is discussed.

The purpose of the supply-side analysis is to evaluate the connectivity of road supply performance, and that of demand-side is to evaluate the concentration of traffic. This analysis uses both supply-side and demand-side indicators of traffic conditions as weights of EC to verify the difference between them. The supply-side indicator is traffic capacity (vehicle/day) and length (km) on each link, same as the previous section. The demand-side indicator is traffic volume (vehicle/day) on each link. The weights for supply-side is written as eq(6.1), and the weights for demand–side is represented as follows,

$$w_e = V_e, \tag{6.2}$$

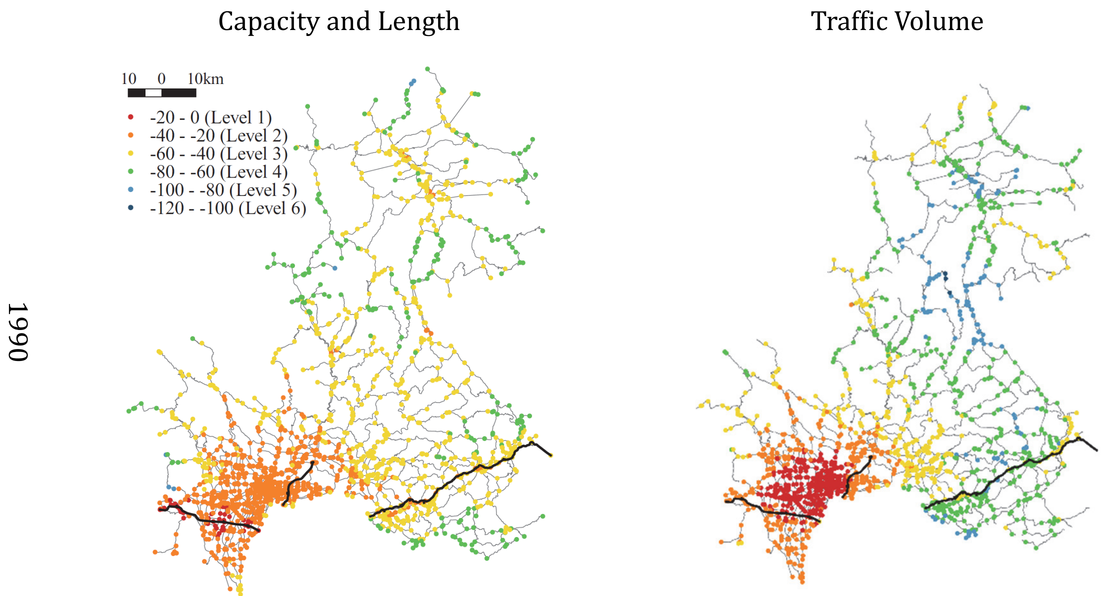
where,

V_e : A traffic volume of link e (vehicle/day).

On the demand-side, to evaluate the level of traffic concentration based on the actual usage, traffic volume is set as weights.

To verify the relationship between the road performance connectivity on the supply side and changes in traffic conditions on the demand side, the EC using weights on both sides is applied to the practical road network. The road networks to analyse in this section focus on 4-year points (1990, 1999, 2005 and 2010). The number of nodes and links are shown in Table 6.2. A Traffic volume, traffic capacity and length in each link used as weights are obtained from the national road traffic census survey for those years in Japan (MLIT, 1990, 1999, 2005, 2010).

Figure 6.1 shows the distribution of the EC at four-year points. On the supply-side evaluation, already described in 5.4.1, the area with many high centrality nodes changes by time along road network improvements. Especially from 1999 to 2005, it is clear that the road network connectivity has become stronger as a result of the new constructions of Tokai Kanjo Expressway, and it seems that an eastern part of Gifu has become more connected with the central part. However, in 2010 the network connectivity did not change much. On the other hand, it is interesting to say that the demand-side based on traffic volume does not have large changes in the centrality distribution by the constructions of new roads. The area where nodes with the highest level (in red) are located in 1990 is the most urbanised area of Gifu Prefecture, and this tendency does not change much over time from 1990 to 2005. The EC values then changed drastically from 2005 to 2010. It may be because of the lagged effect; the impact of previous road investment may have appeared in these periods, or the instantaneous effect; the impact of road investment between 2005 and 2010 is very large. The biggest change between then is the opening of Hida Tunnel, and the centrality on demand-side in the northern area has increased. Therefore, the instantaneous effect may have happened during these periods in the northern area.



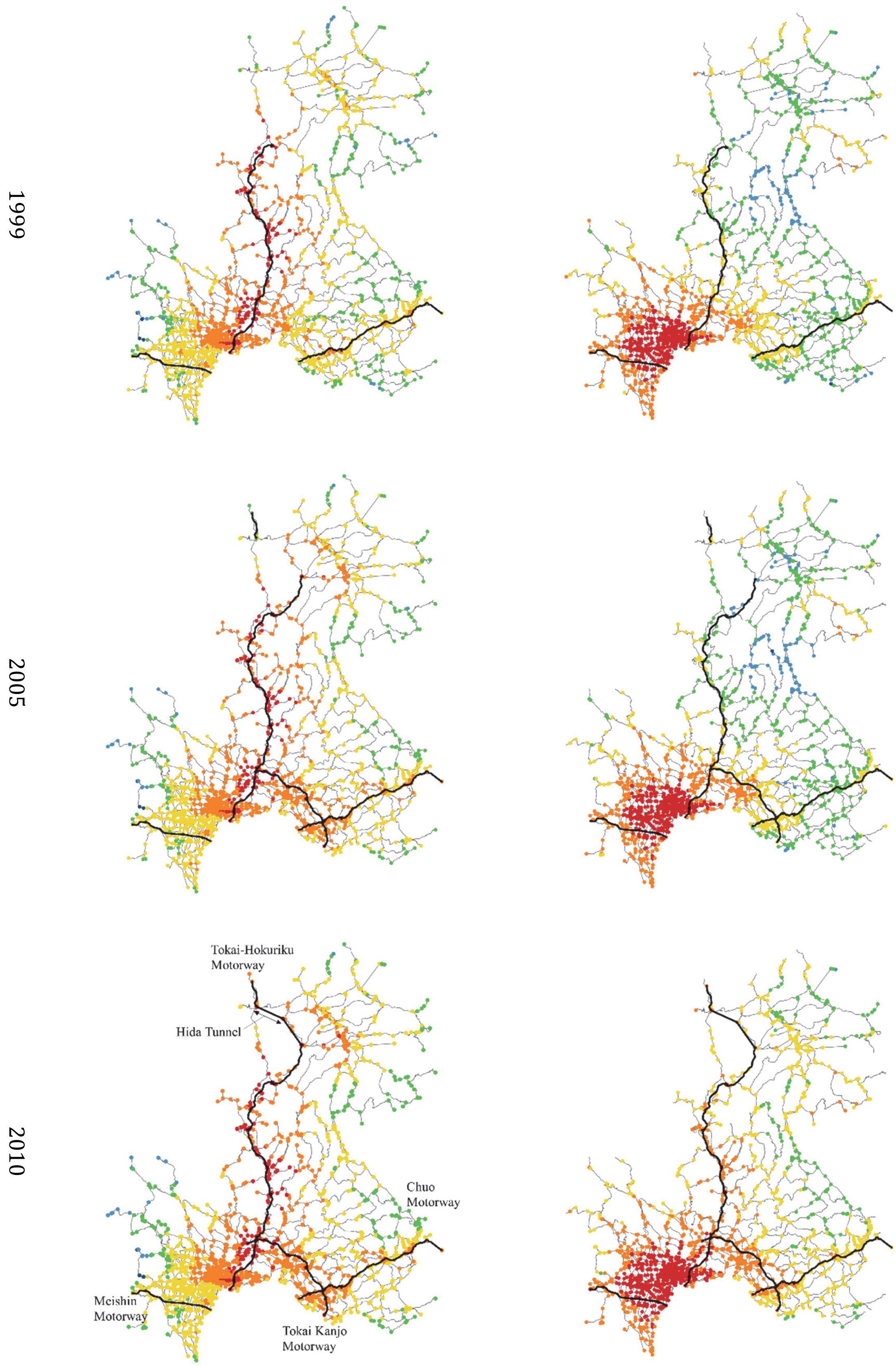


Figure 6.8 The distribution of EC by both weights in the long-term road networks

Further the lagged effect is examined. The scatter plot in Figure 6.9 shows the difference between supply-side and demand-side evaluations at the same nodes in same years. Different colour represents the values in different years. For these plots, the slopes of regression lines passing through the origin of coordinates are 0.808, 0.834, 0.737 and 1.002 for years 1990, 1999, 2005 and 2010, respectively. Therefore, the nodes with high centrality evaluations on the supply-side are often evaluated as low on the demand-side. This means that the supply-side connectivity has been improved, but from the demand-side the connectivity has not become as much as it is in the supply-side. These characteristics are different depends on the year. Table 6.4 shows the correlation coefficients among demand-side and supply-side EC values. It is interesting to say that, although two EC measures are calculated with different weights, there may have strong relationship up to 0.764 (between supply 1990 and demand 2010). This suggests the strong relation of demand and supply sides. From the correlation analysis among different time periods we can also examine the causal relationship; whether road network is improved because of the increase in demand, or the demand increased by the result of road improvements. If the road network was improved because of the increase in demand, there should be a correlation between past demand and future supply. The correlation coefficients continually decrease when we evaluate the value from left to right direction, suggesting that such relationship does not occur. It is unlikely that the road has improved with increasing demand. Conversely, the idea that the demand is generated as a result of road network improvement is considered. If this idea is true, there should be a correlation between past supply and demand in subsequent years; the correlation coefficients should continually increase when we evaluate the value from top to bottom direction. According to Table 6.4, in most cases the correlation coefficients continue increasing as the demand year progresses. Hence, it can be said that there is a lagged effect of road investment; demand-side connectivity increases as supply-side connectivity becomes better by road network improvements.

To examine the change of EC distributions by different year, Figure 6.10 shows the number of nodes included in each level classified by the log-scaled EC values. From 1990 to 2005, although the number of nodes included in Level 1 is large in demand-side, there are many nodes included in Level 4, 5 and 6. In supply-side, while the number of nodes included in Level 1 is small, the nodes included in Level 4, 5 and 6 are also small compared with the demand-side. Although it is a simple comparison of frequency distributions, Figure 6.10 shows that there is a difference between the demand-side distribution where the high centrality nodes increase significantly and the supply-side distribution where the low centrality nodes decrease by the road network improvements. The possible reason for this is that the connectivity of road supply performance evaluated by the supply-side EC and the concentration of traffic evaluated by the demand-side EC represent different aspects of the network. Gifu Prefecture has small cities and towns throughout the network. In terms of demand, the evaluations of EC show almost no impact of such places and the nodes with lower level may remain, so there is little 'induced' demand by new road investment. On the other hand, in terms of supply, the evaluations of EC show that road network improvements have contributed to improve connectivity to such places. As for 2010, the EC distribution drastically changes, and it resembles the supply-side distributions. As is discussed, it may be because of the lagged effect, but other factors such as depopulations, increase of tourist demand and so on should be carefully examined.

The application of changes in the practical road networks for 20 years revealed that the impact of road improvements differs on the supply-side and demand-side. In the case of Gifu Prefecture, the evaluations on demand-side gradually increased according to the improvement of road supply performance connectivity. Also, the change of EC distribution by different year shows the possibility of differences in the aspects of the road performance connectivity evaluated by supply-side and the concentration of traffic evaluated by the demand-side.

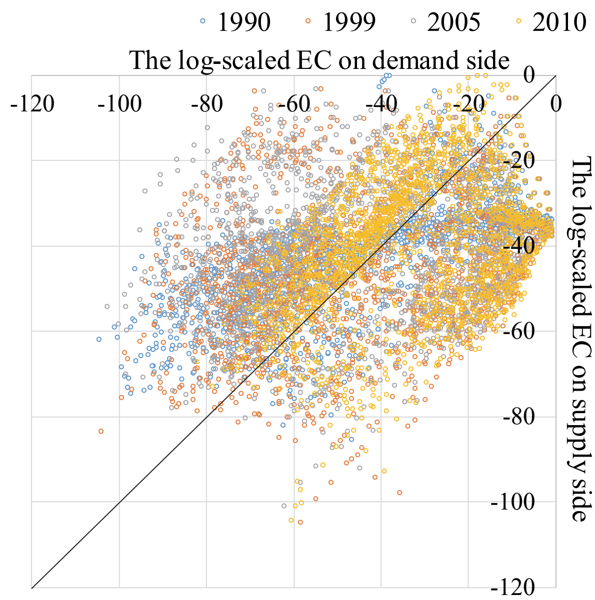


Figure 6.9 Scatter plot in EC on both weights

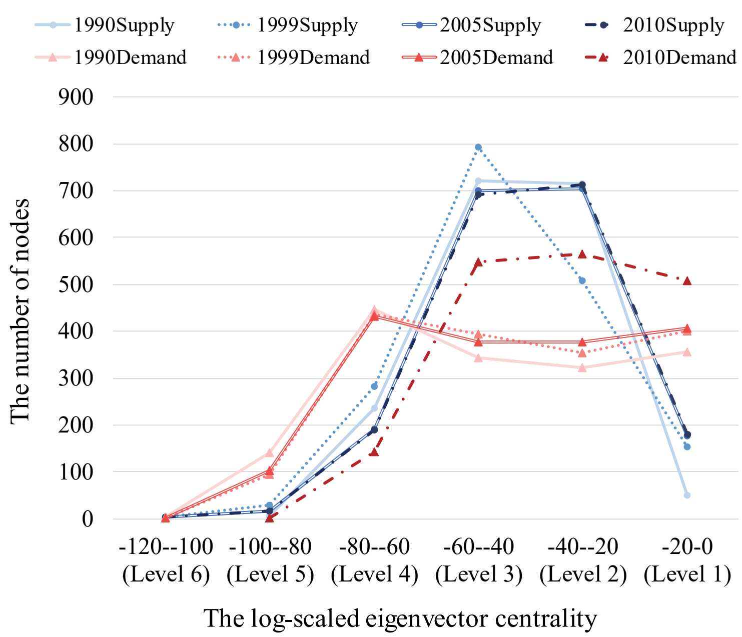


Figure 6.10 The number of nodes in each level

Table 6.4 Correlation coefficients of both weights

		Supply			
	Year	1990	1999	2005	2010
Demand	1990	0.720	0.300	0.140	0.131
	1999	0.715	0.289	0.154	0.140
	2005	0.728	0.289	0.157	0.143
	2010	0.764	0.478	0.361	0.349

6.6. Concluding Remarks

This chapter analysed EC in long-term road networks to confirm the impact of road improvements on the connectivity evaluation. The road network in Gifu prefecture for 30 years has been invested year by year. The road network from 1985 to 2024 are evaluated by the EC weighted by traffic capacity and link length. The result verified that the impact of improvements on the road supply ability can be identified by connectivity analytics. Moreover, the relationship between supply-side and demand-side is analysed by comparing the EC evaluation by weights according to both sides. The result revealed that the impact of road improvements differs on the supply-side and demand-side and the ones on demand-side tends to be delayed. Also, the difference in the changes of trend by network improvements was confirmed. The knowledges obtained from the analysis results are summarised as follows.

The connectivity of road supply performance enhances certainly according to the road improvements. The distribution change of EC evaluations showed the expansion of the high connectivity area by road improvements. In particular, the spread of the impact is large when the expressway with long length and large capacity are connected, like the Tokai-Kanjo Expressway connected Tokai-Hokuriku Expressway and Chuo Expressway. Moreover, the opening of the all Tokai-Kanjo Expressway in 2024 improved the distribution of EC evaluations even though the weight has not increased so much. This means that the opening of all Tokai-Kanjo Expressway has significant effect for the connectivity of supply ability on the whole of Gifu Prefecture.

As an example of capacity expansion rather than the construction of a new road, the impact of the change from provisional 2 lanes to 4 lanes road was verified. A road section that greatly contributes to the network connectivity regardless of the road area after improvements were shown. Furthermore, it was found that the connectivity evaluation results differ depending on the range of the 4 lanes road completed. Hence, this analysis is helpful for prioritising the improvement plans.

The appearance of impact on the connectivity evaluations by the road improvement depends on the demand-side and supply-side. It was concluded that the impact of road improvement on demand-side may have lagged effect by the analysis of these relationships. In the case of Gifu prefecture, the evaluations on demand-side gradually increased after the improvement of road supply performance connectivity. Focusing on the areas where the tendency of the effects by road network changes greatly may lead to finding useful information on improvements. In addition, the target network in this chapter was limited in the Gifu prefecture. There may be improvements that can produce more effective results when the target area is extended to the surrounding prefecture. It is necessary to consider the road network outside the Gifu prefecture to verify such effect.

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Chapter 7

Conclusions

7.1. Summary of Contributions

This study analysed vulnerability and connectivity of road networks by using network topological analytics. The proposed method based on the graph theory enables to evaluate road network from different aspects by setting different link weights. Compared with conventional road network evaluation methods with a large computational load, the proposed method can easily and quickly obtain the road network evaluation results. This leads to an evaluation independent of the level of detail of the network. The findings obtained in each chapter are summarised below.

Chapter 2 summarised related researches on network robustness evaluations. In the existing studies, the traditional road network evaluation methods using traffic assignment and route enumeration is difficult to apply to large-scale networks due to the computational load. Therefore, it is necessary to process such as the link aggregation and restriction of target area. Based on these tasks of existing studies, it was found that the evaluation by network topological vulnerability is effective for analysis independent of network size. However, there is no clear knowledge about the relationship between the evaluation by network topology indicators and traditional vulnerability analysis. Also, the relationship between weight settings and evaluation indicators is required to analyse to use the network topology indicators for road network evaluations. The topological network evaluation methods make it possible analyse the detailed network in wide range areas. In addition to the viewpoint of evaluation revealed by traditional methods, it may be possible to give new findings by using detailed networks. From these backgrounds, the objective of this study is to add a new perspective to the field of road network evaluation by showing that analytics using network topological indicators can extract critical part that was not found due to the limitations of conventional methods.

Chapter 3 organised the measured value of traffic function used as weights and the objectives of evaluation by network topological analytics based on those weights. The spectral partitioning method and eigenvector centrality method were introduced as methods for analysing weighted networks by using graph theory. Both methods were calculated in a simple weighted network and showed the characteristics of each method. The proposed analysis method with weighted network heavily depends on the setting of the weight which is determined by what you want to evaluate. Therefore, the relationship between challenges to be analysed and evaluated and the measured values of traffic function as weights were summarised. This helps to interpret each evaluation result.

Chapter 4 proposed to use capacity weighted spectral partitioning method to identify critical potential bottlenecks. When the traffic demand data is available from the traffic assignment or onsite surveys, the bottleneck can be identified. However, in reality, there are many cases where such accurate demand data is not available at disaster or in the future planning stages. It is very important to identify the parts that are likely to be bottlenecks from topological point of view for extracting potential vulnerable parts in terms of traffic supply performance. The comparison with the conventional vulnerability evaluation method indicated the usefulness of the proposed method. The validity of the method for large-scaled networks was also tested, and it took around 200 minutes to obtain the result for the largest network with 75,359 links. Finding critical bottlenecks easily with the proposed method can immediately provide information of the critical parts that appears by changing the network based on road improvement plan. Furthermore, the spectral partitioning method with other weight settings was applied to the road network according to the evaluation purpose. It was shown that evaluation according weight settings was possible.

Chapter 5 introduced the capacity weighted eigenvector centrality measure as a network topological indicator for connectivity evaluation considering the ease of link disruption. EC identified a relatively weakly and strongly connected part in the network. Same with the spectral partitioning method, the usefulness was verified by comparison with the conventional evaluation method, and the operation in the large-scaled road network was confirmed. Moreover, other weighted EC also analysed the road network. As combinations of these evaluations, node clustering with EC obtained by several weights represented the geographical and performance characteristics of the network. Moreover, the detailed network analysis including small city roads could identify characteristics that cannot be found in the approximate network. For example, there are areas with low connectivity even near large-capacity roads, and identification of easily isolated central city areas that are extremely low connectivity with external cities. These are the contributions of the proposed method that can be analyse detailed networks.

Chapter 6 confirmed the impact of road improvements on the connectivity by analysing capacity-length weighted EC for 20 years. It was shown that the road network improvement in Gifu prefecture for 20 years has definitely increased the road supply performance. Furthermore, link capacity expansion case studies verified which section of the expansion would lead to improve the connectivity of road supply performance for the whole of network. The results showed an effective and efficient improvement section for increasing the connectivity of road supply performance. In this way, the change of connectivity indicators suggests that the proposed method is helpful for prioritising the improvement plan. It was further found that the appearance of impact of road improvement onto the connectivity evaluation was different for the demand-side and supply-side analysis. In Gifu prefecture road network, it was concluded that the impact of road improvement on traffic demand-side may have time lags to emerge.

In the vulnerability analytics, the proposed method could identify the critical potential bottlenecks in a wide and detailed network. In the connectivity analytics, the proposed method evaluated the effect of supply performance to each region by the large capacity roads connecting inter-city, and the connectivity to the outside of area. In conclusion, the contributions of this study are summarised as follows;

Significance of adopting network topological indicators for road network evaluation

This study showed the significance of vulnerability and connectivity evaluation method by road network topological indicators. The methods evaluating the characteristics of network from the structure of connections have been especially studied in recent years, and it is expected to be developed continually. Therefore, the achievement that the road network performance can be evaluated by the network topological indicators will lead to introduction of a new direction for the future road network evaluation methods. The network evaluation methods become more and more simple and diverse, as the optimal network topology according to each objective and network evolution mechanism can be identified. It is a great progress to show that road networks can be included in such a discussion.

Determination of evaluation target by changing weight settings

In the evaluation of vulnerability and connectivity, various weight settings were adopted in the network topological analytics. These weight values are determined according to what we want to evaluate. In other words, changing the weight setting allows for a wide range of evaluation by the same network topological indicator. This study used the measured values of traffic function as weights, however the weight option is very wide. Also, it is possible to set the weight value combining some features. The analytics with several weights showed the possibility to evaluate for different targets.

Applicability to detailed and large road networks

In chapter 4, the analysis of a large network with around 400,000 links were operated easily. The proposed method can thus be applicable, regardless of the level of detail and size. The capability to evaluate the connectivity in a wide-scale and detailed network including small city roads give an advantage that roads with different roles depending on the rank can be evaluated simultaneously. For example, intercity expressways and high capacity national roads are connected and accessible, but there are areas that are easily affected by link disruptions and failures around the residential area. In such case, the supply performance by the large capacity roads cannot be sufficiently secured because the connectivity is weak before reaching the large capacity roads from the residence place. In other words, the connectivity is greatly increased by some road improvements for such areas. It is possible to find candidates for efficient and effective road improvements by evaluating the all rank roads simultaneously. Moreover, the size of the network can be changed freely. For example, evaluation for the whole of the prefecture, evaluation for each city, evaluation only for larger than national roads, evaluation including city roads and so on. Because the analysis target can be determined without depending on the network resolution, there is no need to consider link aggregation methods even if you want to analyse a wide area.

Evaluation of Isolation risk of small villages at the disaster

Gifu Prefecture has urban and mountainous areas. Especially, the analysis considering the city roads in mountainous area provided interesting results. The evaluation of connectivity of the detailed network emphasises each small village in mountainous area. In the case of where these villages are connected by roads with sufficient capacity such as national roads, the connectivity is propagated to other villages.

However, the connectivity evaluations found some isolated villages where they have no connection with surrounding villages. Since these villages are connected to the outside of the study area by roads with very few and small capacity, the risk of isolation may not appear in the evaluation of the number of routes. Nevertheless, it can be said that the essential connectivity of the villages is extremely weak. It may be very useful in disaster countermeasures such as relief material transportation plans.

Effects of using open source network data

The open source network data has been provided recently (ex, open street map, GIS), and anyone can easily obtain data on a large-scaled road network. Establishing a method that can analyse open source network without processing will increase the value of providing network data. For example, real estate business related to land use, tourism business, there are many fields that are affected by road services which are infrastructure facilities closely related daily life. The road network performance has the potential to influence decision making in various fields as one of the elements to understand the characteristics of places. In that respect, it is very effective that the process from obtainment network data, analysis, evaluation is easy and quick.

7.2. Future Works

Lastly, the further challenges of this study are summarised.

Selection of the target area to evaluate

The target area of this study is Gifu prefecture. In the verification of the impact of road improvements in Chapter 5, the impact of Hida tunnel construction in the northern area of Gifu prefecture and the link capacity expansion in that area was not large. The reason for this may be that the areas where connectivity is increased by these road improvements are large not only in Gifu prefecture, but also in Toyama and Ishikawa prefecture. It is necessary to select the target area considering where the improvement to be evaluated will contribute.

Relationship between vulnerability/connectivity evaluation and geographical condition, social situation of cities.

Both two proposed methods of vulnerability and connectivity were applied to large-scaled network of 6 cities. As a result, the distribution of connectivity evaluation by EC and the partitioning are very different from city to city. Road characteristics may depend on factors such as urban land use, geographical structures and so on. Understanding such differences of characteristics may include valuable information related to land use and urban policies, that may also contribute to social sustainability.

Vulnerability analysis by the proposed connectivity evaluation method

The connectivity evaluation method using EC was proposed. It is considered that the vulnerability of the network can be analysed by evaluating how much connectivity is decreased by the link disruption. Also, this method is suitable for repeating the calculation because the computational load is significantly small. Therefore, many combination problems of the link disruption pattern can be applied. It will be a new indicator to define the vulnerability by the change of connectivity evaluation results of whole network.

Relationship of other factors on demand change

It was concluded that the impact of road improvement on demand-side may have lagged effect by the analysis of the relationship between demand and supply. In the case of Gifu prefecture, the evaluations on demand-side gradually increased after the improvement of road service connectivity. However, the reason for the demand change is not limited to the impact of the road improvements. It should include many other social factors such as depopulation, land use change and tourism demand increase. The verification of the demand effect from these factors makes more clear the impact of road improvements.

Is this road improvement really necessary? It is not easy to answer this question because the necessity of road improvement has a deeply enmeshed relationship with regionality of the target area, the situation between adjacent cities, disaster risk, social situations and so on. In the transport system for social sustainability, if it asked whether many roads should be built, there may be regions where roads should be improved and regions where it should not be improved. It is difficult to understand it only by cost-effectiveness and efficiency. In the future, the situation where the direction of road improvement is determined by the unified standard based on each country and local government will change. In this paper, the usefulness of an evaluation method for analysing a road network using any weight was indicated. By setting the weight according to each objective, it is possible to consider those factors in the evaluation based on the network topology. By utilising this knowledge will lead to evaluation of road network that considers many factors comprehensively. Because the weight values can be set flexibly, it is possible to try the evaluation based on not only limited aspects but also multiple viewpoints. Numerous factors are considered to be largely related to road improvement and others are not. In order to clarify these various factors and criteria, the road network evaluation method with small calculation loads and flexible condition setting will greatly help to construct new evaluation standards different in each target area.

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まず，岐阜大学工学部社会基盤工学科 倉内文孝教授には，2011 年の岐阜大学入学時から助言教員としてお世話になり，9 年間にわたってご指導いただきました．特に 2014 年に交通システムデザイン研究室に配属されてから今日までの 6 年間は研究活動に関するご指導から日常生活まで，すべての面において大変お世話になり，深く感謝いたします．研究室配属時，希望者多数のなか“あみだくじ”によって導かれたご縁でありましたが，学部生，修士課程，博士課程と長くにわたり学ばせていただいたことは私にとってすべてが大事な財産です．日々の研究遂行や論文執筆はもちろんのこと，学会やセミナー，国際会議など多くの経験の機会を存分に与えていただいたこと，ここに感謝の意を表します．また，発表練習や論文添削では毎度余裕がない状態で教えを乞う私に対し，ご多忙の中いつも迅速かつ丁寧に対応いただき，どんな状況においても的確なアドバイスを与えてくださいました．将来自分が指導をおこなう立場となったとき，常にこのような姿勢でありたいと思わせられる日々でした．これまでのありとあらゆるご指導すべてに深く感謝するとともに，今後ともご厚誼を賜りたく存じます．

岐阜大学工学部社会基盤工学科 髙木朗義教授には学部 4 年時に地域システムデザイン研究グループに所属してからご指導いただきました．特に学部生，修士課程在学時には減災教室や防災キャンプ，自主防災訓練など実際に街に出て住民の方々とともに知識を共有し，実践的に学ぶという機会を多く経験させていただきました．学術的な研究の成果をどのように住民の方へ伝え，役に立つ形として社会へ貢献するのかという過程を多く学ばせていただいたことにより，常に社会における使われ方を想像して研究するという意識が身についたと思っております．博士課程 3 年目となった最終年には髙木先生の居室が C 棟に移られたこともあり，より多くの会話の機会が得られ，ちょっとしたお喋りの時間はとても楽しい息抜きの時間でもありました．締切の近い日の深夜，学生部屋をふらっと訪れ声をお掛けいただいたことも何度かありました．切羽詰まった時間帯の励ましのお言葉は「よし，もう少し頑張ろう」という勇気になるとともに，髙木先生の体力にはいつも驚かされました．長年のご指導・ご鞭撻，心より感謝申し上げます．そして，今後ともどうぞ宜しくお願いいたします．

岐阜大学工学部社会基盤工学科 出村嘉史准教授には学部 4 年時に地域システムデザイン研究グループに所属してからご指導いただき，特に研究グループ活動である発表ゼミや GURSPs などでは自分では想像のできない方向からの視点によるアドバイスを多くいただきました．発想の柔軟性がいかに重要であるかを幾度となく思い知ることができました．一度，出村先生が普段から使用されているノートを拝見させていただいたことがあり，思ったこと，感じたこと，考えたことをとにかく書き

出してみることの大切さを感じました．研究内容や美殿町ラボ，柳ヶ瀬まち会社などの研究室を超えた活動の起点はこの書き出しであることを実感したうえで，今後の自分の研究者人生においてもとにかく書き出してみることを心掛けようと思うきっかけとなりました．6 年間，多くのタイミングで的確な助言をいただき，大変お世話になりました．深く感謝いたします，今後ともどうぞ宜しくお願いいたします．その際には面白いと感じてもらえるようなアップデートとともにお会いできるよう邁進して参ります．

岐阜大学工学部社会基盤工学科 宮城俊彦特任教授には交通システムデザイン研究室に配属されてから発表ゼミや GURSPs などの発表機会に加え，年に数回宮城ゼミを開催していただき，多くの助言をいただきました．修士課程 2 年時，私にとって初めての国際会議となった International Symposium on Dynamic Traffic Assignment にご一緒させていただいたことはとても思い出に残っております．常に前向きにかつ貪欲に学ぶことの大事さを教えていただきました．博士課程進学後には，特に個別ゼミの場において，いつも多方面から多くのヒントとなる情報を与えていただき大変感謝いたします．宮城先生のゼミの後には毎回やる気がかき立てられ，いつも以上にワクワクした気持ちで研究に取り組むことができました．本学位論文の副査をお受けいただき，研究内容から論文構成などあらゆる面において懇切丁寧なご指導をいただきましたこと，心より感謝申し上げます．そして，今後ともどうぞ宜しくお願いいたします．

The University of Sydney，Michael G H Bell 教授には，2018 年 3 月から 9 月の半年間 Institute of Transport and Logistics Studies において研究する機会を与えていただき，研究グループに温かく迎え入れてくださったこと，深く感謝の意を表します．Bell 教授のもとシドニー大学滞在中におこなった研究成果は本学位論文において重要なパートを構成するものとなり，この半年間の成果や経験は本研究の基盤となりました．また，研究内容のみならず海外における生活経験や同年代研究者との繋がりという面においても，私にとって非常に貴重な時間であったと感じています．Bell 教授の研究グループメンバーとして，Supun Perera 氏，Kam Fung Cheung 氏，Jing-Jing Pan 氏，Shengda Zhu 氏には日常生活から研究まで多くのご助力をいただいたおかげで充実した留学期間となりましたこと，感謝申し上げます．また滞在期間中に関わっていただいた皆様には，その後も本研究に関して常に気を配っていただき，誠に感謝いたします．今後ともどうぞよろしくお願いいたします．

北海道大学大学院工学研究院 杉浦聡志准教授には，特に学部 4 年時に地域システムデザイン研究グループに所属してから，岐阜大学に在籍された昨年までの 5 年間，あらゆる面において大変お世話になりました．日々の小さな疑問質問や悩みなど，気軽に相談できる存在でいてくださったことにより，精神的に救われたことは少なくありません．心より感謝申し上げます．私が研究室に配属された当時博士課程の学生として先輩だった方が研究者として生きていく変化の過程を間近で感じることができたおかげで，自分もしっかり頑張らないとという思いを抱き，目標となる道筋を示してくださいました．北海道大学に異動されたあとも，本学位論文の執筆にあたり中間発表や公聴会などの際には励ましの言葉をいただき，非常に心強かったです．今後もご迷惑をおかけすることもあると思いますが，何卒宜しくお願いいたします．

岐阜大学工学部社会基盤工学科 地域システムデザイン研究グループに在籍した学生の皆様には大変お世話になりました．歴代の皆様のおかげで，困ったことがあれば協力して助け合い乗り越えるということを学び，楽しく賑やかな研究室生活を送ることができました．特に修士課程 2 年にあたる明光就平さん，御村まゆさん，柴田貴文さん，福井彩水さん，堀口拓治さん，北田寛明さん，栗本明宜さん，牧野広誉さん，王思逸さんとは 3 年間研究室生活をともに過ごし，多くの思い出ができました．3 学年の違いがある私に対しても分け隔てなく接していただいて感謝しております．辛いことがあったとき, 苦しい時期, 皆と他愛もない話をする時間が気分転換になり何度も元気をもらいました．これからの皆様のご活躍を心より祈念しております．数年後，皆で集いたくさんのお話ができる機会を持つことができればとても嬉しいです．

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