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A Synthesis of an Optimal File Transfer on a File Transmission Net

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SUMMARY A file transmission net N is a directed communication net with vertex set V and arc set B such that each arc e has positive cost $c_a(e)$ and each vertex u in V has two parameters of positive cost $c_v(u)$ and nonnegative integral demand $d(u)$. Some information to be distributed through N is supposed to have been written in a file and the written file is denoted by J, where the file means an abstract concept of information carrier. In this paper, we define concepts of file transfer, positive demand vertex set U and mother vertex set M, and we consider a problem of distributing $d(v)$ copies of J through a file transfer on N from a vertex v_1 to every vertex v in V. As a result, for N such that $M \subseteqq U$, we propose an $O(nm+n^2 \log n)$ algorithm, where $n=|V|$ and $m=|B|$, for synthesizing a file transfer whose total cost of transmitting and making copies of J is minimum on N.

key words: *minimum spanning tree, shortest path, vertex cost, arc cost, vertex demand*

1. Introduction

We consider a directed communication net, called a file transmission net, which is a connected directed graph N with vertex set V and arc set B such that (1) if $(x, y) \in B$, then $(y, x) \in B$, (2) with each vertex $u \in V$, a positive integral weight $c_v(u)$ as well as a nonnegative integral weight $d(u)$ is associated, (3) with each arc $e \in B$ a positive integral weight $c_a(e)$ is associated, and (4) for each arc (x, y), there holds $c_a((x, y)) = c_a((y, x))$. It should be noted from (1) and (4) that N is represented by an undirected communication net.

Suppose that some information to be distributed through N has been written in a file and the written file is denoted by J, where the file means an abstract concept of information carrier. Then we consider a problem of distributing copies of J through N from a vertex v_1 to every vertex. In this situation, $c_v(u)$ means the cost of making a copy of J at a vertex u, and $c_a(e)$ means the cost of transmitting a copy of J through an arc e. The demand at u, denoted by $d(u)$, is the number of copies of J needed at u. On N, we define a file transfer with which (1) J is first given to v_1 from the outside of N, (2) copies of J are transmitted through arcs, and (3) $d(u)$ copies of J are taken out of each vertex u to the outside of N. We introduce concepts of positive demand vertex set U and mother vertex set M, and for N such that $M \subseteqq U$, we propose an $O(nm+n^2 \log n)$ algorithm, where $n=|V|$ and $m=|B|$, of synthesizing an optimal file transfer by which we mean a file transfer whose total cost of transmitting and making copies of J is minimum on N. The definition of U and M will be made in the preliminaries.

2. Preliminaries

For basic graph-theoretic terms and concepts used in this paper, refer to those in Ref. (1). Let V and B be the sets of vertices and arcs of a file transmission net N. Throughout this paper, any arc is directed but any edge is undirected, and for any arc $e=(x, y)$ and any function f on B, $f((x, y))$ is simply denoted by $f(x, y)$. Let $A(v) = \{w \in V | (v, w) \in B \text{ or } (w, v) \in B\}$ for a vertex v in V. In this paper, every path is simple unless otherwise stated. We simply say a u-w path instead of a directed path from a vertex u to a vertex w in N. The set of u-w paths in N is denoted by $\boldsymbol{P}_{u,w}$. For any path P in N, $V(P)$ and $B(P)$ denote the vertex set and the arc set, respectively, on P. For a path P in N, an x-y path P' is called the x-y subpath of P if $V(P') \subseteqq V(P)$ and $B(P') \subseteqq B(P)$. The total cost of all arcs on a path P is denoted by $c(P)$. If a u-w path P in N satisfies $c(P) \leqq c(P')$ for every other u-w path P' in N, then P is called a minimum cost u-w path in N. The set of minimum cost u-w paths in N is denoted by $\tilde{\boldsymbol{P}}_{u,w}$. In the following, for any two vertices u and w, we simply denote $c_{u,w}$ instead of $c(P)$ for any path P in $\tilde{\boldsymbol{P}}_{u,w}$. For a directed or undirected graph G, $B(G)$ and $E(G)$ denote the arc set and the edge set, respectively, of G. If with all arcs on a path P a uniform number k is associated, then P is called uniformly weighted with k. For a set $\boldsymbol{P}$ of all uniformly weighted paths on N, we superimpose all the paths in $\boldsymbol{P}$ to form a net $N(\boldsymbol{P})$ in such a way that (1) the vertex set of $N(\boldsymbol{P})$ is V and (2) the weight of each arc (x, y) of $N(\boldsymbol{P})$ is the sum of the corresponding arc weights of all the paths containing (x, y) in $\boldsymbol{P}$. Then $N(\boldsymbol{P})$ is called the superimposition net of $\boldsymbol{P}$. For a function f on B, if every arc e on a path P satisfies $f(e) > 0$, then

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P is said to be f-positive.

As is stated in the introduction, J is a file in which some information to be distributed through N has been written, $c_v(u)$ is the cost of making a copy of J at a vertex u, $c_a(e)$ is the cost of transmitting a copy of J through an arc e, and for every arc (x, y) in B, $c_a((x, y))$ is simply denoted by $c_a(x, y)$. Also, $d(u)$ is the number of copies of J needed at a vertex u, called the demand at u. N is sometimes denoted by $N=(V, B, c_v, d, c_a)$.

Let U be the set of vertices where some copies of J are practically needed, i.e., $U=\{v\in V|d(v)>0\}$. We call U the positive demand vertex set of N. Throughout this paper, U means the positive demand vertex set of N and $\boldsymbol{Z}_+$ denotes the set of nonnegative integers.

Suppose that the original of J is first given to some vertex v_1 of N from the outside of N. We take out $d(v)$ copies of J from each vertex v through a file transfer defined below:

Definition 1: In $N=(V, B, c_v, d, c_a)$, let ψ be a function from V into $\boldsymbol{Z}_+$ and let f be a function from B into $\boldsymbol{Z}_+$. Then $D=(\psi, f)$ is called a file transfer on N if ψ and f satisfy the following two conditions:

(C1) The conservation at vertex; there hold

$$\sum_{x\in A(v)} f(x, v)+\psi(v)=\sum_{y\in A(v)} f(v, y)+d(v) \quad (v\in V\backslash\{v_1\}),$$

$$1+\sum_{x\in A(v_1)} f(x, v_1)+\psi(v_1)=\sum_{y\in A(v_1)} f(v_1, y)+d(v_1).$$

(C2) The distribution of J; for any supply vertex v with respect to D, there exists an f-positive v_1-v path in D, where a vertex v is called a supply vertex with respect to D if $\psi(v)>0$. □

Note here that, in (C1), the original of J given to v_1 is regarded as one of $d(v_1)$ copies if $d(v_1)>0$. Since a file transfer D satisfies (C1), $d(v)$ copies of J can be taken out from a vertex v to the outside of N. Any file transfer on N has the following property.

Lemma 1: A file transfer $D=(\psi, f)$ contains an f-positive v_1-u path for any vertex u in U on $N=(V, B, c_v, d, c_a)$.

Proof: If $u=v_1$, then clearly holds this lemma. Hence we consider a vertex in $U\backslash\{v_1\}$. For a vertex u in $U\backslash\{v_1\}$, let $V'=\{v\in V|$ there exists an f-positive v-u path on $D\}$. If $v_1\in V'$, then clearly holds this lemma. Hence we assume that $v_1\notin V'$. Summing equations of (C1) over $v\in V'$, we get

$$\sum_{\substack{x\in V\backslash V'\\ y\in V'\\ (x,y)\in B}} f(x, y)+\sum_{v\in V'}\psi(v)$$

$$=\sum_{\substack{x\in V\backslash V'\\ y\in V'\\ (x,y)\in B}} f(x, y)+\sum_{v\in V'} d(v).$$

By the definition of V', the first term in left-hand side of the above equation is equal to 0. Since $u\in V'\cap U$, there holds $\sum_{v\in V'} d(v)\geqq 1$, which implies that $\sum_{v\in V'}\psi(v)\geqq 1$. Suppose that a vertex v' in V' satisfies $\psi(v')>0$. Then from (C2) there exists an f-positive v_1-v' path on D. However, this means that D contains an f-positive v_1-u path via v' from $v'\in V'$, which contradicts $v_1\notin V'$. Since u is arbitrary, we have this lemma. □

The cost of D, denoted by $C(D)$, is defined to be the sum of the cost of making copies of J at vertices and the cost of transmitting copies of J through arcs.

Definition 2: For a file transfer $D=(\psi, f)$ on $N=(V, B, c_v, d, c_a)$, let

$$C(D)=\sum_{u\in V} c_v(u)\cdot\psi(u)$$
$$+\sum_{(x,y)\in B} c_a(x, y)\cdot f(x, y),$$

which is called the cost of D. A file transfer D on N is said to be *optimal* if $C(D)\leqq C(D')$ for every other file transfer D' on N. □

The following concept of mother vertex is fundamental in order to synthesize an optimal file transfer on a given file transmission net.

Definition 3: A vertex x in $N=(V, B, c_v, d, c_a)$ is called a mother vertex in N if $c_v(x)<c_v(y)+c_{x,y}$ for any vertex y in $V\backslash\{x\}$. The set of all mother vertices in N is called the mother vertex set in N and is denoted by M. □

In the following, unless otherwise stated, we simply denote M instead of the mother vertex set, and we only consider file transmission nets such that $M\subseteqq U$. In relation to each mother vertex in an optimal file transfer, we have the following proposition, which is an expansion of Proposition 1 of Ref. (5).

Proposition 1: Suppose that $M\subseteqq U$ in $N=(V, B, c_v, d, c_a)$. Then a necessary condition for a file transfer $D=(\psi, f)$ to be optimal on N is that there holds $\sum_{x\in A(m)} f(x, m)=1$ for any vertex m in $M\backslash\{v_1\}$, and that if $v_1\in M$, there holds $\sum_{x\in A(v_1)} f(x, v_1)=0$.

Proof: By Lemma 1 and $M\subseteqq U$, there exists an f-positive v_1-v path for any vertex v in M, which implies $\sum_{x\in A(m)} f(x, m)\geqq 1$ for every vertex m in $M\backslash\{v_1\}$. Let $M_f=\{v\in M|\sum_{x\in A(v)} f(x, v)\geqq 2\}$. In order to prove this proposition, it suffices to show that if a file transfer $D=(\psi, f)$ satisfies $M_f\neq\phi$, then there exists another file transfer $\tilde{D}=(\tilde{\psi}, \tilde{f})$ such that $C(\tilde{D})<C(D)$ and $M_{\tilde{f}}\subset M_f$, where $M_{\tilde{f}}=\{v\in M|\sum_{x\in A(v)}\tilde{f}(x, v)\geqq 2\}$. Let S be the supply vertex set with respect to D. Let m be a vertex in M_f and let $k=\sum_{x\in A(m)} f(x, m)$. Then, without loss of generality, we assume that k' copies of J are made at some vertex s in S, are transmitted through an s-m path P, and are sent to m, where $0<k'\leqq k$. Let $j=\min\{k', k-1\}$. For $D=(\psi, f)$, we define a function ψ' on V as well as a function f' on B to be

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$$\psi'(s)=\psi(s)-j,\quad \psi'(m)=\psi(m)+j,$$
$$\psi'(v)=\psi(v)\quad \text{(otherwise)},$$
$$f'(e)=f(e)-j\quad (e\in B(P)),$$
$$f'(e)=f(e)\quad \text{(otherwise)}.$$

For f' and P, let $S'=\{v\in V(P)\cap S \mid \sum_{x\in A(v)} f'(x, v)=0\}$. Here two cases of (i) $S'=\phi$, and (ii) $S'\neq\phi$ are considered. In the case (i), $D'=(\psi', f')$ is a file transfer on N, because ψ' and f' clearly satisfy (C1) and (C2). In the case (ii), let P_1 be the s-u subpath of P such that $u\in S'$ and $V(P_1)\cap S\supseteqq S'$. For $D'=(\psi', f')$, we define a function ψ'' on V as well as a function f'' on B to be

$$\psi''(s)=\psi'(s)+1,\quad \psi''(u)=\psi'(u)-1,$$
$$\psi''(v)=\psi'(v)\quad \text{(otherwise)},$$
$$f''(e)=f'(e)+1\quad (e\in B(P_1)),$$
$$f''(e)=f'(e)\quad \text{(otherwise)}.$$

Clearly ψ'' and f'' satisfy (C1). For any vertex v in S', there exists an f''-positive v_1-v path because there exists an f''-positive s-v path. Then ψ'' and f'' satisfy (C2), which means that $D''=(\psi'', f'')$ is a file transfer on N. In the case (i), we have $C(D)-C(D')=\{c_v(s)+c(P)-c_v(m)\}\cdot j>0$ by Definition 3. In the case (ii), we have $C(D)-C(D'')=(j-1)\cdot\{c_v(s)+c(P)-c_v(m)\}+c_v(u)+c(P_2)-c_v(m)>0$, where P_2 is the u-m subpath of P. As a result, if $k-j\neq 1$, repeating the above operation, we get a file transfer $\tilde{D}=(\tilde{\psi}, \tilde{f})$ such that $C(D)>C(\tilde{D})$ and $M_{\tilde{f}}=M_f\backslash\{m\}\subset M_f$. In the similar way, we can prove $\sum_{x\in A(v_1)} f(x, v_1)=0$ if $v_1\in M$. □

As an example, let us consider a file transmission

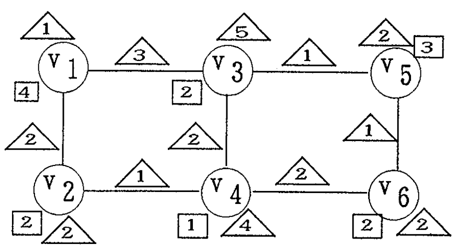

Fig. 1 An example of a file transmission net N.

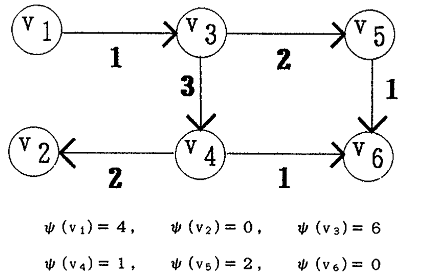

Fig. 2 A file transfer $D=(\psi, f)$ on N.

net N shown in Fig. 1, where N is represented by the corresponding undirected graph for simplicity and the symbol △ denotes the cost of vertex or arc and the symbol □ denotes the demand of vertex. In this example, N satisfies $U=V$, and a file transfer $D=(\psi, f)$ is shown in Fig. 2, where a number beside an arc e denotes $f(e)$. Throughout this paper, arcs with zero weights are omitted in all figures but those of illustrating file transmission nets. The set of supply vertices with respect to D is $\{v_1, v_3, v_4, v_5\}$. The cost $C(D)$ of D is

$$C(D)=1\cdot 4+5\cdot 6+4\cdot 1+2\cdot 2+3+2+6+1+2+2=58.$$

3. Delivery Transfer

In this section, we show that it suffices to consider a file transfer D such that the supply vertex set with respect to D is a subset of M in N in order to synthesize an optimal file transfer on N.

Definition 4: For a vertex u in V in $N=(V, B, c_v, d, c_a)$, let $H(u)=\{w\in V \mid c_v(w)<+\infty$ and $c_v(w)+c_{w,u}\leqq c_v(w')+c_{w',u}$ for any vertex w' in $V\}$. □

In the following, a function H on V indicates the function H in Definition 4, unless otherwise stated. Let w be a vertex in $H(u)$ for a vertex u. Then the total cost of making copies of J at w and sending the copies to u through a minimum cost u-w path is not more than the total cost of making copies of J at a vertex w' and sending the copies to u through some w'-u path. It turns out from Definitions 3 and 4 that a vertex v is a mother vertex if and only if $H(v)=\{v\}$. In relation to H and M, the following lemmas hold.

Lemma 2: For a vertex x in $V\backslash M$, suppose that $H(x)\cap M\neq\phi$ in $N=(V, B, c_v, d, c_a)$. Then $H(y)\cap M\neq\phi$ for a vertex y such that $x\in H(y)$.

Proof: Let z be a vertex in $H(x)\cap M$. Then we have $c_v(z)+c_{z,x}\leqq c_v(x)$, because $z\in H(x)$. Clearly we have $c_{z,y}\leqq c_{z,x}+c_{x,y}$. From these equalities, we get $c_v(z)+c_{z,y}\leqq c_v(x)+c_{x,y}$, which implies that $z\in H(y)$ because $x\in H(y)$. Hence this lemma. □

Lemma 3: In $N=(V, B, c_v, d, c_a)$, for a vertex x in $V\backslash M$, $H(x)$ contains a vertex y such that $c_v(x)>c_v(y)$.

Proof: By $x\notin M$ and Definition 3, N contains a vertex z but x such that $c_v(z)+c_{z,x}\leqq c_v(x)$, which implies that if $x\in H(x)$ then $z\in H(x)$ because of Definition 4. Then $H(x)\backslash\{x\}\neq\phi$. It is clear that $c_v(y)+c_{y,x}\leqq c_v(x)$ for a vertex y in $H(x)\backslash\{x\}$. Then $c_v(x)>c_v(y)$ because $y\neq x$ and $c_{y,x}>0$. Hence this lemma. □

Lemma 4: In $N=(V, B, c_v, d, c_a)$, we have $H(u)\cap M\neq\phi$ for any vertex u in V.

Proof: It is clear that $H(m)\cap M\neq\phi$ for any vertex m in M because $H(m)=\{m\}$. Then we should prove

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that for any vertex u in $V\backslash M$, there holds $H(u)\cap M \neq \phi$. Let u_1 be a vertex in $V\backslash M$. By Lemma 3, we have a vertex u_2 in $H(u_1)$ such that $c_v(u_1)>c_v(u_2)$. If $u_2 \notin M$, then we can repeat using Lemma 3, and we get $c_v(u_1)>c_v(u_2)>\cdots>c_v(u_j)$ for an integer j more than 2 such that $u_{i+1}\in H(u_i)$ with $i=1,2,\cdots,j-1$. Since $|V\backslash M|<\infty$, there exists an integer k such that $u_k\in H(u_{k-1})\cap M$. Then we apply Lemma 2 in order of u_k, $u_{k-1},\cdots u_1$, and we get $H(u_1)\cap M\neq\phi$. Hence this lemma. □

Using this lemma, we have proven the following proposition.

Proposition 2: If, in a file transfer $D=(\phi, f)$ on N such that $M\subseteqq U$, there hold

(1) $\sum_{x\in A(m)} f(x,m)=1$ for any vertex m in $M\backslash\{v_1\}$, and if $v_1\in M$, then $\sum_{x\in A(v_1)} f(x,v_1)=0$, and

(2) $S\backslash M\neq\phi$ for the supply vertex set S with respect to D, then N contains a file transfer $\tilde{D}=(\tilde{\phi},\tilde{f})$ such that

(1) $\sum_{x\in A(m)} \tilde{f}(x,m)=1$ for any vertex m in $M\backslash\{v_1\}$, and if $v_1\in M$, then $\sum_{x\in A(v_1)} \tilde{f}(x,v_1)=0$,

(2) $\tilde{S}\subseteqq M$ for the supply vertex set $\tilde{S}$ with respect to $\tilde{D}$, and

(3) $C(\tilde{D})\leqq C(D)$.

Proof: Lemma 4 says that $H(v)\cap M\neq\phi$ for each vertex v in V. Let s be a vertex in $S\backslash M$ and let m be a vertex in $H(s)\cap M$. Let P be a minimum cost m-s path in N. Note that $V(P)\cap M=\{m\}$. Then for $D=(\phi,f)$, we define a function ϕ' on V as well as a function f' on B to be

$$\phi'(m)=\phi(m)+\phi(s),\quad \phi'(s)=0,$$

$$\phi'(v)=\phi(v)\quad \text{(otherwise)},$$

$$f'(e)=f(e)+\phi(s)\quad (e\in B(P)),$$

$$f'(e)=f(e)\quad \text{(otherwise)}.$$

Clearly, ϕ' and f' satisfy (C1). Let $S'=\{v\in V|\phi'(v)>0\}$. Since $D=(\phi,f)$ is a file transfer, D contains an f-positive v_1-v path for every vertex v in S by (C2). There also exists an f-positive v_1-m path by $M\subseteqq U$ and Lemma 1. Hence for every vertex v in S' there exists an f'-positive v_1-v path, which implies that ϕ' and f' satisfy (C2). Then, we can say that $D'=(\phi', f')$ is a file transfer on N. Since $V(P)\cap M=\{m\}$, $P\in \boldsymbol{P}_{m,s}$ and $s\notin M$, we can say that $B(P)$ has no arc whose end vertex is a mother vertex. Then from the above condition (1) of D, we can say that $\sum_{x\in A(m)} f'(x, m)=1$ for any vertex m in $M\backslash\{v_1\}$, and that if $v_1\in M$ then $\sum_{x\in A(v_1)} f'(x,v_1)=0$. We also have $S'=S\cup\{m\}\backslash\{s\}$. Moreover it follows from Definition 2, $\phi(s)>0$, and $m\in H(s)$ that $C(D')-C(D)=\phi(s)\cdot\{c_v(m)+c(P)-c_v(s)\}\leqq 0$. If $S\backslash M\neq\phi$, then for every vertex in $S\backslash M$ we can repeat this operation and we get a file transfer $\tilde{D}$ satisfying the above three conditions. Hence this proposition. □

In order to synthesize an optimal file transfer, it turns out from Proposition 2 that we should consider a file transfer D such that the supply vertex set S with respect to D satisfies $S\subseteqq M$. Using this property, we define the following net.

Definition 5: For each vertex u in U in $N=(V, B, c_v, d, c_a)$, select a vertex w in $H(u)\cap M$ and let $h(u)=w$, where h is a function from U into M. Select a minimum cost $h(u)$-u path P and associate with P a number $d(u)$. Let $\boldsymbol{P}$ be the set of such $|U|$ uniformly weighted paths and let N_h be the superimposition net of $\boldsymbol{P}$ and let $w(e)$ be the weight of each arc e in $B(N_h)$. $D_h=(\phi_h, f_h)$ is called a delivery transfer on N, if a function ϕ_h from V into $\boldsymbol{Z}_+$ as well as a function f_h from B into $\boldsymbol{Z}_+$ satisfies

$$\phi_h(v)=\sum_{u\in S(v)} d(u)\quad (v\in M),$$

$$\phi_h(v)=0\quad \text{(otherwise)},$$

$$f_h(e)=w(e)\quad (e\in B(N_h)),$$

$$f_h(e)=0\quad \text{(otherwise)},$$

where $S(v)=\{u\in U|h(u)=v\}$. Moreover, the cost, denoted by $C(D_h)$, of D_h is defined as

$$C(D_h)=\sum_{u\in V} c_v(u)\cdot\phi_h(u)+\sum_{e\in B} c_a(e)\cdot f_h(e).\quad \Box$$

On a file transmission net N in Fig. 1, we have $M=\{v_1, v_2, v_5, v_6\}$. Since $U=V$, we have $U\backslash M=\{v_3, v_4\}$. There hold $H(v_3)\cap M=\{v_5\}$ and $H(v_4)\cap M=\{v_2\}$, which imply $h(v_3)=v_5$ and $h(v_4)=v_2$. Then we get a delivery transfer shown in Fig. 3, where a number beside an arc e denotes $f_h(e)$. Any delivery transfer has the following properties.

Lemma 5: For any delivery transfer $D_h=(\phi_h, f_h)$ on $N=(V, B, c_v, d, c_a)$, there hold

$$\sum_{x\in A(v)} f_h(x,v)=\sum_{y\in A(v)} f_h(v,y)+d(v)\quad (v\in V\backslash M), \tag{1}$$

$$\sum_{x\in A(v)} f_h(x,v)=0\quad (v\in M), \tag{2}$$

Proof: It is clear that we have Eq. (1) from Definition 5. Suppose that a vertex m in M does not satisfy Eq. (2), which implies that for two vertices m'

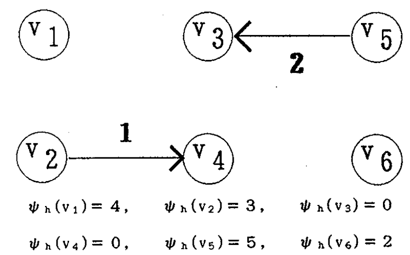

Fig. 3 A delivery transfer $D_h=(\phi_h, f_h)$.

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in M and v in $U\backslash M$, there exists an arc (x, m) in B such that $f_h(x, m) \geqq d(v)$ in D_h. Since $(x, m) \in B(P)$ for a path P in $\tilde{\boldsymbol{P}}_{m',v}$, we have $c_{m',v}=c_{m',m}+c_{m,v}$. From $m' \in H(v)$ and Definition 4, we have $c_v(m')+c_{m',v} \leqq c_v(m)+c_{m,v}$. Then from these inequalities, we get $c_v(m')+c_{m',m} \leqq c_v(m)$, which contradicts $m \in M$. Hence Eq. (2) holds. □

Lemma 6: The cost $C(D_h)$ of any delivery transfer $D_h=(\psi_h, f_h)$ on $N=(V, B, c_v, d, c_a)$ is constant independently of the choice of both a vertex $h(u)$ and a path in $\tilde{\boldsymbol{P}}_{h(u),u}$ for each vertex u in U.

Proof: In order to get a delivery transfer $D_h=(\psi_h, f_h)$, we superimpose a path P in $\tilde{\boldsymbol{P}}_{h(u),u}$ with which an integer $d(u)$ is associated for each vertex u in U. It takes $\{c_v(h(u))+c_{h(u),u}\} \cdot d(u)$ for superimposing P to get D_h, which is constant independently of the choice of both $h(u)$ and P from Definitions 4 and 5. Hence this lemma. □

We show some relation between a file transfer and a delivery transfer.

Proposition 3: For any file transfer D and any delivery transfer $D_h=(\psi_h, f_h)$ on $N=(V, B, c_v, d, c_a)$ such that $M \subseteqq U$, there exists on N a file transfer $\tilde{D}=(\tilde{\psi}, \tilde{f})$ such that

(1) $C(D) \geqq C(\tilde{D})$ and

(2) each arc e in B satisfies $\tilde{f}(e) \geqq f_h(e)$.

Proof: From Propositions 1 and 2, it suffices to consider a file transfer $D=(\psi, f)$ such that

(1) $\sum_{x \in A(m)} f(x, m)=1$ $(m \in M\backslash\{v_1\})$, and if $v_1 \in M$, then $\sum_{x \in A(v_1)} f(x, v_1)=0$, and

(2) $\{v \in V | \psi(v)>0\} \subseteqq M$.

For a vertex u in $U\backslash M$, let w be a vertex in $H(u) \cap M$. Select a path P in $\tilde{\boldsymbol{P}}_{w,u}$. Note that $V(P) \cap M=\{w\}$. Unless $d(u)$ copies are made at w, are transmitted through P and are sent to u, then by the above property of D, we can say that (1) $k(1 \leqq k \leqq d(u))$ copies of J are made at a vertex m in $M\backslash\{w\}$, and (2) k copies of J are transmitted through a path P' in $\boldsymbol{P}_{w,u}\backslash\{P\}$ such that $V(P') \cap M=\{m\}$. Using ψ and f, we define a function ψ' on V as well as a function f' on B to be

$$\psi'(m)=\psi(m)-k, \quad \psi'(w)=\psi(w)+k,$$

$$\psi'(v)=\psi(v) \quad \text{(otherwise)},$$

$$f'(e)=f(e)-k \quad (e \in B(P')\backslash B(P)),$$

$$f'(e)=f(e)+k \quad (e \in B(P)\backslash B(P')),$$

$$f'(e)=f(e) \quad \text{(otherwise)}.$$

Clearly ψ' and f' satisfy (C1). By Lemma 1 and $M \subseteqq U$, there exists an f-positive v_1-v path for every vertex v in M. Then, since $V(P') \cap M=\{m\}$, there also exists an f'-positive v_1-v path for every vertex v in M, which implies that ψ' and f' satisfy (C2). Hence $D'=(\psi', f')$ is a file transfer on N. Since $V(P) \cap M=\{w\}$, $P \in \tilde{\boldsymbol{P}}_{w,u}$, $V(P') \cap M=\{m\}$, and $P' \in \boldsymbol{P}_{m,u}$, we can say that $B(P) \cup B(P')$ contains no arc whose end vertex is a mother vertex. Then from the above property (1) of D, we have $\sum_{x \in A(m)} f'(x, m)=1$ for every vertex m in $M\backslash\{v_1\}$ and if $v_1 \in M$, then $\sum_{x \in A(v_1)} f'(x, v_1)=0$. From $w \in M$ and the above property (2) of D, we have $\{v \in V | \psi'(v)>0\} \subseteqq M$. Moreover by $w \in H(u)$ and Definition 5, we have $C(D)-C(D')=\{c_v(m)+c(P')-c_v(w)-c(P)\} \cdot k \geqq 0$. Note that we have $f'(e) \geqq f(e)+k$ for every arc e in $B(P)$. If necessary, we repeat the above operation and we get a file transfer $D''=(\psi'', f'')$ where $d(u)$ copies are made at w, are transmitted through P and are sent to u, where $f''(e) \geqq f(e)+d(u)$ for every arc e in $B(P)$. Similarly in the case of u, repeating the above operation for each vertex in $U\backslash M$, we finally get a file transfer $\tilde{D}$ satisfying the proposition. □

Lemma 7: Suppose that for any file transfer $D=(\psi, f)$ and any delivery transfer $D_h=(\psi_h, f_h)$ on $N=(V, B, c_v, d, c_a)$, there holds (1) $\{v \in V | \psi(v)>0\} \subseteqq M$, and (2) every arc e in B satisfies $f(e) \geqq f_h(e)$. Let

$$f_+(e)=f(e)-f_h(e) \quad (e \in B). \tag{3}$$

Then, we have

$$\sum_{x \in A(v)} f_+(x, v)=\sum_{y \in A(v)} f_+(v, y) \quad (v \in V\backslash M'),$$

where $M'=M \cup \{v_1\}$. If $v_1 \notin M$, we have

$$1+\sum_{x \in A(v_1)} f_+(x, v_1)=\sum_{y \in A(v_1)} f_+(v_1, y).$$

Proof: From the condition (1) of D, there holds $\psi(v)=0$ for every vertex v in $V\backslash M$. Then we get the first equation from (C1), Eq. (1), and Eq. (3). Similarly, we get the other equation from (C1), Eq. (2), and Eq. (3). □

4. Supply Transfer

Definition 6: In $N=(V, B, c_v, d, c_a)$ and let T be an arborescence with root v_1 and vertex set $M'=M \cup \{v_1\}$. Then for each arc $e=(x, y)$ in $B(T)$, select a path P in $\tilde{\boldsymbol{P}}_{x,y}$, and associate with P a number 1. Let $\boldsymbol{P}$ be the set of such $|M'|-1$ uniformly weighted paths and let N_T be the superimposition net of $\boldsymbol{P}$, where a number $w(e)$ is associated with each arc e. $D_T=(\psi_T, f_T)$ is called a supply transfer on N if a function ψ_T from V into $\boldsymbol{Z}_+$ as well as a function f_T from B into $\boldsymbol{Z}_+$ satisfies

$$\psi_T(v)=\delta_+(v;T)-1 \quad (v \in M'),$$

$$\psi_T(v)=0 \quad \text{(otherwise)},$$

$$f_T(e)=w(e) \quad (e \in B(N_T)),$$

$$f_T(e)=0 \quad \text{(otherwise)},$$

where $\delta_+(v;T)$ denotes the out-degree of a vertex v in T. Moreover, the cost, denoted by $C(D_T)$, of D_T is defined as

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$$C(D_T)=\sum_{u\in V}c_v(u)\cdot\psi_T(u)+\sum_{e\in B}c_a(e)\cdot f_T(e).\ \square$$

Any supply transfer has the following property.

Lemma 8: Let $D_T=(\psi_T, f_T)$ be a supply transfer on $N=(V, B, c_v, d, c_a)$, where T is an arborescence with root v_1 and vertex set $M'=M\cup\{v_1\}$. Then there hold

$$\delta_+(v_1;T)=\sum_{y\in A(v_1)}f_T(v_1, y) \text{ and } \sum_{x\in A(v_1)}f_T(x, v_1)=0,$$

$$\delta_+(v;T)=\sum_{y\in A(v)}f_T(v, y)$$

$$\text{and } \sum_{x\in A(v)}f_T(x, v)=1 \quad (v\in M\backslash\{v_1\}),$$

$$\sum_{x\in A(v)}f_T(x, v)=\sum_{y\in A(v)}f_T(v, y) \quad (v\in V\backslash M'), (4)$$

where $\delta_+(v;T)$ denotes the out-degree of a vertex v in T. Moreover, let P_e be a path in $\tilde{\boldsymbol{P}}_{x,y}$ with which we get D_T for an arc $e=(x, y)$ in $B(T)$. Then we have

$$C(D_T)=-c_v(v_1)+\sum_{e=(x,y)\in B(T)}\{c_v(x)+c(P_e)-c_v(y)\}. \quad (5)$$

Proof: Since we superimpose a path P_e with which an integer 1 is associated for each arc e in $B(T)$ and both the start vertex and end vertex of P_e are in M', it is clear that we have Eq. (4). Let $\boldsymbol{P}$ be the set of such $|M'|-1$ paths. Then there holds

$$\sum_{e\in B}c_a(e)\cdot f_T(e)=\sum_{P\in\boldsymbol{P}}c(P).$$

Since T is an arborescence with root v_1, we have

$$\sum_{u\in M'}\{\delta_+(v;T)-1\}\cdot c_v(u)$$
$$=-c_v(v_1)+\sum_{(x,y)\in B(T)}\{c_v(x)-c_v(y)\}.$$

From these two equations and Definition 6, we easily get Eq. (5). Hence this lemma. $\square$

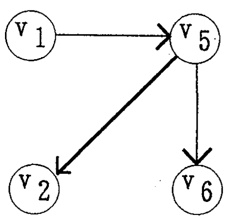

Fig. 4 An arborescence T with root v_1.

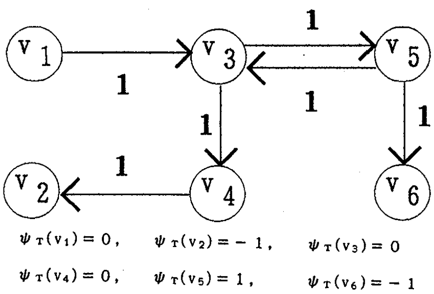

Fig. 5 A supply transfer $D_T=(\psi_T, f_T)$.

As an example, for $M=\{v_1, v_2, v_5, v_6\}$ on N in Fig. 1, we choose an arborescence T with root v_1 shown in Fig. 4. Then we have a supply transfer D_T shown in Fig. 5, where a number beside an arc e denotes $f_T(e)$. In relation to a supply transfer and a delivery transfer, the following lemma holds.

Lemma 9: For any delivery transfer $D_h=(\psi_h, f_h)$ and any supply transfer $D_T=(\psi_T, f_T)$ on $N=(V, B, c_v, d, c_a)$, if a function ψ on V as well as a function f on B satisfies

$$\begin{aligned}\psi(v)&=\psi_T(v)+\psi_h(v) \quad (v\in V),\\ f(e)&=f_T(e)+f_h(e) \quad (e\in B),\end{aligned} \quad (6)$$

then $D=(\psi, f)$ is a file transfer on N. Moreover, there holds

$$C(D)=C(D_h)+C(D_T). \quad (7)$$

Proof: It turns out from Definitions 5, 6 and Eq. (6), that ψ and f satisfy (C1). From Definitions 5, 6 and Eq. (6), we have $\{v\in V|\psi(v)>0\}\backslash\{v_1\}\subseteqq M$, and from Definition 6 we have an f_T-positive v_1-m path for each vertex m in M. Then clearly this implies that ψ and f satisfies (C2). Therefore we can say that $D=(\psi, f)$ is a file transfer on N. We can easily get Eq. (7) from Definitions 5, 6 and Eq. (6). $\square$

In the following, if a file transfer D satisfies Eq. (6) for a supply transfer D_T and a delivery transfer D_h, then we simply write $D=D_T+D_h$.

Lemma 10: Suppose that a file transfer $D=(\psi, f)$ on $N=(V, B, c_v, d, c_a)$ such that $M\subseteqq U$ satisfies Propositions 1, 2, and 3. Namely, there hold

(1) $\sum_{x\in A(v)}f(x, v)=1$ for any vertex v in $M\backslash\{v_1\}$,

(2) $\sum_{x\in A(v_1)}f(x, v_1)=0$, if $v_1\in M$,

(3) $S\backslash\{v_1\}\subseteqq M$ for the supply vertex set S with respect to D, and

(4) each arc e in B satisfies $f(e)\geqq f_h(e)$ for some delivery transfer $D_h=(\psi_h, f_h)$.

For a function f_+ on B defined as Eq. (3), let G be a directed multiple graph where vertex set is V and $f_+((x, y))$ arcs exist from a vertex x to a vertex y. Then if G contains no path from v_1 to a mother vertex, then there exists in N a file transfer $\tilde{D}=(\tilde{\psi}, \tilde{f})$ which satisfies the above four conditions, $C(D)\geqq C(\tilde{D})$, and $\tilde{G}$ contains a path from v_1 to any mother vertex, where $\tilde{G}$ is obtained from $\tilde{f}$ in the same way that G is obtained from f.

Proof: Let $M'=M\cup\{v_1\}$ and let $R=\{m\in M'|G$ contains a v_1-m path$\}$ for the above directed multiple graph G. In order to prove this lemma, it suffices to show that if $R\subset M'$, then there exists a directed multiple graph G' such that $R\subset R'$ where $R'=\{m\in M'|G'$ contains a v_1-m path$\}$ and G' is obtained from f' in the same way that G is obtained from f such that a file transfer $D'=(\psi', f')$ satisfies the above 4 conditions and $C(D)\geqq C(D')$. From the above (1) and

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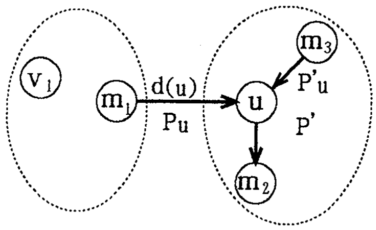

Fig. 6 An illustration for Lemma 10.

Eq. (3), we have $\sum_{x\in A(m)} f_+(x, m)=1$ for every mother vertex m, which implies that G contains at least $|M'|-1$ paths where end vertex is m, the start vertex is in M', and every vertex but the start and end vertex is in $V\backslash M$ by Lemma 7. Although $R\subset M'$, there exists in D an f-positive m_1-m_2 path P such that $V(P)\cap M'=\{m_1, m_2\}$ for a vertex m_1 in R and a vertex m_2 in $M'\backslash R$. Let m_3 be the start vertex of P' where m_2 is the end vertex of P' and $V(P')\cap M'=\{m_3, m_2\}$. Since G has no m_1-m_2 path, we can say that D contains an f-positive m_1-u subpath P_u of P for a vertex u in $V(P)\cap V(P')$. Let P'_u be the m_3-u subpath of P'. (See Fig. 6) For $D=(\psi, f)$, we define a function ψ' on V as well as a function f' on B as

$$\begin{aligned} &\psi'(m_1)=\psi(m_1)+1,\\ &\psi'(m_3)=\psi(m_3)-1,\\ &\psi'(v)=\psi(v) \quad (\text{otherwise}),\\ &f'(e)=f(e)+1 \quad (e\in B(P_u)\backslash B(P'_u)),\\ &f'(e)=f(e)-1 \quad (e\in B(P'_u)\backslash B(P_u)),\\ &f'(e)=f(e) \quad (\text{otherwise}). \end{aligned} \tag{8}$$

Clearly ψ' and f' satisfy (C1). Since ψ and f satisfy (C2), $V(P'_u)\cap M'=\{m_3\}$ and m_3 is the start vertex of P'_u, we can say that ψ' and f' satisfy (C2). Then $D'=(\psi', f')$ is a file transfer on N. It is clear that D' satisfies the above conditions (1), (2), and (3). Moreover every arc e in $B(P'_u)$ exists in G, which implies $f(e)>f_h(e)$. Then from Eq. (8), f' satisfies the above Eq. (4). Let G' be a directed multiple graph obtained from f' in the same way that G is obtained from f. Let $R'=\{m\in M'|G'$ contains a v_1-m path$\}$. We can say a relation between G and G' as follows; G' is obtained from G by deleting an m_3-u path and adding an m_1-u path. This means that $R\subseteqq R'$ because $m_1\in R$. It turns out from $m_2\in R'$ and $m_2\notin R$ that $R'\supseteqq R\cup\{m_2\}\supset R$, which implies $R'\supset R$. By $m_1\in H(u)$, $P_u\in \boldsymbol{P}_{m_1,u}$ and Definition 5, we get

$$\begin{aligned} &C(D)-C(D')\\ &=c_v(m_2)+c(P'_u)-\{c_v(m_1)+c(P_u)\}\\ &=c_v(m_2)+c(P'_u)-\{c_v(m_1)+c_{m_1,u}\}\geqq 0. \end{aligned}$$

Hence this lemma. □

The following proposition shows the relation between a file transfer written in Definition 6 and any file transfer.

Proposition 4: For any file transfer D and any delivery transfer D_h on $N=(V, B, c_v, d, c_a)$ such that $M\subseteqq U$, there exists on N a supply transfer D_T such that (1) $C(D)\geqq C(D')$ and (2) $D'=D_T+D_h$.

Proof: By Lemma 10, it is no problem that we assume that a file transfer $D=(\psi, f)$ on N satisfies the four conditions of Lemma 10, and assume that G contains a v_1-v path for every mother vertex v where G is a directed multiple graph obtained as in Lemma 10. In this proof, let $M'=M\cup\{v_1\}$ and let $\delta_+(u;G_r)$ and $\delta_-(u;G_r)$ denote the out-degree and the in-degree of a vertex u in a directed graph G_r, respectively. Then from the above assumption and Lemma 7, we have

$$\begin{aligned} &\delta_-(v;G)=1 && (v\in M\backslash\{v_1\}),\\ &\delta_+(v;G)=\delta_-(v;G) && (v\in V\backslash M'). \end{aligned} \tag{9}$$

If $v_1\notin M$, then

$$1+\delta_-(v_1;G)=\delta_+(v_1;G), \tag{10}$$

Otherwise, $\delta_-(v_1;G)=0$. By the above assumption, G contains a v_1-m path P_1 such that $V(P_1)\cap M=\{m_1\}$. Let G_1 be a directed graph which is obtained from G by deleting P_1. In the following, we consider the case of $v_1\notin M$, because we can similarly prove the case of $v_1\in M$. If $v_1\notin M$, then from Eqs. (9) and (10), we have

$$\begin{aligned} &\delta_-(m_1;G_1)=0\\ &\delta_-(v;G_1)=1 && (v\in M\backslash\{m_1\}),\\ &\delta_+(v;G_1)=\delta_-(v;G_1) && (v\in V\backslash M). \end{aligned} \tag{11}$$

It turns out from Eq.(11) that for some vertex m_2 G_1 contains an elementary m'_2-m_2 path P_2 such that $V(P_2)\cap M=\{m'_2, m_2\}$ and $m'_2\in M$. Let G_2 be a directed graph which is obtained from G_1 by deleting P_2. Then repeating the similar way that we get G_2 from G_1, finally we have a directed graph G' such that

$$\begin{aligned} &\delta_-(v;G')=0 && (v\in M),\\ &\delta_+(v;G')=\delta_-(v;G') && (v\in V\backslash M), \end{aligned} \tag{12}$$

with $|M|$ paths set $\boldsymbol{P}$ such that $\boldsymbol{P}$ satisfies

(P1) every path P in $\boldsymbol{P}$ satisfies the end vertex of P is each vertex in M, the start vertex of P is in M', and every vertex but the start and the end vertex in $V(P)$ is in $V\backslash M$.

Note that we obtain G from G' by adding all paths in $\boldsymbol{P}$. From Eq. (12), we have $\delta_+(m;G')=0$ for every mother vertex m, which implies that $\delta_+(v;G')=\delta_-(v;G')$ for any vertex v in G'. Then there exists a circuit set $\boldsymbol{L}$ such that G' is identical with the superimposition net $N(\boldsymbol{L})$ of $\boldsymbol{L}$ and that every circuit L in $\boldsymbol{L}$ satisfies $V(L)\subseteqq V\backslash M$. As a result, G is identical with $N(\boldsymbol{P}\cup\boldsymbol{L})$. For a set $\boldsymbol{P}$ let $s(P)$ and $e(P)$ be the start and end vertex of P in $\boldsymbol{P}$, respectively, and for $\boldsymbol{P}$ which satisfies (P1) we define a directed graph T as

(P2) $V(T)=M'$ and $B(T)=\{(s(P), e(P))|P\in\boldsymbol{P}\}$.

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In order to prove this proposition it suffices to show that there exists a path set $\boldsymbol{P}$ and a circuit set $\boldsymbol{L}$ such that
(1) G is identical with $N(\boldsymbol{P}\cup\boldsymbol{L})$,
(2) $\boldsymbol{P}$ satisfies (P1), and
(3) A directed graph defined as (P2) is connected and an arborescence whose root is v_1,
because it turns out from the above (2), (3), and Definition 6 that $N(\boldsymbol{P})$ is a supply transfer on N, denoted by D_T, which implies from Lemma 9 and the above (1) that we can say $D'=D_T+D_h$ is a file transfer on N and

$$C(D)=C(D_T)+C(D_h)+\sum_{L\in\boldsymbol{L}} c(L)\geqq C(D').$$

For a directed graph T defined as (P2), let $R=\{v\in M'|T$ contains a v_1-v path$\}$. Unless T satisfies the above (3), then a connected component including v_1 of T is an arborescence whose root is v_1, every other connected component is a circuit, and there holds $R\subset M'$. In order to prove the existence of a path set $\boldsymbol{P}$ and a circuit set $\boldsymbol{L}$ which satisfy the above 3 conditions, it suffices to show that for such T there exists a directed graph T'' such that $R\subset R''$ where $R''=\{v\in M'|T''$ contains a v_1-v path$\}$ and T'' is defined as (P2) from a path set $\boldsymbol{P}''$ satisfying the above (1) and (2) for a circuit set $\boldsymbol{L}''$. From the above assumption, G contains an m'-m path P_3 such that $m'\in R$, $m\in M'\backslash R$, and $V(P_3)\cap M'=\{m', m\}$. Let P_4 be a path in $\boldsymbol{P}$ whose end vertex is m and let s be the start vertex of P_4. Note here that an arc $(v, m)\in B(P_3)$ is in $B(P_4)$ because $\boldsymbol{P}$ satisfies (P1). Then the following two cases are considered if an arc $e=(x, y)$ is not contained in $B(P_4)$.
(Case 1) $e\in B(L)$ for a circuit in $\boldsymbol{L}$, and
(Case 2) $e\in B(P_5)$ for a path P_5 but P_4 in $\boldsymbol{P}$.
For each case we consider the following operations. In the Case 1, let P_4' be an s-m path whose s-y subpath is identical with that of P_4, whose y-y path is identical with L, and whose y-m subpath is identical with that of P_4, let $\boldsymbol{P}'=\boldsymbol{P}\backslash\{P_4\}\cup\{P_4'\}$, and let $\boldsymbol{L}'=\boldsymbol{L}\backslash\{L\}$. Then $\boldsymbol{P}'$ and $\boldsymbol{L}'$ satisfy the above (1) and (2).

In the Case 2, let t and w be the start and end vertex of P_5. Let P_4' be a t-m path whose t-y subpath is identical with that of P_5 and whose y-m subpath is identical with that of P_4 and let P_5' be an s-w path whose s-y subpath is identical with that of P_4 and whose y-w subpath is identical with that of P_5, and let $\boldsymbol{P}'=\boldsymbol{P}\backslash\{P_4, P_5\}\cup\{P_4', P_5'\}$. Then $\boldsymbol{P}'$ and $\boldsymbol{L}$ satisfy the above (1) and (2).

Note that in both cases, we have $(x, y)\in B(P_4')$. Clearly, we can repeat the above operation for P_4' instead of P_4 unless $e\in B(P_3)$ is contained in $B(P_4')$. Suppose that vertices on P_3 appear in order of $m'=u_1$, $u_2, \cdots, u_k=m$. We repeat the above operation in order of $i=k, k-1, \cdots, 2$, for every arc (u_{i-1}, u_i) in $B(P_3)$, and finally get a path set $\boldsymbol{P}''$ which satisfies (P1). Clearly, there exists a circuit set $\boldsymbol{L}''$ which satisfies the above (1) and (2) for this $\boldsymbol{P}''$. We get a directed graph T'' defined as (P2) from this $\boldsymbol{P}''$, and let $R''=\{v\in M'|T''$ contains a v_1-v path$\}$. Since every connected component but one including v_1 in T is a circuit and $m\in R''\backslash R$, we have $R\subset R''$. Hence this proposition. □

It turns out from Lemma 6 and Eq. (7) that we should consider to minimize the cost of a supply transfer in order to synthesize an optimal file transfer.

5. Minimum Cost Supply Transfer

A supply transfer whose cost is minimum among all supply transfers is called a minimum cost supply transfer, which we aim to construct in this section. The following associated net is very useful for our purpose.
Definition 7: Let $M'=M\cup\{v_1\}$ in $N=(V, B, c_v, d, c_a)$. Then the associated net AN is defined to be an undirected complete graph with vertex set M', in which to each edge (x, y) a value of $c_v(x)+c_v(y)+c_{x,y}$ is assigned. □

In relation to AN, we have the following lemma.
Lemma 11: Let G be a spanning tree on AN. Then there holds

$$\sum_{e\in E(G)} w(e)=\sum_{(x,y)\in E(G)} c_{x,y}+\sum_{u\in M'}\delta(u;G)\cdot c_v(u), \tag{13}$$

where $\delta(u;G)$ denotes the degree of a vertex u in G.
Proof: From Definition 7, for each vertex u in M', $c_v(u)$ appears $\delta(u;G)$ times in left-hand side of Eq. (13). Hence it is clear that Eq. (13) holds. □
Proposition 5: In $N=(V, B, c_v, d, c_a)$, let T be an arborescence with root v_1 and vertex set $M\cup\{v_1\}$. Then the cost of a supply transfer D_T is minimum if and only if there hold
(A1) For each arc (x, y) in $B(T)$, a path in $\tilde{\boldsymbol{P}}_{x,y}$ is selected to form D_T, and
(A2) the structure of T is identical with that of a minimum spanning tree on the associated net AN.
Proof: Clearly holds (A1). Then we should prove (A2). Let G be the underlying graph of T. Note that $\sum_{(x,y)\in E(G)} c_{x,y}=\sum_{(x,y)\in B(T)} c_{x,y}$. From Definition 7, it is clear that AN contains a spanning tree whose structure is identical with G. Let $\delta_+(u;T)$ and $\delta_-(u;T)$ denote the out-degree and the in-degree of a vertex u in T, respectively, and let $\delta(u;G)$ denote the degree of a vertex u in G and let $M'=M\cup\{v_1\}$. With the use of T and (A1), we make a supply transfer D_T. Since T is an arborescence with root v_1. then every vertex u in $M\backslash\{v_1\}$ satisfies $\delta_-(u;T)=1$ and $\delta(u;G)=\delta_+(u;T)+1$. Then, we see from $\sum_{(x,y)\in E(G)} c_{x,y}=\sum_{(x,y)\in B(T)} c_{x,y}$ and Lemma 11 that

$$C(D_T)=\sum_{(u,w)\in B(T)}\{c_v(u)+c_{u,w}-c_v(w)\}$$
$$=\sum_{(u,w)\in B(T)} c_{u,w}+\sum_{u\in M'} c_v(u)\{\delta_+(u;T)$$

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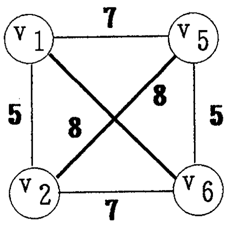

Fig. 7 The associated net AN of N.

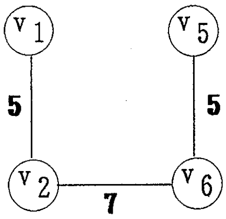

Fig. 8 A minimum spanning tree T_m on AN.

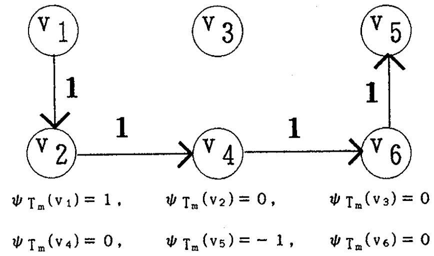

Fig. 9 A minimum cost supply transfer $D_{T_m}=(\psi_{T_m}, f_{T_m})$.

$$-\delta_-(u;T)\}$$

$$=\sum_{(u,w)\in B(T)} c_{u,w}+\sum_{u\in M'} c_v(u)\{\delta(u;G)-2\}$$

$$+2c_v(v_1)$$

$$=\sum_{e\in E(G)} w(e)-2\sum_{u\in M'} c_v(u)+4c_v(v_1),$$

where $-2\sum_{u\in M'} c_v(u)+4c_v(v_1)$ is constant independently of the structure of T. Thus, D_T is a minimum cost supply transfer if and only if G is a minimum spanning tree on AN. Hence this proposition. □

For a file transmission net N in Fig. 1, we get the associated net AN in Fig. 7 from Definition 7. For a minimum spanning tree on AN shown in Fig. 8, we have a minimum cost supply transfer shown in Fig. 9 from Proposition 5. Note that a number beside an arc e denotes $f_{T_m}(e)$.

6. Algorithm to Synthesize an Optimal File Transfer

From Proposition 4 as well as Proposition 5, we get the following theorem.

Theorem: In $N=(V, B, c_v, d, c_a)$ such that $M\subseteqq U$, let D_h be a delivery transfer. Let T be an arborescence with root v_1 and vertex set $M\cup\{v_1\}$. If the structure of T is identical with that of a minimum spanning tree on the associated net AN, then a file transfer $D=D_T+D_h$ is optimal on N, where D_T is a supply transfer. □

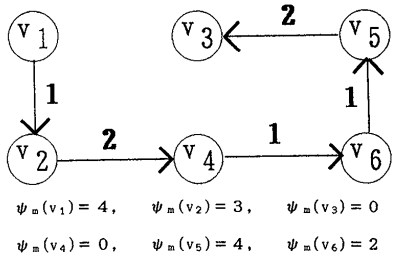

Fig. 10 An optimal file transfer $D_m=(\psi_m, f_m)$ on N.

Based on this theorem, an algorithm for synthesizing an optimal file transfer on N is given as follows.

Algorithm

Step1: In a given file transmission net $N=(V, B, c_v, d, c_a)$, search a minimum cost path and its cost between two vertices in U. Let $f(e)\leftarrow 0$ for each arc e in B.

Step2: Sort vertices in U with a nondecreasing order of their costs, i.e., $c_v(u_1)\leqq c_v(u_2)\leqq\cdots\leqq c_v(u_k)$ $(k=|U|)$.

Step3: $i\leftarrow 2$, $M\leftarrow\{u_1\}$.

Step4: If $i=k+1$, then go to Step 6.

Step5: For $j=1, 2, \cdots, i-1$, if $c_v(u_i)<c_v(u_j)+c_{u_j,u_i}$, then $M\leftarrow M\cup\{u_i\}$. Otherwise let $i\leftarrow i+1$ and go to Step 4.

Step6: $U\leftarrow U\backslash M$. Let $\psi(v)\leftarrow d(v)$ for each vertex v in M and let $\psi(v)\leftarrow 0$ for each vertex v in $V\backslash M$.

Step7: If $U=\phi$, then go to Step 9.

Step8: Select a vertex u from U. For u, search a vertex m' in M such that

$$c_v(m')+c_{m',u}=\min_{m\in M}\{c_v(m)+c_{m,u}\},$$

for every other vertex m in M. Then $\psi(m')\leftarrow\psi(m')+d(u)$, and for every arc e on path P in $\tilde{\boldsymbol{P}}_{m',u}$, let $f(e)\leftarrow f(e)+d(u)$. $U\leftarrow U\backslash\{u\}$ and go to Step 7.

Step9: $M'\leftarrow M\cup\{v_1\}$. Construct the associated net AN whose edge (m_1, m_2) is weighted with $c_v(m_1)+c_v(m_2)+c_{m_1,m_2}$ for any two distinct vertices m_1 and m_2 in M'.

Step10: Find a minimum spanning tree G of AN. Let T be an arborescence with root v_1 whose structure is identical with that of G.

Step11: For every arc (x, y) in $B(T)$, let (1) $\psi(x)\leftarrow\psi(x)+1$, $\psi(y)\leftarrow\psi(y)-1$, and (2) for every arc e on a path in $\tilde{\boldsymbol{P}}_{x,y}$, let $f(e)\leftarrow f(e)+1$.

Step12: $\psi(v_1)\leftarrow\psi(v_1)-1$. Then we get $D=(\psi, f)$, which is an optimal file transfer on N.

Step13: Terminate. □

We can get an optimal file transfer from the above proposed algorithm, which needs an algorithm for finding a shortest path such as Dijkstra's[(2)] and an algorithm for finding a minimum spanning tree such as Prim's.[(3)] Our algorithm takes $O(nm+n^2\log n)$ time-complexity if we use Fibonacci heap[(4)] in searching a shortest path from one vertex to every other vertex in V

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with $O(m+n\log n)$ time-complexity, where $n=|V|$ and $m=|B|$.

On a file transmission net N in Fig. 1, by our algorithm, we get an optimal file transfer $D_m=(\psi_m, f_m)$ shown in Fig. 10, where a number beside an arc e denotes $f_m(e)$. The cost of D_m is

$$C(D_m)=1\cdot4+2\cdot3+2\cdot4+2\cdot2+2+2+2+1+2$$
$$=31.$$

7. Conclusions

In this paper, we have proposed a problem of distributing copies of some information J through a file transfer from a vertex v_1 to every vertex on a file transmission net N. As a result, for the special situation where the mother vertex set M and the positive demand vertex set U satisfy $M \subseteqq U$ on N, we have shown a method of synthesizing an optimal file transfer whose total cost of transmitting and making copies of J is minimum on N.

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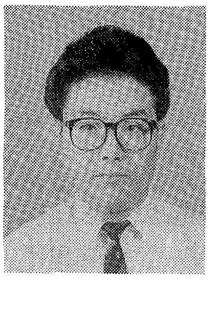
References

(1) Iri, R., Shirakawa, I., Kajitani, Y. and Shinoda, S., eds., *Graph Theory with Exercise*, Corona Co. Ltd., 1983.
(2) Dijkstra, E. W., "A Note on two problems in connection with graphs," *Numerische Math.*, vol. 1, pp. 269–271, 1959.
(3) Prim, R. C., "Shortest connection networks and some generalizations," *Bell System Technical J.*, vol. 36, pp. 1389–1401, 1957.
(4) Fredman, M. L., and Tarjan, R. E., "Fibonacci Heaps and Their Uses in Improved Network Optimization Algorithms," *Proc. of the 25th Annual IEEE Symposium on Foundations of Computer Science*, Singer Island, Florida, pp. 338–346, 1984.
(5) Kaneko, Y., Tashiro, R., Shinoda, S. and Horiuchi, K., "A Linear-Time Algorithm for Designing an Optimal File Transfer through an Arborescence-Net," *IEICE Trans.*, vol. E75-A, no. 7, pp. 901–904, Jul. 1992.



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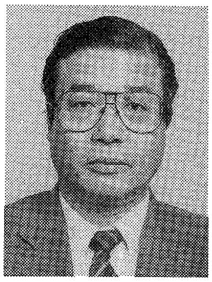

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