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AN ELASTO-VISCOPLASTIC MODEL FOR SOFT SEDIMENTARY ROCK BASED ON TIJ CONCEPT AND SUBLOADING YIELD SURFACE

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ABSTRACT

In this paper, an elasto-viscoplastic constitutive model with strain softening is developed for soft sedimentary rock using a newly proposed evolution equation for subloading yield surface originally invented by Hashiguchi (1980). In the model, associated flow rule is adopted and tij concept, which can take into consideration the influence of intermediate stress on deformation and strength of geomaterials, is used. In the model, as is the same as Cam-clay model, plastic volumetric strain is used as hardening parameter, which is widely accepted by the researchers who specialize in the constitutive model for geomaterials. The application of the model to the experimental results of soft sedimentary rock indicates that the model not only can describe the time dependency, such as strain rate dependency and creep, but also the strain softening behavior of geologic materials. The material parameters involved in the model have clear physical meanings and can be easily determined with triaxial compression tests and creep tests.

Key words: constitutive model, creep, soft rock, strain-rate dependency, strain softening (**IGC**: G6)

INTRODUCTION

Generally speaking, mechanical behavior of soft sedimentary rock is elasto-plastic, strain hardening-strain softening and time dependent. Physically, soft sedimentary rock has an unconfined compressive strength of $1 \sim 20$ MPa and its mechanical behavior is between the behavior of soil and rock.

The softening behavior of soft rock, which is usually thought to be merely a consequence of non-uniform deformation of an "element test", can hardly be linked to the concept of "strain softening" strictly, because it is almost impossible to conduct a pure element test to identify if the strain softening is an inherent property of geomaterials. Nevertheless, this does not prevent researchers from trying to establish a strain-softening model. For an engineering approach, the model, which provides a procedure to describe a drop in shear stress with shear strain on "element level", can be used to describe the macro behavior of the soft rock on convenience. Some researchers insist that softening is related to localization where a decrease of apparent stress occurs not by strain softening but by a reduction of effective area due to the coalescence of void and micro cracks, and that strain hardening is the inherent characteristics of geologic materials. Since the localization of deformation is the main reason causing softening, the softening behavior can be illustrated by solving boundary value problems based on strain hardening models. Unfortunately, up to now, few successful examples of the boundary value problems based on strain hardening models can be found in the literature because of the difficulty in dealing with the geometry of a shear band.

Many developments can be found in recent years related to constitutive model with strain softening and their application to boundary value problems. Oka and Adachi (1985) proposed an elastoplastic model with strain softening for soft rock, based on which a finite element analysis can lead to an unique solution (Adachi and Oka, 1995) for initial value and boundary value problems, and the analytical results have only a very small dependency on the mesh size (Adachi et al., 1991).

Based on the concept of subloading yield surface (Hashiguchi, 1980) and tij concept (Nakai and Mihara, 1984), Nakai and Hinokio (2004) proposed a state variable 'ρ' related to overconsolidated soils and developed a strain hardening and strain softening model that can describe correctly the mechanical behaviors of normally consolidated and overconsolidated soils under monotonic and cyclic loadings. Based on the concept of subloading yield surface, Asaoka et al. (1999) proposed a superloading concept by which the newly developed elastoplastic model not only can describe the mechanical

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behaviors of normally consolidated and overconsolidated soils but also structured soils that most widely be found in natural state. The works by Nakai and Asaoka provide us a foundation for the numerical simulation of boundary value problems of soft soils in an applicable way with a satisfactory accuracy.

As for the time dependent behavior, it involves three aspects, namely, the strain rate effect, creep and stress relaxation. Many models have been proposed to describe the time dependency of geologic materials. For soft clay, typical researches related to time dependency can be found in the works by Sekiguchi (1977) and Adachi and Oka (1982). Sekiguchi (1977) proposed an elasto-viscoplastic constitutive model for normally consolidated clay using a non-stationary viscoplastic potential surface, by which the creep behavior of normally consolidated clay under undrained condition can be well simulated. Based on the concept of over-stress, Adachi and Oka (1982) also proposed an elasto-viscoplastic constitutive model for normally consolidated clay, by which it is possible to describe the creep and strain-rate dependency of normally consolidated clay under undrained condition. Hashiguchi and Okayasu (2000) proposed a time-dependent model for soils based on the concept of subloading yield surface, in which the yield surface is not affected by time at all and an additional viscoplastic strain is added to total strain. In recent years, however, the necessity of using an elasto-viscoplastic model to describe the time dependent behaviors of soft clay becomes questionable because by some studies conducted by Nakano et al. (1998) and Asaoka et al. (1999), it is understood that the time dependent behaviors of soft clay can be described by soil-water coupled analysis based on an elastoplastic model.

In general, there are two viewpoints on the time-dependent behavior of geologic materials, one is the so called apparent viscosity in which the viscosity is due to the coupling of the soil skeleton with pore water, and the other is an inherent viscosity of soil skeleton. For clay, authors agree with the first viewpoint that the time-dependent behavior is merely an apparent viscosity that can be properly described by a suitable elastoplastic model together with soil-water coupled analysis. For soft rock and other granular materials, e.g., Oya Tuff (Okubo and Chu, 1994), chalk, talc and quarts particles (Takei et al., 2003), the viscosity of the materials in real is an inherent one. Field observations also verified the pure creep behavior of grounds. A natural slope failure caused by tunnel excavation was reposted by Yashima et al. (2001), in which it is reported that the slope failed several days after an arriving entrance of the tunnel was cut through. Progressive failure was clearly observed while no evidence of rainfall or underground water movement could be observed. Therefore, for those materials with inherent viscosity, it is still necessary to develop elasto-viscoplastic model to simulate time-dependent behavior of geomaterials.

Oka (1985) proposed a viscoplastic model with memory at first and then Adachi et al. (1990) proposed a viscoplastic model which can describe both the strain rate effect and strain softening of frozen sand. By some modifications, a revised viscoplastic model is proposed by Adachi et al. (1998), which can describe all three aspects of time-dependency and the strain softening behavior of soft rock. Based on the models, numerical analyses related to boundary value problems with FEM were conducted to simulate long-term stability and progressive failure of excavation problems (Adachi et al., 1994; Zhang et al., 2003). The analyses succeeded in simulating both the time dependent and strain softening behaviors of geomaterials in boundary value problems that are regarded as the main reasons of progressive failure and long-term instability of ground. The disadvantages of the models are that the value of the stress history tensor (Oka and Adachi, 1985) should be calculated continually to integrate the past stresses in all time, which makes the numerical analysis based on the model more difficult than those analyses based on Cam-clay model. The second disadvantages of the models is that the physical meaning of some parameters involved in the models are rather vague and can only be determined by curve-fitting method.

The purpose of this research is to develop an elasto-viscoplastic model for soft sedimentary rock. It should have the features that, (a) an associated flow rule is used; (b) the influence of intermediate stress on deformation and strength of geomaterials should be taken into consideration, which is very important because if without considering the influence, constitutive models like Cam-clay model cannot describe the stress-strain relation well under general stress conditions except axisymmetric condition (Zhang et al., 2003); (c) plastic volumetric strain is used as hardening parameter, which is widely accepted by the researchers who specialize in the constitutive model for geomaterials; (d) the material parameters involved in the model should have clear physical meanings and can be easily determined with triaxial compression tests and creep tests. For these reasons, a new evolution equation containing time effect for subloading yield surface, originally proposed by Hashiguchi (1980), is introduced in the model based on tij concept.

DERIVATION OF ELASTO-VISCOPLASTIC MODEL

Nakai and Hinokio (2004) proposed a simple model that can take into consideration the influence of the density and/or confining pressure on the deformation and strength of soil, as well as the influence of intermediate principal stress on the deformation and strength and the influence of stress path on the direction of plastic-strain increment. The model can describe correctly the mechanical behaviors of normally consolidated and overconsolidated soils under monotonic and cyclic loadings, not only for clay but also for sand. In the model, a state variable 'ρ' related to overconsolidation ratio was introduced and the model is based on the concept of tij and subloading yield surface. In the present

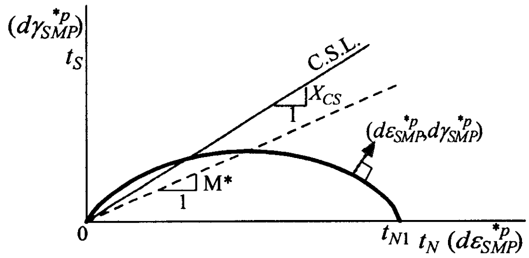

Fig. 1. Yield surface on $t_N - t_S$ plane (After Nakai and Hinokio, 2004)

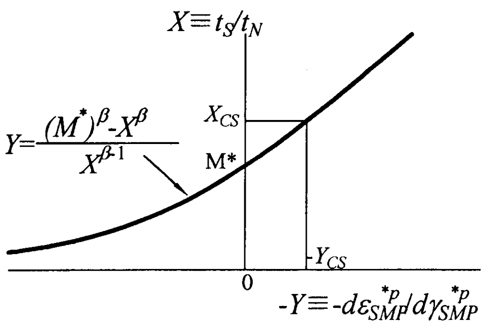

Fig. 2. Relationship between stress ratio and plastic strain increment ratio (After Nakai and Hinokio, 2004)

study, the state variable 'ρ', the concept of tij and subloading yield surface, are adopted in the same way described in previous study (Nakai and Hinokio, 2004). As shown in Fig. 1, yield surface is defined in a two-dimensional modified stress space $t_S - t_N$. Detailed description about the modified stress tensor can be obtained by referring to the reference (Nakai and Matsuoka, 1986). Figure 2 shows the relationship between stress ratio and plastic strain increment ratio. From normality condition that holds between the stress ratio and the plastic strain increment ratio, the following relation can be obtained:

$$dt_N \cdot d\varepsilon^{*p}_{SMP} + dt_S \cdot d\gamma^{*p}_{SMP} = 0 \tag{1}$$

where, the directions of $d\varepsilon^{*p}_{SMP}$ and $d\gamma^{*p}_{SMP}$ coincide with those of dt_N and dt_S, respectively, because coaxialty between stress and plastic strain increment are assumed. Substituting Eq. (1) into the relation shown in Fig. 2, it is derived that

$$Y = \frac{d\varepsilon^{*p}_{smp}}{d\gamma^{*p}_{smp}} = -\frac{dt_s}{dt_n} = \frac{M^{*\beta} - X^\beta}{X^{\beta-1}} \tag{2}$$

Integrating Eq. (2), the following relation can be obtained:

$$f = \ln\frac{t_N}{t_{N1}} + \varsigma(X) = \ln\frac{t_N}{t_{N0}} + \varsigma(X) - \ln\frac{t_{N1}}{t_{N0}} = 0 \tag{3}$$

$$\varsigma(X) = \frac{1}{\beta}\left(\frac{X}{M^*}\right)^\beta \tag{4}$$

where, t_N and $X \equiv t_S/t_N$ are the mean stress and the stress ratio based on tij concept, and t_{N1} determines the size of the yield surface (the value of t_N at $t_S = 0$). t_{N0}, a reference mean stress, is an arbitrary value and is taken as 98 kPa in this paper. From the equation expressed in Fig. 2, the value of M^* in Eq. (4) can be easily expressed using principal stress ratio $X_{CS} \equiv (t_S/t_N)_{CS}$ and plastic strain increment ratio $Y_{CS} \equiv (d\varepsilon^{*p}_{SMP}/d\gamma^{*p}_{SMP})_{CS}$ at critical state:

$$M^* = (X^\beta_{CS} + X^{\beta-1}_{CS} Y_{CS})^{1/\beta} \tag{5}$$

and these ratios X_{CS} and Y_{CS} are represented by the principal stress ratio at critical state in triaxial compression R_{CS} (Nakai and Matsuoka, 1986):

$$X_{CS} = \frac{\sqrt{2}}{3}\left(\sqrt{R_{CS}} - \frac{1}{\sqrt{R_{CS}}}\right) \tag{6}$$

$$Y_{CS} = \frac{1 - \sqrt{R_{CS}}}{\sqrt{2}\,(\sqrt{R_{CS}} + 0.5)} \tag{7}$$

Equation (3) just gives a normal yield surface that can be used for normally consolidated clay. For overconsolidated soils, however, the concept of subloading yield surface by Hashiguchi (1980) is preferred. Therefore, Eq. (3) can be rewritten as:

$$\begin{aligned} f &= \ln\frac{t_N}{t_{N0}} + \varsigma(X) - \ln\frac{t_{N1}}{t_{N0}} \\ &= \ln\frac{t_N}{t_{N0}} + \varsigma(X) - \left(\ln\frac{t_{N1e}}{t_{N0}} - \ln\frac{t_{N1e}}{t_{N1}}\right) = 0 \end{aligned} \tag{8}$$

where, as is the same in Fig. 2, t_N is the present stress state and t_{N1} is the cross point of the axis of $t_S = 0$ with the subloading yield surface that passes through the present stress state. t_{N1e} is the value of mean stress t_N at the cross point of the axis of $t_S = 0$ with the normal yield surface, which is uniquely determined by the present plastic volumetric strain (or void ratio) in such a way that,

$$\varepsilon^p_v = C_p \ln\frac{t_{N1e}}{t_{N0}} \tag{9}$$

where,

$$C_p = \frac{E_p}{1 + e_0} \tag{10}$$

and e_0 is the void ratio at $t_N = t_{N0} = 98$ kPa under isotropic normal consolidated condition. E_p is a plastic modulus and physically it equals the value of $\lambda - \kappa$, that is, swelling index minus compression index. As shown in Fig. 3, points A and B on $e - \ln t_N$ diagram indicate the void ratios of overconsolidated soil and normally consolidated soil at the same mean stress t_{N1}. The difference of the void ratios ρ between A and B can be regarded as a state valuable, because the stress ratio-strains relations of soils with the same ρ are identical regardless of the value of the mean stress based on the experimental results (Nakai and Hinokio, 2004). From Fig. 3, there exists a relation between ρ and the ratio t_{N1e}/t_{N1}:

$$\begin{aligned} \rho &= E_p \cdot \ln\left(\frac{t_{N1e}}{t_{N1}}\right) = (1 + e_0) \cdot C_p \cdot \ln\left(\frac{t_{N1e}}{t_{N1}}\right) \\ &= (1 + e_0) \cdot C_p \cdot \ln \text{OCR} \end{aligned} \tag{11}$$

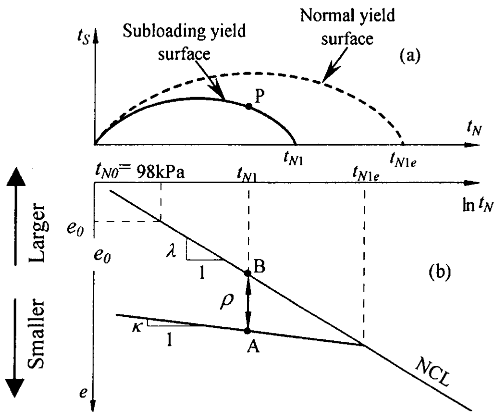

Fig. 3. Shape of yield surface and normally yield surface, and definition of ρ (After Nakai and Hinokio, 2004)

ρ, which represents the value related to overconsolidated ratio (OCR), is a state valuable and can be used to represent the relation between the subloading yield surface and the normal yield surface. Substituting Eqs. (9), (10), (11) into Eq. (8), the following yield function can be obtained:

$$f=\ln\left(\frac{t_{\rm N}}{t_{\rm N0}}\right)+\xi(X)-\frac{1}{C_{\rm p}}\left(\varepsilon_{\rm v}^{\rm p}-\frac{\rho}{1+e_0}\right)=0 \quad (12)$$

In the above yield function, plastic volumetric strain is used as the hardening function. Associated flow rule is adopted in the proposed model and therefore the viscoplastic potential is expressed as,

$$\dot{\varepsilon}_{\rm ij}^{\rm p}=\Lambda\frac{\partial f}{\partial t_{\rm ij}} \quad (13)$$

$$\dot{\varepsilon}_{\rm v}^{\rm p}=\Lambda\frac{\partial f}{\partial t_{\rm kk}} \quad (14)$$

In a constitutive model, consistency equation must be obeyed, by which the value of Λ can be is determined:

$$\dot{f}=0 \quad \Rightarrow \frac{\partial f}{\partial \sigma_{\rm ij}}\dot{\sigma}_{\rm ij}-\frac{1}{C_{\rm p}}\left(\dot{\varepsilon}_{\rm v}^{\rm p}-\frac{\dot{\rho}}{1+e_0}\right)=0 \quad (15)$$

In the works by Hashiguchi (1980), Asaoka et al. (1998), and Nakai and Hinokio (2004), the evolution equation of the variable related to the size of subloading yield surface is proportional to the variable Λ, that is, a homogeneous function of Λ. In the proposed model, however, the evolution equation of ρ is assumed to be a non-homogeneous function of Λ in such a way that,

$$\frac{\dot{\rho}}{1+e_0}=-\Lambda\cdot\frac{G(\rho,t)}{t_{\rm N}}+h(t) \quad (16)$$

$$h(t)=\dot{\varepsilon}_{\rm v}^{0}[1+t/t_1]^{-\alpha} \quad (17)$$

where $\dot{\varepsilon}_{\rm v}^{0}$ is the volumetric strain rate at time $t=0$, in which the time $t=0$ represents the time when shearing begins. t_1 is a unit time used to standardized the time. One should keep in mind that a creep stress is usually not applied to a sample abruptly and the loading process to the creep stress should be taken into consideration. A new parameter α is a parameter of time dependency which mainly controls the gradient of strain rate vs. time in logarithmic axes. The reason for using the non-homogeneous function of Λ expressed in Eq. (16) for the evolution equation is very easily understood because under a creep state $(\partial f/\partial\sigma_{\rm ij})\dot{\sigma}_{\rm ij}=0$, if a homogeneous function of Λ is adopted, then Λ will always be zero, which means that viscoplastic strain will never develop in creep state, which is not interesting at all. Let $\dot{f}_\sigma=(\partial f/\partial\sigma_{\rm ij})\dot{\sigma}_{\rm ij}$, the function $G(\rho, t)$ is then assumed to be the form as,

$$G(\rho,t)=a\cdot\rho\cdot\rho^{C_{\rm n}\ln(1+t/t_1)} \quad (18)$$

where, a new parameters $C_{\rm n}$, which controls the stain rate dependency of soft rock, is introduced. From Eqs. (15) and (16), it is easy to obtain the following relation:

$$\Lambda=\frac{\dot{f}_\sigma+h(t)/C_{\rm p}}{\dfrac{1}{C_{\rm p}}\left(\dfrac{\partial f}{\partial t_{\rm kk}}+\dfrac{G(\rho,t)}{t_{\rm N}}\right)}=\frac{\dot{f}_\sigma+h(t)/C_{\rm p}}{\dfrac{h^{\rm p}}{C_{\rm p}}} \quad (19)$$

$$h^{\rm p}=\left(\frac{\partial f}{\partial t_{\rm kk}}+\frac{G(\rho,t)}{t_{\rm N}}\right) \quad (20)$$

Under a creep state $\dot{f}_\sigma=(\partial f/\partial\sigma_{\rm ij})\dot{\sigma}_{\rm ij}=0$, the following relation can be obtained:

$$\Lambda=\frac{C_{\rm p}}{h^{\rm p}}\cdot h(t)/C_{\rm p}=h(t)/h^{\rm p} \quad (21)$$

Therefore, the rate of viscoplastic strain tensor can be expressed as,

$$\dot{\varepsilon}_{\rm ij}^{\rm p}=\Lambda\cdot\frac{\partial f}{\partial t_{\rm ij}}=\frac{h(t)}{h^{\rm p}}\cdot\frac{\partial f}{\partial t_{\rm ij}} \quad (22)$$

and,

$$\dot{\varepsilon}_{\rm v}^{\rm p}=\frac{h(t)}{h^{\rm p}}\cdot\frac{\partial f}{\partial t_{\rm kk}} \quad (23)$$

The rate of total strain tensor can be divided into elastic and plastic components in the way as,

$$\dot{\varepsilon}_{\rm ij}=\dot{\varepsilon}_{\rm ij}^{\rm e}+\dot{\varepsilon}_{\rm ij}^{\rm p} \quad (24)$$

The rate of elastic strain tensor can be calculated by Hooke's theory as,

$$\dot{\varepsilon}_{\rm kl}^{\rm e}=E_{\rm ijkl}^{-1}\dot{\sigma}_{\rm ij},\quad E_{\rm ijkl}=\Gamma\delta_{\rm ij}\delta_{\rm kl}+G(\delta_{\rm ik}\delta_{\rm jl}+\delta_{\rm il}\delta_{\rm jk}) \quad (25)$$

$$\Gamma=\nu E/(1+\nu)(1-2\nu),\quad G=E/2/(1+\nu) \quad (26)$$

where E is Young's modulus and ν is Poisson's ratio.

From Eqs. (13), (15), (16), (24) and (25), it is easy to obtain the following equations:

$$\frac{\partial f}{\partial\sigma_{\rm ij}}E_{\rm ijkl}\dot{\varepsilon}_{\rm kl}-\frac{\partial f}{\partial\sigma_{\rm ij}}E_{\rm ijkl}\cdot\Lambda\cdot\frac{\partial f}{\partial t_{\rm kl}} -\frac{1}{C_{\rm p}}\left(\Lambda\cdot\frac{\partial f}{\partial t_{\rm mm}}+\Lambda\cdot\frac{G(\rho,t)}{t_{\rm N}}-h(t)\right)=0 \quad (27)$$

$$\Lambda=\frac{\dfrac{\partial f}{\partial\sigma_{\rm ij}}E_{\rm ijkl}\dot{\varepsilon}_{\rm kl}+h(t)/C_{\rm p}}{\dfrac{h^{\rm p}}{C_{\rm p}}+\dfrac{\partial f}{\partial\sigma_{\rm ij}}E_{\rm ijkl}\dfrac{\partial f}{\partial t_{\rm kl}}} \quad (28)$$

The loading criteria are given in a new way as:

$$\|\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{ij}}\|>0 \quad \text{if } \Lambda>0 \quad \text{and} \begin{cases}\dot{f}_\sigma>0 & \text{hardening}\\ \dot{f}_\sigma<0 & \text{softening}\\ \dot{f}_\sigma=0 & \text{pure creep}\end{cases}$$

$$\|\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{ij}}\|=0 \quad \text{if } \Lambda\leq 0 \quad \text{elastic} \tag{29}$$

Different from any other constitutive models, this loading criteria not only can judge the strain hardening and strain softening, but also a pure creep state which is regarded as a loading process. The reason to define a pure creep state as $\dot{f}_\sigma=0$ is that in ordinary models, $\dot{f}_\sigma=0$ means neutral loading, however, in present viscoplastic model, pure creep state may exist. The rate of viscoplastic strain tensor can then be expressed as,

$$\begin{aligned}\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{ij}}&=\Lambda\cdot\frac{\partial f}{\partial t_{\mathrm{ij}}}\\ &=\frac{\dfrac{\partial f}{\partial\sigma_{\mathrm{mn}}}\cdot E_{\mathrm{mnkl}}\cdot\dot{\varepsilon}_{\mathrm{kl}}+h(t)/C_{\mathrm{p}}}{\dfrac{h^{\mathrm{p}}}{C_{\mathrm{p}}}+\dfrac{\partial f}{\partial\sigma_{\mathrm{mn}}}\cdot E_{\mathrm{mnkl}}\cdot\dfrac{\partial f}{\partial t_{\mathrm{kl}}}}\cdot\frac{\partial f}{\partial t_{\mathrm{ij}}}\\ &=\frac{\dfrac{\partial f}{\partial\sigma_{\mathrm{mn}}}\cdot E_{\mathrm{mnkl}}\cdot\dot{\varepsilon}_{\mathrm{kl}}+h(t)/C_{\mathrm{p}}}{D}\cdot\frac{\partial f}{\partial t_{\mathrm{ij}}}\end{aligned} \tag{30}$$

$$D=\frac{h^{\mathrm{p}}}{C_{\mathrm{p}}}+\frac{\partial f}{\partial\sigma_{\mathrm{mn}}}\cdot E_{\mathrm{mnkl}}\cdot\frac{\partial f}{\partial t_{\mathrm{kl}}} \tag{31}$$

As to the rate of stress tensor, it can be evaluated as,

$$\dot{\sigma}_{\mathrm{ij}}=E_{\mathrm{ijkl}}\cdot(\dot{\varepsilon}_{\mathrm{kl}}-\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{kl}})=E_{\mathrm{ijkl}}\cdot\dot{\varepsilon}_{\mathrm{kl}}-E_{\mathrm{ijqr}}\cdot\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{qr}} \tag{32}$$

$$\dot{\sigma}_{\mathrm{ij}}=E_{\mathrm{ijkl}}\cdot\dot{\varepsilon}_{\mathrm{kl}}-\frac{E_{\mathrm{ijqr}}\cdot E_{\mathrm{mnkl}}\cdot\dfrac{\partial f}{\partial\sigma_{\mathrm{mn}}}\cdot\dfrac{\partial f}{\partial t_{\mathrm{qr}}}\cdot\dot{\varepsilon}_{\mathrm{kl}}}{D}-\frac{E_{\mathrm{ijqr}}\cdot h(t)/C_{\mathrm{p}}}{D}\cdot\frac{\partial f}{\partial t_{\mathrm{qr}}} \tag{33}$$

$$\dot{\sigma}_{\mathrm{ij}}=[E_{\mathrm{ijkl}}-E^{\mathrm{p}}_{\mathrm{ijkl}}]\cdot\dot{\varepsilon}_{\mathrm{kl}}-\Lambda\cdot E_{\mathrm{ijqr}}\cdot\frac{\partial f}{\partial t_{\mathrm{qr}}} \tag{34}$$

where

$$E^{\mathrm{p}}_{\mathrm{ijkl}}=\frac{E_{\mathrm{ijqr}}\cdot E_{\mathrm{mnkl}}\cdot\dfrac{\partial f}{\partial\sigma_{\mathrm{mn}}}\cdot\dfrac{\partial f}{\partial t_{\mathrm{qr}}}}{D},\quad \Lambda=\frac{h(t)}{D\cdot C_{\mathrm{p}}} \tag{35}$$

Equations (22) and (23) are used for calculating the rate of creep strain under creep state while Eq. (34) is used for calculating the rate of stress tensor from the rate of strain tensor.

DETERMINATION OF THE PARAMETERS IN THE MODEL

In normally consolidated state, the state variable ρ will be zero and therefore $G(\rho, t)$ will also be zero. From Eq. (23), it is easy to obtain the relation in creep state:

$$\dot{\varepsilon}^{\mathrm{p}}_{\mathrm{v}}=h(t) \tag{36}$$

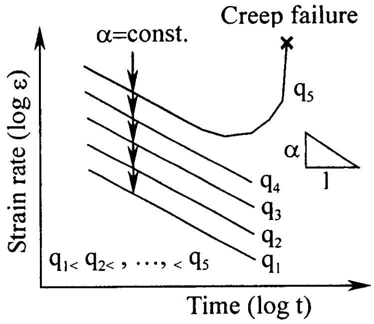

Fig. 4. Creep behavior of soft rock under different creep stresses (drained condition)

which means that function $h(t)$ is equivalent to the volumetric creep strain rate in so-called secondary consolidation of normally consolidated clay. As has be pointed out that for normally consolidated clay, the creep behavior including the secondary consolidation, can be described properly by a suitable elastoplastic model together with the soil-water coupled analysis. Therefore, the authors do not wish to express that the proposed model can describe the so-called secondary consolidation of normally consolidated clay in spite of the fact that it can. The only purpose to give Eq. (36) is just to give a physical meaning to the function $h(t)$. This model is only for soft rock, which can be regarded as a heavily overconsolidated soil.

The physical meaning of function of $\rho^{C_{\mathrm{n}}\ln(1+t)}$ is just the influence of the time effect on the state variable ρ that represents the state of overconsolidation of the geomaterials. In most cases, ρ is much less than 1.0 and the function of $\rho^{C_{\mathrm{n}}\ln(1+t)}$ will diminish to zero along with time. By introducing this time effect, it is possible to describe the rate dependency for overconsolidated geomaterials.

Compared to the model proposed by Nakai and Hinokio (2004), two new parameters C_{n}, and α are added in the new time dependent model. Parameter α is the gradient of strain rate vs. time in logarithmic axes as shown in Fig. 4 and can be uniquely determined with drained triaxial creep test. Parameter C_{n} can also be easily and uniquely determined by the difference in peak strengths of two rock samples under drained triaxial compression tests with different constant shear strain rates. Young's modulus E and Poisson's ratio ν can be determined with conventional way by drained triaxial compression tests. It should be pointed out that for soft clay, Young's modulus E is dependent on confining pressure, or mean stress, while for soft rock, it is independent of the confining pressure. Therefore, in present model, instead of using swelling index κ, Young's modulus E is a constant and is used as a parameter. Plastic modulus E_{p}, which is used to evaluate the relation between the plastic volumetric strain and mean stress, can be easily determined with isotropic consolidation test of soft rock and physically it equals to $\lambda-\kappa$. In other words, instead of using λ and κ, E and E_{p} are used in the new model. The stress ratio at failure state R_{f}, can also be easily determined with triaxial compression test, and can be calculated by the equation $R_f=(1+\sin\phi')/(1-\sin\phi')$, where ϕ' is an effective internal frictional angle. Parameter a, which controls the magnitude of evolution of the overconsolidated ratio, can be determined from the peak strength of soft rock. Detailed description about the determination of the parameter a can be found in the reference by Nakai and Hinokio (2004).

PERFORMANCE OF THE MODEL ON SOFT SEDIMENTARY ROCKS

Application to Triaxial Compression and Creep Tests on Ohya Stone

In order to check the performance of the proposed model, the drained triaxial compression tests and creep tests for saturated samples of Ohya stone, a porous tuff deposited in the Miocene Epoch of Tertiary Period, are analyzed. The drained triaxial compression tests were conducted by Adachi and Ogawa (1980). The physical properties of Ohya stone are listed in Table 1. The tests were conducted under four different confining pressure, $\sigma'_{30}=0.1$ MPa, 0.5 MPa, 1.0 MPa and 2.0 MPa, with a constant strain-rate controlled loading, that is, $\dot{\varepsilon}=0.025\%/\text{min}$. Table 2 lists the material parameters involved in the model.

Figure 5 shows the comparison between the predicted and the observed stress-strain relations and stress-dilatancy relations. It is seen from this figure that the model can predict the mechanical behavior of the soft rock to a satisfactory accuracy, with the only exception that in the case of 0.1 MPa, the theoretical volumetric strain is much bigger than the test result. In this case, the peak and residual strengths are simulated well while the volumetric strain was overestimated by the theory. The reason is that plastic volumetric strain is used as the hardening parameter of the model and therefore it is difficult to describe both the strength and the dilatancy well in small confining pressure. This is a limitation of the model. As we know, in small confining pressure, soft rock behaves more brittle, which means a sharp drop of strength after peak strength. Another point that should be emphasized is that R_f, the stress ratio at critical state, takes different values at different confining pressures, which was confirmed experimentally by Adachi and Ogawa (1980).

Table 1. Physical properties of Ohya stone

Void ratio e	0.72
Specific gravity of soil particle G_s	2.48
Initial yielding stress of consolidation p'_C (MPa)	15.0

Table 2. Material parameters involved in the model for Ohya stone

σ'_{30} (MPa)	E (MPa)	E_p	a	α	β	C_n	ν	R_f
0.1	900.0	0.040	500.0	0.70	1.50	0.025	0.0864	11.0
0.5	900.0	0.010	500.0	0.70	1.50	0.025	0.0864	6.75
1.0	900.0	0.010	500.0	0.70	1.50	0.025	0.0864	5.14
2.0	900.0	0.010	500.0	0.70	1.50	0.025	0.0864	4.18

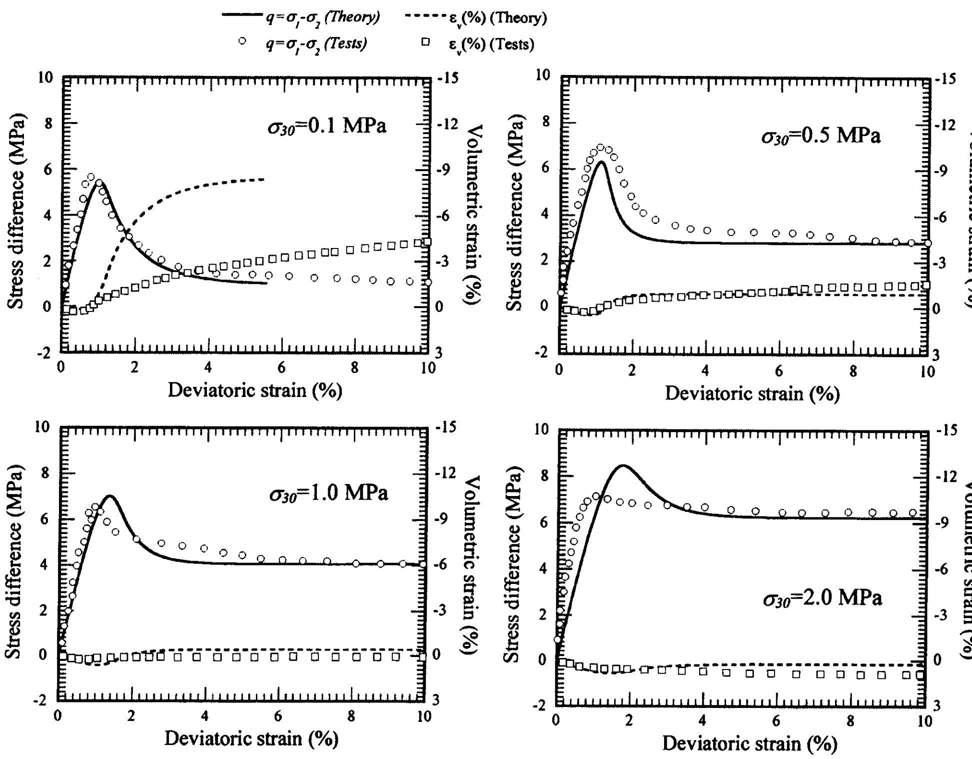

Fig. 5. Verification of the predicted stress-strain relation and stress-dilatancy relation for Ohya stone under constant strain-rate controlled ($\dot{\varepsilon}=0.025\%/\text{min}$) drained triaxial compression tests (tests by Adachi and Ogawa, 1980)

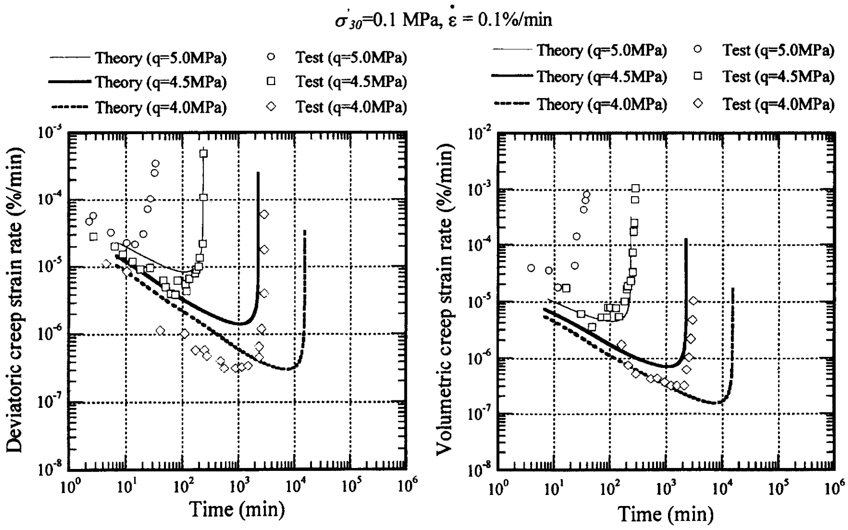

Fig. 6. Verification of the predicted strain rate-time relations for Ohya stone under drained triaxial creep tests (tests by Adachi and Takase, 1981)

The detailed relation between the value of the stress ratio at critical state and confining pressure is:

$$\left(\frac{q}{p'}\right)_{\text{critical}}=\alpha_{\mathrm{R}}^{*}\left(\frac{p'}{p'_0}\right)^{\beta_{\mathrm{R}}^{*}} \tag{37}$$

where β_{R}^{*} usually takes a value between 0.7 to 0.90. This property has already been solved in $p'-q$ space in a constitutive model for soft rock (Zhang, 1994). In $t_{\mathrm{S}}-t_{\mathrm{N}}$ space, however, it is still a problem that needs to be solved in future study. In the present work, the value of R_{f} takes different values for different confining pressures according to Eq. (37).

In the tests, shear bands were always observed in post peak region. As aforementioned, the softening behavior of the soft rock is merely a consequence of non-uniform deformation of an "element test". The "strain softening" fitted by the strain-softening model here is just an engineering approach to describe a drop in shear stress in macro behavior of the soft rock on convenience.

The drained triaxial creep tests on the same samples, which were conducted by Adachi and Takase (1981), are also analyzed. In the simulation, the results of drained creep tests under confining pressure of 0.1 MPa are considered. In the creep tests, the constant strain-rate in the process of loading to the creep stresses is different from the tests of drained triaxial compression tests by Adachi and Ogawa (1980), and is $\dot{\varepsilon}=0.10\%/\mathrm{min}$. The tests at three different creep stresses are simulated. Figure 6 shows the comparison between the theoretical and the experimental results of deviatoric and volumetric creep strain rates at a confining pressure of 0.1 MPa. It is found that the general characteristics of creep behavior, such as the initial creep rate, the steady creep stage and the creep rupture, can be simulated. The theoretical creep failure times, however, are one-order longer than the test results, which is thought to the reason that though the same samples were used in the two tests, there was a one-year time difference between them which might affect the time dependent behavior of the samples. It was reported by Koike (1997) that if the tuff like Ohya stone was placed on air for a period of time and then submerged into water to a saturated state, the peak strength of the tuff would decrease, which means that the creep failure time will be shortened. The parameters used in the simulation of creep tests are the same as the compression tests. If breaking this limitation, it is easy to fit the theoretical results to the test results by adjusting some parameters.

Table 3. Physical properties of Tomuro stone

Void ratio e	0.63
Specific gravity of soil particle G_{s}	2.51
Initial yielding stress of consolidation p'_{C} (MPa)	10.0

Table 4. Material parameters involved in the model for Tomuro stone

σ'_{30} (MPa)	E (MPa)	E_{p}	a	α	β	C_{n}	ν	R_{f}
0.1	350.0	0.023	500	0.80	1.50	0.050	0.0864	10.0

Application to Triaxial Compression Tests at Different Shear Strain Rates on Tomuro Stone

The proposed model is also used to analyze the behavior of triaxial compression tests at different shear strain rates on Tomuro stone (Koike, 1997), a type of Ohya stone. The tests were conducted by strain-controlled conventional triaxial compression tests at different strain rates under the confining pressure of $\sigma'_{30}=0.1$ MPa. The physical properties of Tomuro stone are listed in Table 3. Table 4 lists the material parameters involved in the model.

Figure 7 shows the relations of stress difference ($q=\sigma_{11}-\sigma_{33}$) vs. deviatoric strain. It is found from the figure that the rate dependency of sedimentary soft rock can be simulated well. Both the peak value of strength

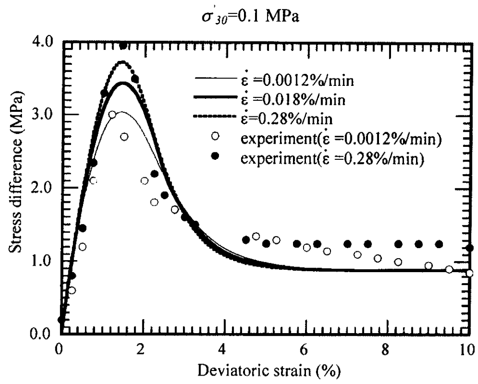

Fig. 7. Verification of the predicted stress-strain relation for Tomuro stone under constant strain-rate controlled drained triaxial compression tests (tests by Koike, 1997)

and the residual value of strength can be simulated. The theoretical results are also compared with the experimental results at constant strain rates of 0.28%/min and 0.0012%/min. It is found that the theoretical results are in good agreement with the experimental results.

CONCLUSIONS

In this paper, an elasto-viscoplastic constitutive model with strain softening is developed for soft sedimentary rock. The features of the model are listed as below:

(1) In order to consider the influence of intermediate stress on deformation and strength of geomaterials, tij concept is adopted.

(2) The concept of subloading yield surface proposed by Hashiguchi (1980) is used to describe the behavior of overconsolidated geomaterials, in which a state variable ρ related to overconsolidation, introduced by Nakai and Hinokio (2004) for overconsolidated soils, is also adopted for soft rocks.

(3) An associated flow rule, physically a much smarter formula than a non-associated flow rule, is adopted in the model, which was thought to be almost impossible in the previous models for soft rocks.

(4) In the evolution equation of the state variable ρ related to the size of subloading yield surface, a non-homogeneous function is newly proposed, which makes it possible to describe the strain rate dependency and creep behavior simultaneously.

(5) In the model, as is the same as Cam-clay model, instead of shearing strain, plastic volumetric strain is used as the hardening parameter, which is natural and is widely accepted by the researchers who specialize in the constitutive model for geomaterials. Since plastic volumetric strain is always used in defining critical state, it is more natural to use it as a hardening parameter than any other variable. Besides, it is directly related to void ratio that is most familiar to all geotechnical engineers and can be easily measured.

(6) Different from any other constitutive models, the loading criteria defined in this model not only can judge the strain hardening and strain softening, but also the creep state uniquely.

(7) The material parameters involved in the model have clear physical meanings and can be easily determined with triaxial compression tests and creep tests.

(8) The application of the model to the experimental results of soft sedimentary rock indicates that the model not only can describe the time dependency, such as strain rate dependency and creep, but also the strain softening behavior of overconsolidated geologic materials.

(9) Further works on the verification of the model by tests results is necessary, especially the tests under different loading rate conditions and drained or undrained conditions.

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