Fully Coupled Channel Approach to Doubly Strange s-Shell Hypernuclei

H. Nemura,[1,*] S. Shinmura,[2] Y. Akaishi,[3] and Khin Swe Myint[4]

[1]*Institute of Particle and Nuclear Studies, KEK, Tsukuba 305-0801, Japan*
[2]*Department of Information Science, Gifu University, Gifu 501-1193, Japan*
[3]*College of Science and Technology, Nihon University, Funabashi 274-8501, Japan*
[4]*Department of Physics, Mandalay University, Mandalay, Union of Myanmar*

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We describe *ab initio* calculations of doubly strange, $S = -2$, s-shell hypernuclei (${}^{4}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$He, and ${}^{6}_{\Lambda\Lambda}$He) as a first attempt to explore the few-body problem of the *full*-coupled channel scheme for these systems. The wave function includes $\Lambda\Lambda$, $\Lambda\Sigma$, $N\Xi$, and $\Sigma\Sigma$ channels. Minnesota NN, D2′ YN, and simulated YY potentials based on the Nijmegen hard-core model are used. Bound-state solutions of these systems are obtained. We find that a set of phenomenological B_8B_8 interactions among the octet baryons in $S = 0, -1$, and -2 sectors, which is consistent with all of the available experimental binding energies of $S = 0, -1$, and -2 s-shell (hyper)nuclei, can predict a particle stable bound state of ${}^{4}_{\Lambda\Lambda}$H. For ${}^{5}_{\Lambda\Lambda}$H and ${}^{5}_{\Lambda\Lambda}$He, ΛN-ΣN and ΞN-$\Lambda\Sigma$ potentials significantly affect the net $\Lambda\Lambda$-$N\Xi$ coupling, and a large Ξ probability is obtained even for a weaker $\Lambda\Lambda$-$N\Xi$ potential.

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Both recent experimental and theoretical studies of doubly strange ($S = -2$) s-shell hypernuclei (${}^{4}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$He, and ${}^{6}_{\Lambda\Lambda}$He) are of utmost interest in the field of hypernuclei [1–11]. An experimental report [1] on a new observation of ${}^{6}_{\Lambda\Lambda}$He has had a significant impact on strangeness nuclear physics. The *Nagara* event provides unambiguous identification of ${}^{6}_{\Lambda\Lambda}$He production, and suggests that the $\Lambda\Lambda$ interaction strength is rather weaker than that expected from an older experiment [12].

The BNL-AGS E906 experiment [2] has conjectured a formation of ${}^{4}_{\Lambda\Lambda}$H, in accordance with our earlier predictions [13,14] that ${}^{4}_{\Lambda\Lambda}$H would exist as a particle stable bound state against strong decay. If this is the case, the ${}^{4}_{\Lambda\Lambda}$H would be the lightest bound state among doubly strange hypernuclei. However, a theoretical study [3] of the weak-decay modes from ${}^{4}_{\Lambda\Lambda}$H does not support this conjecture, and our earlier studies should be reanalyzed by taking account of the new datum, Nagara event.

A recent Faddeev-Yakubovsky search for ${}^{4}_{\Lambda\Lambda}$H [4] found no bound-state solution over a wide range of $\Lambda\Lambda$ interaction strengths, although this conclusion has been in conflict with the result calculated by authors using a variational method [5]. The total binding energy is more sensitive to the ${}^{3}S_{1}$ channel of the ΛN interaction than to the ${}^{1}S_{0}$ $\Lambda\Lambda$ interaction, because the number of the ${}^{3}S_{1}$ ΛN pairs is 3 times larger than the number of the ${}^{1}S_{0}$ $\Lambda\Lambda$ pair, as was discussed in Ref. [4]. Therefore, the spin-dependent part of the ΛN interaction has to be determined very carefully. The algebraic structure of the $(\sigma_\Lambda \cdot \sigma_N)$ interaction for the $S = -2$ systems is similar to the structure for the ${}^{5}_{\Lambda}$He [15]. Namely, the ΛN interaction, which is utilized in the theoretical search for ${}^{4}_{\Lambda\Lambda}$H, has to reproduce the experimental $B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$ as well as the B_Λ's of $A = 3, 4$, $S = -1$ hypernuclei. However, there is a long-standing problem known as the ${}^{5}_{\Lambda}$He anomaly [16], since the publication by Dalitz *et al.* in 1972 [17]. Recently, Akaishi *et al.* [18] successfully resolved the anomaly by explicitly taking account of ΛN-ΣN coupling.

Considering the fact that the $\Lambda\Lambda$ system couples to the $N\Xi$ and $\Sigma\Sigma$ states, and also the ΛN system couples to the ΣN states, a theoretical search for ${}^{4}_{\Lambda\Lambda}$H should be made in a fully coupled channel formulation with a set of interactions among the octet baryons. The ${}^{5}_{\Lambda\Lambda}$H-${}^{5}_{\Xi}$H (or ${}^{5}_{\Lambda\Lambda}$He-${}^{5}_{\Xi}$He) mixing due to $\Lambda\Lambda$-$N\Xi$ coupling is also an interesting topic, since the α-formation effect could be significant [7,9]. Thus, the purpose of this study is threefold: First, it is to describe a systematic study for the complete set of s-shell hypernuclei with $S = -2$ in a framework of a full-coupled channel formulation. Second, it is to make a conclusion if a set of baryon-baryon interactions, which is consistent with the experimental data, predicts a particle stable bound state of ${}^{4}_{\Lambda\Lambda}$H. The third is to explore the fully hyperonic mixing of ${}^{5}_{\Lambda\Lambda}$H, including the ΛN-ΣN transition potential in addition to $\Lambda\Lambda$-$N\Xi$-$\Sigma\Sigma$.

The wave function of a system with $S = -2$, comprising $A(= N + Y)$ octet baryons, has four isospin-basis components. For example, ${}^{6}_{\Lambda\Lambda}$He has four components as $ppnn\Lambda\Lambda$, $NNNNN\Xi$, $NNNN\Lambda\Sigma$, and $NNNN\Sigma\Sigma$. We abbreviate these components as $\Lambda\Lambda$, $N\Xi$, $\Lambda\Sigma$, and $\Sigma\Sigma$, referring to the last two baryons. The Hamiltonian of the system is hence given by 4×4 components as

$$H = \begin{pmatrix} H_{\Lambda\Lambda} & V_{N\Xi-\Lambda\Lambda} & V_{\Lambda\Sigma-\Lambda\Lambda} & V_{\Sigma\Sigma-\Lambda\Lambda} \\ V_{\Lambda\Lambda-N\Xi} & H_{N\Xi} & V_{\Lambda\Sigma-N\Xi} & V_{\Sigma\Sigma-N\Xi} \\ V_{\Lambda\Lambda-\Lambda\Sigma} & V_{N\Xi-\Lambda\Sigma} & H_{\Lambda\Sigma} & V_{\Sigma\Sigma-\Lambda\Sigma} \\ V_{\Lambda\Lambda-\Sigma\Sigma} & V_{N\Xi-\Sigma\Sigma} & V_{\Lambda\Sigma-\Sigma\Sigma} & H_{\Sigma\Sigma} \end{pmatrix}, \quad (1)$$

where $H_{B_1B_2}$ operates on the B_1B_2 component, and $V_{B_1B_2-B_1'B_2'}$ is the sum of all possible two-body transition potentials connecting the B_1B_2 and $B_1'B_2'$ components:

$$V_{\Lambda\Lambda-N\Xi} = v_{\Lambda\Lambda-N\Xi}, \tag{2}$$

$$V_{\Lambda\Lambda-\Lambda\Sigma} = \sum_{i=1}^{N} v_{N_i\Lambda-N_i\Sigma}, \qquad V_{N\Xi-\Lambda\Sigma} = v_{N\Xi-\Lambda\Sigma}, \tag{3}$$

$$V_{\Lambda\Lambda-\Sigma\Sigma} = v_{\Lambda\Lambda-\Sigma\Sigma}, \qquad V_{N\Xi-\Sigma\Sigma} = v_{N\Xi-\Sigma\Sigma}, \tag{4}$$

$$V_{\Lambda\Sigma-\Sigma\Sigma} = \sum_{i=1}^{N} v_{N_i\Lambda-N_i\Sigma} + v_{\Lambda\Sigma-\Sigma\Sigma}. \tag{5}$$

Note that we take account of *full-coupled* channel potentials including the ΛN-ΣN and $N\Xi$-$\Lambda\Sigma$ transitions [Eq. (3)] in the $^3S_1$ channel, while other full-coupled channel approaches (e.g., Refs. [10,11]) take only $\Lambda\Lambda$-$N\Xi$-$\Sigma\Sigma$ in the $^1S_0$ channel [Eqs. (2) and (4)] into account.

In the present calculations, we use the Minnesota potential [19] for the NN interaction and D2$'$ for the YN interaction. The Minnesota potential reproduces reasonably well both the binding energies and the sizes of few-nucleon systems, such as ^{2}H, ^{3}H, ^{3}He, and ^{4}He [20]. The D2$'$ potential is a modified potential from the original D2 potential [18]. The strength of the long-range part (V_b in Table I of Ref. [18]) of the D2$'$ potential in the ΛN-ΛN $^3S_1$ channel is reduced by multiplying by a factor (0.954) in order to reproduce the experimental $B_\Lambda(^5_\Lambda\mathrm{He})$ value. The calculated B_Λ values for the Λ hypernuclei ($^3_\Lambda$H, $^4_\Lambda$H, $^4_\Lambda$He, $^4_\Lambda$H*, $^4_\Lambda$He*, and $^5_\Lambda$He) are 0.056, 2.23, 2.17, 0.91, 0.89, and 3.18 MeV, respectively. For the YY interaction, we use a full-coupled channel potential among the octet baryons in both the spin triplet and the spin singlet channels. We assume that the YY potential consists of only the central component, and the effect due to the noncentral force (e.g., tensor force) should be included into the central part effectively. The YY potential has Gaussian form factors, whose parameters are set to reproduce the low-energy S matrix of the Nijmegen hard-core model D (ND) or F (NF) [21]. We take the hard-core radius to be $r_c = 0.562\,71$ $(0.449\,15)$ fm in the spin singlet (triplet) channel for the ND, whereas $r_c = 0.529\,72$ $(0.524\,33)$ fm is used in the singlet (triplet) channel for the NF. Each number is the same as the hard-core radius of the YN sector in each channel for each model. The strength parameters are first determined on a charge basis, and then the strength parameters on an isospin basis are constructed from the charge-basis parameters. We denote ND$_S$ (NF$_S$) for the simulating ND (NF) potential [22].

The calculations are made by using the stochastic variational method [23,24]. This is essentially along the lines of Ref. [25], except for the isospin function. The isospin function consists of four components, in accordance with Eq. (1). The reader is referred to Refs. [24,25] for the details of the method.

Table I lists the $B_{\Lambda\Lambda}$ values for $S = -2$ hypernuclei. Using ND$_S$ or NF$_S$ YY potential, we have obtained the bound-state solutions of $^4_{\Lambda\Lambda}$H, $^5_{\Lambda\Lambda}$H, $^5_{\Lambda\Lambda}$He, and $^6_{\Lambda\Lambda}$He. In the case of the ND$_S$ YY potential, we have

$$\Delta B^{(\mathrm{calc})}_{\Lambda\Lambda}(^{\,6}_{\Lambda\Lambda}\mathrm{He}) = B^{(\mathrm{calc})}_{\Lambda\Lambda}(^{\,6}_{\Lambda\Lambda}\mathrm{He}) - 2B^{(\mathrm{calc})}_{\Lambda}(^5_\Lambda\mathrm{He}) = 1.59\ \mathrm{MeV}, \tag{6}$$

which is slightly larger than the experimental value $[\Delta B^{(\mathrm{exp})}_{\Lambda\Lambda}(^{\,6}_{\Lambda\Lambda}\mathrm{He}) = 1.01 \pm 0.20^{+0.18}_{-0.11}$ MeV [1]], while the NF$_S$ reproduces the experimental value fairly well. We have also calculated these systems using a modified ND$_S$ (mND$_S$) YY potential; the strength of the $\Lambda\Lambda$ diagonal part of the mND$_S$ potential is reduced by multiplying by a factor of 0.8 from the original ND$_S$ in order to reproduce the experimental $\Delta B_{\Lambda\Lambda}(^{\,6}_{\Lambda\Lambda}\mathrm{He})$. The scattering length and effective range parameters for the ND$_S$, mND$_S$, and NF$_S$ are (given in units of fm) $(a_s, r_s) = (-1.37, 4.98)$, $(-0.91, 6.25)$, and $(-0.40, 12.13)$, respectively. The scattering length for the mND$_S$ or NF$_S$ is consistent with the other analyses [4,10] concerning the Nagara event. We should note that the mND$_S$ potential predicts the particle stable bound state of $^{\,4}_{\Lambda\Lambda}$H; the obtained energy is very close to, but still (0.002 MeV) lower than, the $^3_\Lambda\mathrm{H} + \Lambda$ threshold. Therefore, due to the result for mND$_S$ or NF$_S$, we should come to the following novel conclusion: A set of phenomenological baryon-baryon interactions among the octet baryons in $S = 0, -1$, and -2 sectors, which is consistent with the Nagara event as well as all the experimental binding energies of $S = 0$ and -1 s-shell (hyper)-nuclei, can predict a particle stable bound state of $^{\,4}_{\Lambda\Lambda}$H.

Figure 1 schematically displays the present results of the full-coupled channel calculations of $A = 3 - 6, S = -1, -2$ hypernuclei, using the mND$_S$. Since the present calculation has been made on the isospin basis, the results for $^{\,5}_{\Lambda\Lambda}$He are qualitatively similar to the results for $^{\,5}_{\Lambda\Lambda}$H, so that we omit the explicit result for $^{\,5}_{\Lambda\Lambda}$He. Figure 1 also displays the probabilities of the $N\Xi$, $\Lambda\Sigma$, and $\Sigma\Sigma$ components for the $S = -2$ hypernuclei. In the case of NF$_S$, the probabilities are (given in percentage) $(P_{N\Xi}, P_{\Lambda\Sigma}, P_{\Sigma\Sigma}) = (0.58, 0.38, 0.03)$ for $^{\,4}_{\Lambda\Lambda}$H, $(3.10, 2.10, 0.10)$ for $^{\,5}_{\Lambda\Lambda}$H, and $(1.34, 1.14, 0.10)$ for $^{\,6}_{\Lambda\Lambda}$He, respectively. In the present calculations, the $\Lambda\Lambda$ component is the main part of the wave function. No unrealistic bound states were found for the YY subsystem, since the hard-core model hardly incorporates an unrealistic strong attractive force in the short-range region, in contrast to the soft core model, such as the Nijmegen soft-core potentials NSC97e or NSC97f [11]. This is one of the reasons why we used the YY potential

TABLE I. $\Lambda\Lambda$ separation energies, given in units of MeV, of $A = 4 - 6$, $S = -2$ s-shell hypernuclei.

YY	$B_{\Lambda\Lambda}(^{\,4}_{\Lambda\Lambda}\mathrm{H})$	$B_{\Lambda\Lambda}(^{\,5}_{\Lambda\Lambda}\mathrm{H})$	$B_{\Lambda\Lambda}(^{\,5}_{\Lambda\Lambda}\mathrm{He})$	$B_{\Lambda\Lambda}(^{\,6}_{\Lambda\Lambda}\mathrm{He})$
ND$_S$	0.107	4.05	3.96	7.94
mND$_S$	0.058	3.75	3.66	7.54
NF$_S$	0.128	3.84	3.77	7.53
Expt.				$7.25 \pm 0.19^{+0.18}_{-0.11}$

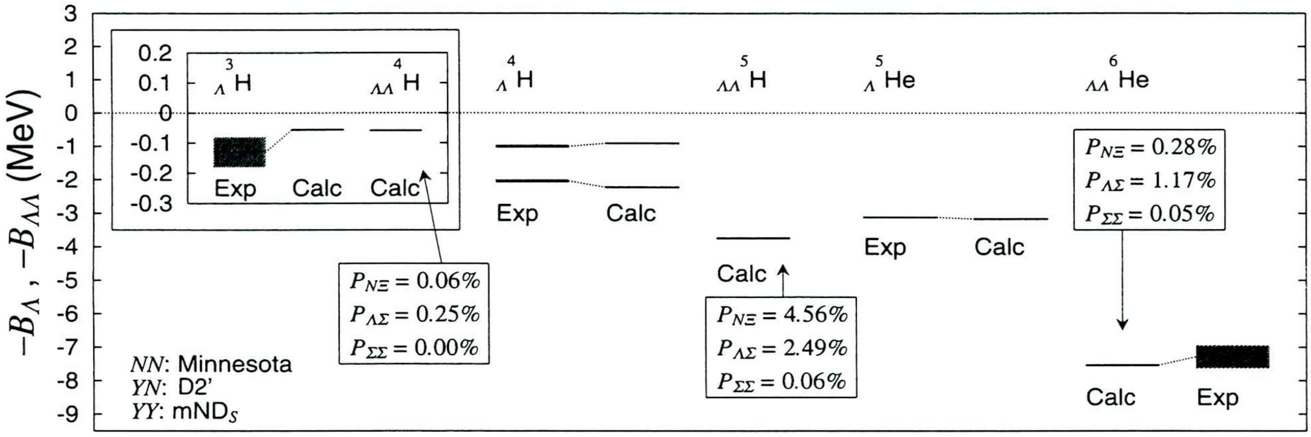

FIG. 1. Λ and $\Lambda\Lambda$ separation energies of $A = 3-6$, $S = -1$ and -2 s-shell hypernuclei. The Minnesota NN, D2$'$ YN, and mND$_S$ YY potentials are used. The width of the line for the experimental B_Λ or $B_{\Lambda\Lambda}$ value indicates the experimental error bar. The probabilities of the $N\Xi$, $\Lambda\Sigma$, and $\Sigma\Sigma$ components are also shown for the $\Lambda\Lambda$ hypernuclei.

constructed from the hard-core model, for the first attempt to the full-coupled channel calculation. $P_{\Sigma\Sigma}$'s are very small for both systems, due to a large mass difference between the $\Lambda\Lambda$ and $\Sigma\Sigma$ channels ($m_{\Sigma\Sigma} - m_{\Lambda\Lambda} \cong 155$ MeV).

We should emphasize that the $P_{N\Xi}({}^{\,5}_{\Lambda\Lambda}\mathrm{H})$ obtained by the mND$_S$ has a surprisingly large value (4.56%), which is larger than the $P_{N\Xi}({}^{\,5}_{\Lambda\Lambda}\mathrm{H})$ obtained by the NF$_S$ (3.10%), in spite of the fact that the strength of the $\Lambda\Lambda$-$N\Xi$ coupling potential of the ND is rather weaker than that of the NF. This does not imply that a stronger $\Lambda\Lambda$-$N\Xi$ coupling potential means a larger $P_{N\Xi}$ probability. This is in remarkable contrast with other calculations based on the $(t + \Lambda + \Lambda)$ and $(\alpha + \Xi^-)$ two-channel model [7,9].

Although the present calculation assumes no simplified structures, such as $(t + \Lambda + \Lambda)$ and $(\alpha + \Xi^-)$, this kind of model is useful to make a clear explanation of the complicated full coupling dynamics of the $A = 5$, $S = -2$ hypernucleus. Let us consider a set of simple *core nucleus* + $Y(+Y)$ model wave functions for the ${}^{\,5}_{\Lambda\Lambda}\mathrm{H}$:

$$|{}^{\,5}_{\Lambda\Lambda}\mathrm{H}\rangle = \psi_t \times \psi_{\Lambda\Lambda} \times \psi_{\Lambda\Lambda-t}, \tag{7}$$

$$|{}^{\,5}_{\Xi}\mathrm{H}\rangle = \psi_\alpha \times \psi_{\Xi^-} \times \psi_{\Xi^- -\alpha}, \tag{8}$$

$$|{}^{\,5}_{\Lambda\Sigma}\mathrm{H}\rangle_{S_{\Lambda\Sigma}} = \sqrt{\tfrac{1}{3}}[\psi_t \times [\psi_{\Lambda\Sigma^0}]_{S_{\Lambda\Sigma}}] \times \psi_{\Lambda\Sigma^0-t} - \sqrt{\tfrac{2}{3}}[\psi_h \times [\psi_{\Lambda\Sigma^-}]_{S_{\Lambda\Sigma}}] \times \psi_{\Lambda\Sigma^- -h} \quad (\text{for } S_{\Lambda\Sigma} = 0 \text{ or } 1), \tag{9}$$

where $\psi_c(c = t, h, \alpha)$ is the wave function (WF) of the core nucleus, $\psi_{YY}(YY = \Lambda\Lambda, \Xi^-, \Lambda\Sigma)$ is the WF of the hyperon(s), and ψ_{YY-c} is the WF that describes the relative motion between YY and c. We assume that all of the baryons occupy the same $(0s)$ orbit. For the ${}^{\,5}_{\Lambda\Sigma}\mathrm{H}$ state, we have two independent states for the WF $\psi_{\Lambda\Sigma}$, that the spin of two hyperons ($S_{\Lambda\Sigma}$) is either a singlet or a triplet. Since the $\Sigma\Sigma$ component plays a minor role, we omit the ${}^{\,5}_{\Sigma\Sigma}\mathrm{H}$ state. Using these WFs, we can obtain the algebraic factors for each averaged coupling potential of the allowed spin state, $\bar{v}^s$ or $\bar{v}^t$:

$$\langle V_{\Lambda\Lambda-N\Xi}\rangle = \sqrt{\tfrac{1}{2}}\bar{v}^s_{\Lambda\Lambda-N\Xi}, \tag{10}$$

$$\langle V_{\Lambda\Lambda-\Lambda\Sigma}\rangle = \begin{cases} \sqrt{\tfrac{9}{8}}\bar{v}^t_{N\Lambda-N\Sigma} + \sqrt{\tfrac{1}{8}}\bar{v}^s_{N\Lambda-N\Sigma} & (\text{for } S_{\Lambda\Sigma} = 0), \\ \sqrt{\tfrac{3}{8}}\bar{v}^t_{N\Lambda-N\Sigma} - \sqrt{\tfrac{3}{8}}\bar{v}^s_{N\Lambda-N\Sigma} & (\text{for } S_{\Lambda\Sigma} = 1), \end{cases} \tag{11}$$

$$\langle V_{N\Xi-\Lambda\Sigma}\rangle = \begin{cases} -\sqrt{\tfrac{3}{4}}\bar{v}^s_{N\Xi-\Lambda\Sigma} & (\text{for } S_{\Lambda\Sigma} = 0), \\ \tfrac{3}{2}\bar{v}^t_{N\Xi-\Lambda\Sigma} & (\text{for } S_{\Lambda\Sigma} = 1). \end{cases} \tag{12}$$

The $v_{\Lambda\Lambda-N\Xi}$ potential is suppressed by a factor of $\sqrt{1/2}$ for the $A = 5$ hypernucleus. The $v_{N\Lambda-N\Sigma}$ and $v_{N\Xi-\Lambda\Sigma}$ potentials, particularly in the spin triplet channel, play significant roles instead. Namely, these equations imply that the $\Lambda\Sigma$ component strongly couples both to the $\Lambda\Lambda$ and to the $N\Xi$ components, and the $\Lambda\Sigma$ component plays a crucial role in the hypernucleus.

The normalized energy expectation values of the Hamiltonian (1) for ${}^{\,5}_{\Lambda\Lambda}\mathrm{H}$ are (given in units of MeV)

$$h = \begin{pmatrix} \frac{\langle H_{\Lambda\Lambda}\rangle}{P_{\Lambda\Lambda}} & \frac{\langle V_{N\Xi-\Lambda\Lambda}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{N\Xi}}} & \frac{\langle V_{\Lambda\Sigma-\Lambda\Lambda}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{\Lambda\Sigma}}} \\ \frac{\langle V_{\Lambda\Lambda-N\Xi}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{N\Xi}}} & \frac{\langle H_{N\Xi}\rangle}{P_{N\Xi}} & \frac{\langle V_{\Lambda\Sigma-N\Xi}\rangle}{\sqrt{P_{N\Xi}P_{\Lambda\Sigma}}} \\ \frac{\langle V_{\Lambda\Lambda-\Lambda\Sigma}\rangle}{\sqrt{P_{\Lambda\Lambda}P_{\Lambda\Sigma}}} & \frac{\langle V_{N\Xi-\Lambda\Sigma}\rangle}{\sqrt{P_{N\Xi}P_{\Lambda\Sigma}}} & \frac{\langle H_{\Lambda\Sigma}\rangle}{P_{\Lambda\Sigma}} \end{pmatrix}$$

$$= \begin{cases} \begin{pmatrix} -9.12 & -1.82 & -14.52 \\ -1.82 & 5.01 & -10.39 \\ -14.52 & -10.39 & 92.41 \end{pmatrix} & (\text{for the mND}_S), \\ \begin{pmatrix} -6.09 & -20.51 & -14.92 \\ -20.51 & 115.4 & -10.01 \\ -14.92 & -10.01 & 101.6 \end{pmatrix} & (\text{for the NF}_S). \end{cases} \tag{13}$$

Here, we display only the 3×3 components of the Hamiltonian (1), comprising $\Lambda\Lambda$, $N\Xi$, and $\Lambda\Sigma$, since the contributions from the $\Sigma\Sigma$ component are not large. The first 2×2 components for the mND$_S$ are quite different from those for the NF$_S$. These numbers reflect the nature of each YY potential model (ND or NF). The ND has weak $\Lambda\Lambda$-$N\Xi$ coupling and weakly attractive $N\Xi$-$N\Xi$ potential, which is consistent with recent experimental data [26–28], while the NF gives strong $\Lambda\Lambda$-$N\Xi$ and strongly repulsive $N\Xi$-$N\Xi$ potential in the $I = 0$, ${}^1S_0$ channel.

If we solve the eigenvalue problem, $\det(h - \lambda I) = 0$, we obtain the ground state energy, $E = -11.82$ MeV (-11.83 MeV), and the probability, $P_{N\Xi} = 3.99\%$ (2.84%), for the mND$_S$ (NF$_S$). On the other hand, if we ignore the last row and the last column, the eigenenergy of only the first 2×2 subspace becomes $E = -9.35$ MeV (-9.46 MeV), and the probability, $P_{N\Xi} = 1.57\%$ (2.63%), for the mND$_S$ (NF$_S$). This clearly means that the couplings between the ($\Lambda\Lambda$, $N\Xi$) and $\Lambda\Sigma$ components play crucial roles to make ${}^{5}_{\Lambda\Lambda}$H (and ${}^{5}_{\Lambda\Lambda}$He) bound. In the case of mND$_S$, the large coupling potentials, $\langle V_{\Lambda\Lambda-\Lambda\Sigma}\rangle$ and $\langle V_{N\Xi-\Lambda\Sigma}\rangle$, also enhance the $P_{N\Xi}$ probability. On the other hand, for NF$_S$, these coupling potentials hardly enhance $P_{N\Xi}$, though the total binding energy significantly increases.

In summary, we have performed full-coupled channel *ab initio* calculations for the complete set of doubly strange s-shell hypernuclei. Two kinds of YY interactions, mND$_S$ and NF$_S$, reproduce the $\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$ of the Nagara event. We obtained bound-state solutions for ${}^{4}_{\Lambda\Lambda}$H, ${}^{5}_{\Lambda\Lambda}$H, and ${}^{5}_{\Lambda\Lambda}$He by using these YY interactions. We thus conclude that a set of phenomenological B_8B_8 interactions among the octet baryons in the $S = 0, -1, -2$ sectors, which is consistent with all of the available experimental binding energies of the $S = 0, -1, -2$ s-shell (hyper)nuclei, can predict a particle stable bound state of ${}^{4}_{\Lambda\Lambda}$H. For the ${}^{5}_{\Lambda\Lambda}$H (and ${}^{5}_{\Lambda\Lambda}$He), we found that the ΛN-ΣN and $N\Xi$-$\Lambda\Sigma$ potentials are considerably important to make the system bound, and affect the net effect of the $\Lambda\Lambda$-$N\Xi$ coupling potential. One-boson-exchange potential models for the B_8B_8 interactions have sound bases of the SU(3) symmetry and are widely accepted, though uncertainties of the interactions in the $S = -2$ sector are still large because of the limitation of experimental information. The NSC97 models have a crucial defect in $\Lambda\Lambda$-$N\Xi$-$\Sigma\Sigma$ couplings. Therefore, we have attempted only two possible cases with the models ND and NF, which have different characters in the $S = -2$ sector (strengths of $\Lambda\Lambda$-$N\Xi$ coupling and $N\Xi$-$N\Xi$ diagonal potentials), though the recent Ξ-nucleus data are in favor of only the ND, and the strongly repulsive $N\Xi$-$N\Xi$ interaction of the NF might be suspect. We do hope that a future experimental facility (e.g., J-PARC) develops our knowledge of the $S = -2$ interactions.

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*Present address: Advanced Meson Science Laboratory, DRI, RIKEN, Wako, Saitama, 351-0198, Japan.

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