

Decentralized Adaptive Coordinated Control of Multiple Robot Arms Handling One Object*

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This paper presents an adaptive decentralized coordinated control method for multiple robot arms grasping a common object. In the proposed controller, the dynamic parameters of both object and robot arms are estimated adaptively. The desired motions of the robot arms are generated by an estimated object reference model and each robot controller works independently as hybrid adaptive controller. The asymptotic stability of position and internal force of the object is proven by the Lyapunov-Like Lemma. Experimental comparisons between adaptive control with force sensor and one without force sensor for the two robot arms grasping a common object are shown.

Key Words: Robot, Adaptive Control, Manipulator, Coordinated Control, Force and Position Control, Stability

1. Introduction

The robotics system consisting of multiple robots has more capacity than the single robot system for the tasks such as handling of a heavy material and assembly. Many researchers have studied the coordinated control of multiple robot arms actively. When multiple robots grasp an object, the robotics system forms a closed chain mechanism that is extremely nonlinear and coupled. The control of such system is more complicated because a set of holonomic equality constraints is imposed and the number of actuators exceeds the mobility of the system. The control objective is both of the motion of the grasped object and the internal force exerted on the object by robot arms, which does not affect its motion.

Most of the coordinated controls are based on the impedance control^{(1),(2)} and the hybrid position/force control⁽³⁾⁻⁽⁵⁾, for which the dynamic parameters of the object and the robot arms are known. It is well known that it is very difficult and time consuming to identify the dynamic parameters of the object and the robot arms precisely. Moreover, the dynamic parameters of the object

often vary according to the task, which is variable. To control multiple robot arms whose dynamic parameters are unknown, a control method using neural networks⁽⁶⁾, a learning control⁽⁷⁾, and sliding mode controls^{(8),(9)} have been respectively proposed. Pagilla and Tomizuka⁽¹⁰⁾ presented an adaptive control, in which the parameters of the robot arms were known and the parameters of the object were estimated adaptively. Liu et al.⁽¹¹⁾ also proposed an adaptive control, in which the dynamic parameters of the robot arms were estimated based on the orthogonality of force and velocity in the joint spaces.

The centralized controller suffers from its complicated architecture because the state space of the dynamics of multiple robots has a high dimension. The method of decentralized control⁽¹²⁾, the impedance control method⁽¹³⁾ and the force feedback control method in the task space⁽¹⁴⁾, in which each robot is controlled separately by its own controller, are desirable approaches because of their simple architecture. However, these decentralized controllers are not adaptive. Kawasaki et al.⁽¹⁵⁾ have presented an adaptive coordinated control without force sensor, in which each robot controller works independently, but asymptotic convergence of the internal force has not been guaranteed.

In this paper, an adaptive decentralized coordinated control method with force sensor for multiple robot arms, which guarantees the asymptotic convergence of the motion and internal force of a grasped object, is proposed. In the control, the dynamic parameters of both the object and the robot arms are estimated. The desired motions

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of the robot arms are generated by an estimated object reference model, which is evaluated using parameter estimates and desired motion of the object. The motion of each robot arm is controlled independently and adaptively without any communication between the arms. The global asymptotic stability of motion and internal force of the object is proved by the Lyapunov-Like Lemma. Moreover, experimental comparisons between adaptive control with force sensor and one without force sensor for two robot arms grasping an object are shown.

2. Modeling of System Dynamics

Consider the dynamic equations of k non-redundant arms holding a common rigid object as shown in Fig. 1, in which all robot end-effectors hold the same object moving in three-dimensional space. The coordinate systems are defined as follows: Σ_p is the task coordinate system, Σ_o is the object coordinate system, fixed on the object, Σ_i is the i -th arm coordinate system, fixed on the i -th end-effector located at the grasping point, and Σ_{s_i} is the i -th arm base coordinate system. We also use the notations defined as: $p_o \in R^3$ is the position vector of the origin of the object coordinate system Σ_o with respect to Σ_p , $\Phi_o \in R^3$ is the orientation vector of the object coordinate system Σ_o with respect to Σ_p , $r_o = [p_o^T \ \Phi_o^T]^T$ is the position and orientation vector of the object with respect to the task coordinate system Σ_p , $p_i \in R^3$ is the position vector of the origin of Σ_i with respect to Σ_p , $\Phi_i \in R^3$ is the orientation vector of Σ_i with respect to Σ_p , $p_{oc_i} \in R^3$ is the position vector from the origin of Σ_o to the origin of Σ_i with respect to Σ_p and $r_i = [p_i^T \ \Phi_i^T]^T$ is the position and orientation vector of the i -th arm with respect to the task coordinate system Σ_p . Hereafter, a time derivative of any vector a is described by \dot{a} .

To facilitate the dynamic formulation, the following assumptions are made.

A1: All the end-effectors of the robot arms are rigidly attached to the common object so that no relative mo-

tion occurs between the object and any end-effector, and it is possible to generate arbitrary force and moment at any contact.

A2: $r_o, \dot{r}_o, r_i,$ and \dot{r}_i are measurable.

A3: The desired position r_o^d , desired velocity \dot{r}_o^d , desired acceleration \ddot{r}_o^d , and vector to generate desired internal force f_{int}^d of the object are time-continuous and bounded.

2.1 Object dynamics

Each robot arm applies a force f_i and a moment n_i through the contact point C_i to the object. Then, the resultant force and moment, f_o, n_o , applied to the object by multiple robot arms are given by

$$f_o = \sum_{i=1}^k f_i \tag{1}$$

$$n_o = \sum_{i=1}^k (n_i + p_{oc_i} \times f_i). \tag{2}$$

Equations (1) and (2) can be rewritten as

$$F_o = \sum_{i=1}^k W_i F_i \tag{3}$$

where $F_o = [f_o^T \ n_o^T]^T$ is the external force applied to the object, $F_i = [f_i^T \ n_i^T]^T$ is the force and moment applied to the object by the i -th robot arm, and W_i is a grasp-form matrix at the contact point C_i given by

$$W_i = \begin{bmatrix} I & 0 \\ \hat{p}_{oc_i} & I \end{bmatrix} \tag{4}$$

where I is the identity matrix and \hat{p} is defined as such the relation $\hat{p}a = p \times a$ is satisfied for any vectors a, p . Equation (3) is represented in a more compact form as follows:

$$F_o = WF \tag{5}$$

where $W = [W_1 \ \dots \ W_k]$ and $F = [F_1^T \ \dots \ F_k^T]^T$.

In general, the dynamic equation of the object⁽¹⁸⁾ is represented by

$$M_o(r_o)\ddot{r}_o + C_o(r_o, \dot{r}_o)\dot{r}_o + g_o(r_o) = F_o \tag{6}$$

where M_o is the symmetric positive definite inertia matrix, $C_o(r_o, \dot{r}_o)\dot{r}_o$ is the vector of Coriolis and centrifugal forces, and g_o is the gravitational force term. It is well known that: 1) a suitable definition of C_o makes matrix $\dot{M}_o - 2C_o$ skew-symmetric, and 2) the dynamic model of the object is linear regarding with the dynamic parameter vector, as follow:

$$M_o(r_o)\dot{v}_o + C_o(r_o, \dot{r}_o)v_o + g_o(r_o) = Y_o(r_o, \dot{r}_o, v_o, \dot{v}_o)\sigma_o \tag{7}$$

where v_o is an arbitrarily defined velocity, σ_o is the dynamic parameter vector of the object, and $Y_o(r_o, \dot{r}_o, v_o, \dot{v}_o)$ is a regressor matrix with respect to σ_o . These structural properties are utilized in our controller.

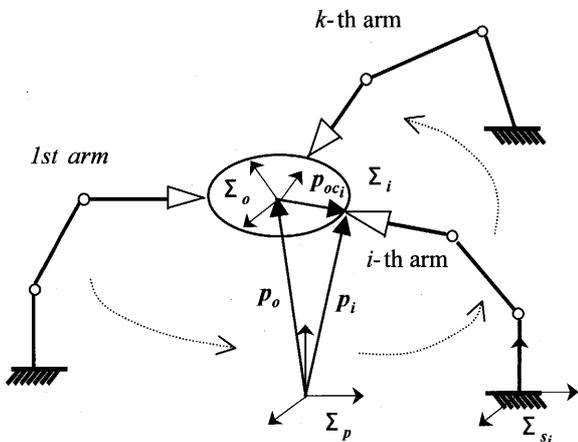


Fig. 1 Grasped object and coordinate systems

2.2 Robot arm dynamics

The dynamic equation of the i -th robot arm is represented by

$$\mathbf{M}_i(\mathbf{r}_i)\ddot{\mathbf{r}}_i + \mathbf{C}_i(\mathbf{r}_i, \dot{\mathbf{r}}_i)\dot{\mathbf{r}}_i + \mathbf{g}_i(\mathbf{r}_i) = \mathbf{u}_i - \mathbf{F}_i \quad (8)$$

where $\mathbf{r}_i = [\mathbf{p}_i^T \quad \Phi_i^T]^T$ is the position and orientation vector of the i -th arm with respect to the task coordinate system Σ_p , \mathbf{M}_i is the symmetric positive definite inertia matrix, \mathbf{C}_i is the damping coefficient matrix, \mathbf{g}_i is the gravitational force term, and \mathbf{u}_i is the control input given in the task coordinate system. It is well known that the robot dynamic model is linear regarding with the dynamic parameters, as

$$\mathbf{M}_i(\mathbf{r}_i)\dot{\mathbf{v}}_i + \mathbf{C}_i(\mathbf{r}_i, \dot{\mathbf{r}}_i)\mathbf{v}_i + \mathbf{g}_i(\mathbf{r}_i) = \mathbf{Y}_i(\mathbf{r}_i, \dot{\mathbf{r}}_i, \mathbf{v}_i, \dot{\mathbf{v}}_i)\boldsymbol{\sigma}_i \quad (9)$$

where \mathbf{v}_i is an arbitrarily defined velocity, $\boldsymbol{\sigma}_i$ is the dynamic parameter vector of the i -th robot arm and $\mathbf{Y}_i(\mathbf{r}_i, \dot{\mathbf{r}}_i, \mathbf{v}_i, \dot{\mathbf{v}}_i)$ is a regressor matrix with respect to $\boldsymbol{\sigma}_i$. The robot dynamics can be rewritten more concisely as follows:

$$\mathbf{M}(\mathbf{r})\ddot{\mathbf{r}} + \mathbf{C}(\mathbf{r}, \dot{\mathbf{r}})\dot{\mathbf{r}} + \mathbf{g}(\mathbf{r}) = \mathbf{u} - \mathbf{F} \quad (10)$$

where $\mathbf{r} = (\mathbf{r}_1^T \cdots \mathbf{r}_k^T)^T$, $\mathbf{u} = (\mathbf{u}_1^T \cdots \mathbf{u}_k^T)^T$, $\mathbf{F} = (\mathbf{F}_1^T \cdots \mathbf{F}_k^T)^T$, $\mathbf{M} = \text{blockdiag}(\mathbf{M}_1 \cdots \mathbf{M}_k)$, $\mathbf{C} = \text{blockdiag}(\mathbf{C}_1 \cdots \mathbf{C}_k)$, and $\mathbf{g} = (\mathbf{g}_1^T \cdots \mathbf{g}_k^T)^T$.

The robot model (10) is characterized by the following structural properties, which are utilized in our controller design.

- P1: \mathbf{M} is symmetric positive definite.
- P2: Suitable definitions of \mathbf{C}_i for all i make matrix $\dot{\mathbf{M}} - 2\mathbf{C}$ skew-symmetric.
- P3: The robot model is linear regarding the dynamic parameters, as

$$\mathbf{M}(\mathbf{r})\dot{\mathbf{v}} + \mathbf{C}(\mathbf{r}, \dot{\mathbf{r}})\mathbf{v} + \mathbf{g}(\mathbf{r}) = \mathbf{Y}(\mathbf{r}, \dot{\mathbf{r}}, \mathbf{v}, \dot{\mathbf{v}})\boldsymbol{\sigma} \quad (11)$$

where $\mathbf{v} = (\mathbf{v}_1^T \cdots \mathbf{v}_k^T)^T$, $\boldsymbol{\sigma} = (\boldsymbol{\sigma}_1^T \cdots \boldsymbol{\sigma}_k^T)^T$ and $\mathbf{Y} = \text{blockdiag}(\mathbf{Y}_1, \cdots, \mathbf{Y}_k)$.

2.3 Integrated dynamics

By eliminating \mathbf{F}_o and \mathbf{F} from the object dynamics and the robot dynamics by the relation (5), the integrated dynamics are obtained as follows:

$$\begin{aligned} \mathbf{M}_o(\mathbf{r}_o)\ddot{\mathbf{r}}_o + \mathbf{C}_o(\mathbf{r}_o, \dot{\mathbf{r}}_o)\dot{\mathbf{r}}_o + \mathbf{g}(\mathbf{r}_o) \\ + \mathbf{W}\{\mathbf{M}(\mathbf{r})\ddot{\mathbf{r}} + \mathbf{C}(\mathbf{r}, \dot{\mathbf{r}})\dot{\mathbf{r}} + \mathbf{g}(\mathbf{r})\} = \mathbf{W}\mathbf{u} \end{aligned} \quad (12)$$

The integrated dynamics doesn't include the force and moment at the contact points.

3. Adaptive Coordinated Control 1

The dynamic equations of the object and the robot arms contain dynamic parameters such as the inertia tensor of links. If these dynamic parameters are unknown, it is very difficult to control the motion of the object precisely. The control objective is to provide a set of input joint torques as such the motion of the object converges to the desired trajectory asymptotically for the case where

the dynamic parameters of the object and the robot arms are unknown. An adaptive coordinated control is proposed; it is designed following three steps:

(1) Compute the desired external force \mathbf{F}_o^d using an estimated reference model of the object, which is generated by the desired trajectories of the object, \mathbf{r}_o^d , $\dot{\mathbf{r}}_o^d$, and $\ddot{\mathbf{r}}_o^d$.

(2) For all i , compute the desired trajectory of the i -th robot arm, \mathbf{r}_i^d , and $\ddot{\mathbf{r}}_i^d$, and the desired contact force \mathbf{F}_i^d using the obtained desired external force \mathbf{F}_o^d , and vector to generate the desired internal force of the object \mathbf{f}_{int}^d .

(3) For all i , compute an adaptive control law of the i -th robot arm by a reference model of the i -th robot arm.

3.1 Desired external force applied to the object

Let us define a reference velocity of the object by

$$\dot{\mathbf{r}}_{or} = \dot{\mathbf{r}}_o^d + \rho \mathbf{e}_o \quad (13)$$

where $\mathbf{e}_o = \mathbf{r}_o^d - \mathbf{r}_o$ is the position error vector and $\rho > 0$ is a constant scalar. Then, desired external force is generated by an estimated reference model of the object as follows:

$$\begin{aligned} \mathbf{F}_o^d = \hat{\mathbf{M}}_o(\mathbf{r}_o)\ddot{\mathbf{r}}_{or} + \hat{\mathbf{C}}_o(\mathbf{r}_o, \dot{\mathbf{r}}_o)\dot{\mathbf{r}}_{or} + \hat{\mathbf{g}}_o(\mathbf{r}_o) - \mathbf{K}_o \mathbf{s}_o \\ = \mathbf{Y}_o(\mathbf{r}_o, \dot{\mathbf{r}}_o, \dot{\mathbf{r}}_{or}, \ddot{\mathbf{r}}_{or})\hat{\boldsymbol{\sigma}}_o - \mathbf{K}_o \mathbf{s}_o \end{aligned} \quad (14)$$

where $\hat{\boldsymbol{\sigma}}_o$ is a parameter estimate of $\boldsymbol{\sigma}_o$, and $\hat{\mathbf{M}}_o$, $\hat{\mathbf{C}}_o$, and $\hat{\mathbf{g}}_o$ are estimates of \mathbf{M}_o , \mathbf{C}_o , and \mathbf{g}_o respectively, they are computed using $\hat{\boldsymbol{\sigma}}_o$, and \mathbf{K}_o is a positive definite gain matrix. An adaptive law of the dynamic parameters of object is given by

$$\dot{\hat{\boldsymbol{\sigma}}}_o = -\boldsymbol{\Gamma}_o \mathbf{Y}_o^T(\mathbf{r}_o, \dot{\mathbf{r}}_o, \dot{\mathbf{r}}_{or}, \ddot{\mathbf{r}}_{or})\mathbf{s}_o \quad (15)$$

where $\boldsymbol{\Gamma}_o > 0$ is an adaptive gain matrix and $\mathbf{s}_o \in \mathbb{R}^6$ is a residual error given by

$$\mathbf{s}_o = \dot{\mathbf{r}}_o - \dot{\mathbf{r}}_{or} = -\dot{\mathbf{e}}_o - \rho \mathbf{e}_o. \quad (16)$$

The desired external force applied to the object is updated based on the parameter estimates of the object dynamic parameters.

3.2 Desired force and moment at contact point

It is assumed that the force and moment at contact points equilibrating the external force, exist. Then, the desired force and moment at contact points equilibrating the desired external force should satisfy the relation of Eq. (5). Moreover, the force and moment at contact points generate the internal force in the object allowing a stable grasp. Hence, the general solution of the desired force and moment at contact points is given by

$$\mathbf{F}^d = \left((\mathbf{F}_1^d)^T \cdots (\mathbf{F}_k^d)^T \right)^T = \mathbf{W}^+ \mathbf{F}_o^d + (\mathbf{I} - \mathbf{W}^+ \mathbf{W}) \mathbf{f}_{int}^d \quad (17)$$

where \mathbf{I} is the identity matrix, \mathbf{f}_{int}^d is a vector to generate the desired internal force applied to the object, and \mathbf{W}^+ is a pseudo-inverse of \mathbf{W} given by

$$\mathbf{W}^+ = \mathbf{W}^T (\mathbf{W}\mathbf{W}^T)^{-1} \quad (18)$$

By denoting the force error at contact point $\Delta \mathbf{F} = \mathbf{F}^d - \mathbf{F}$, where \mathbf{F}_i is the i -th force and moment at contact point and

$F = ((F_1)^T \dots (F_k)^T)^T$, the force error applied to the object is given by

$$\Delta F_o = W \Delta F \tag{19}$$

3.3 Control law of robot arms

The duality of force and velocity yields a relationship between the velocities of the object and the i -th robot arm as follows:

$$\dot{r}_i = W_i^T \dot{r}_o \tag{20}$$

Similarly, the desired velocities of the i -th robot arm is given by

$$\dot{r}_i^d = W_i^T \dot{r}_o^d \tag{21}$$

Then, the velocity error of the i -th robot arm, $\dot{e}_i (= \dot{r}_i^d - \dot{r}_i)$, is represented by

$$\dot{e}_i = W_i^T (\dot{r}_o^d - \dot{r}_o) = W_i^T \dot{e}_o \tag{22}$$

Let us define a reference velocity of the i -th arm by

$$\dot{r}_{ir} = \dot{r}_i^d + \rho W_i^T e_o + \Omega_i \Delta F_i + \Psi_i \eta_i \tag{23}$$

where ΔF_i is the i -th sub-vector of ΔF , $\Omega_i > 0$ and $\Psi_i > 0$ are positive symmetric gain matrices, and η_i is the integral of the contact force error given by

$$\eta_i = \int_0^t \Delta F_i dt \tag{24}$$

A reference model of the i -th arm is given by

$$M_i(r_i) \ddot{r}_{ir} + C_i(r_i, \dot{r}_i) \dot{r}_{ir} + g_i(r_i) = Y_i(r_i, \dot{r}_i, \ddot{r}_{ir}) \sigma_i \tag{25}$$

where σ_i is the dynamic parameters vector of the i -th robot arm. Then, a control law of the i -th robot arm in task space is given by

$$u_i = Y_i(r_i, \dot{r}_i, \ddot{r}_{ir}) \hat{\sigma}_i + F_i^d - K_i s_i \tag{26}$$

where $\hat{\sigma}_i$ is an estimate of σ_i , $K_i > 0$ is a feedback gain matrix, and $s_i (= \dot{r}_i - \dot{r}_{ir})$ is the residual error between the

reference velocity and the actual velocity, which is rewritten as

$$s_i = W_i^T s_o - \Omega_i \Delta F_i - \Psi_i \eta_i \tag{27}$$

An adaptive law of parameter estimate of the i -th robot arm is given by

$$\dot{\hat{\sigma}}_i = -\Gamma_i Y_i^T(r_i, \dot{r}_i, \ddot{r}_{ir}, \ddot{r}_i) s_i \tag{28}$$

where $\Gamma_i > 0$ is an adaptive gain matrix. The joint input torque of the i -th robot arm τ_i is given by

$$\tau_i = J_i^T u_i \tag{29}$$

where J_i is the Jacobean of the i -th robot arm. In the right side of Eq. (26), the first term is a feed forward input based on an estimated reference model, the second term is a feed forward input for the desired force and moment at the contact point, and the third term is a feedback input for the trajectory and force errors. It is noted that the control law does not need the desired position of the end-effector of the arm.

The integrated control law and adaptive law are represented by

$$u = Y(r, \dot{r}, \ddot{r}_r, \ddot{r}_r) \hat{\sigma} + F^d - Ks \tag{30}$$

$$\dot{\hat{\sigma}} = -\Gamma Y^T(r, \dot{r}, \ddot{r}_r, \ddot{r}_r) s \tag{31}$$

where $Y(r, \dot{r}, \ddot{r}_r, \ddot{r}_r) = \text{blockdiag}(Y_1(r_1, \dot{r}_1, \ddot{r}_{1r}, \ddot{r}_{1r}), \dots, Y_k(r_k, \dot{r}_k, \ddot{r}_{kr}, \ddot{r}_{kr}))$, $K = \text{blockdiag}(K_1, \dots, K_k)$, $r_r = (r_{1r}^T \dots r_{kr}^T)^T$, $s = (s_1^T \dots s_k^T)^T$, $\hat{\sigma} = (\hat{\sigma}_1^T \dots \hat{\sigma}_k^T)^T$, $F^d = ((F_1^d)^T \dots (F_k^d)^T)^T$, and $\Gamma = \text{blockdiag}(\Gamma_1, \dots, \Gamma_k)$

From Eq. (26), the following relation can be written:

$$s = W^T s_o - \Omega \Delta F - \Psi \eta \tag{32}$$

where $\Psi = \text{blockdiag}(\Psi_1, \dots, \Psi_k)$ and $\eta = (\eta_1^T \dots \eta_k^T)^T$.

A scheme of the proposed adaptive coordinated control is shown in Fig. 2. The desired external force of the object is generated using the desired trajectory of the

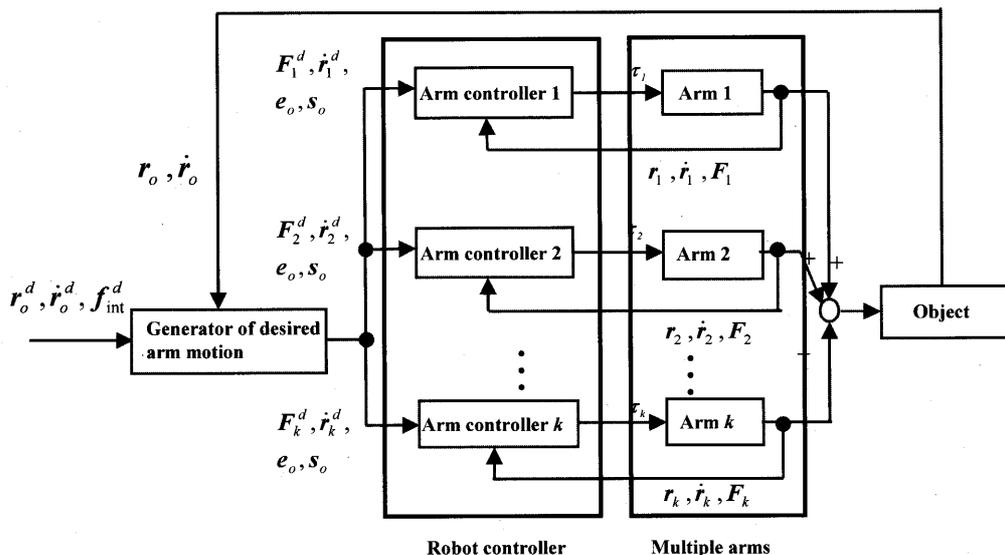


Fig. 2 Robot arm controller

object and its measured trajectory. The desired velocity and the desired contact force of each robot arm are computed using the desired external and internal forces and the desired trajectory of the object, and each robot arm is controlled independently; that is, communication among robot arms is not needed.

3.4 Asymptotic stability

For the proposed controller, the following theorem is proved.

Theorem 1: Consider a rigid object attached by k robot arms. For a system (12) using the integrated control law (30) with the integrated adaptation law (31), in which the desired external force of the object is given by (14), the closed-loop system is asymptotically stable in the sense that

$$\begin{aligned} (1) \quad & \mathbf{r}_o \rightarrow \mathbf{r}_o^d, \quad \dot{\mathbf{r}}_o \rightarrow \dot{\mathbf{r}}_o^d, \text{ as } t \rightarrow \infty \\ (2) \quad & \mathbf{F} \rightarrow \mathbf{F}^d, \quad \mathbf{F}_o \rightarrow \mathbf{F}_o^d, \text{ as } t \rightarrow \infty \end{aligned}$$

Proof: Using Eqs. (16), (6), (7), and (14) sequentially, the following relation can be easily deduced:

$$\mathbf{M}_o \dot{\mathbf{s}}_o + \mathbf{C}_o \mathbf{s}_o = -\Delta \mathbf{F}_o + \mathbf{Y}_o(\mathbf{r}_o, \dot{\mathbf{r}}_o, \ddot{\mathbf{r}}_{or}, \ddot{\mathbf{r}}_{or}) \Delta \boldsymbol{\sigma}_o - \mathbf{K}_o \mathbf{s}_o \quad (33)$$

where $\Delta \boldsymbol{\sigma}_o = \hat{\boldsymbol{\sigma}}_o - \boldsymbol{\sigma}_o$ is an estimate error vector of the object dynamic parameters. Similarly, using Eqs. (8), (9), and (26), it is shown that

$$\mathbf{M} \dot{\mathbf{s}} + \mathbf{C} \mathbf{s} = \Delta \mathbf{F} + \mathbf{Y}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}_r, \ddot{\mathbf{r}}_r) \Delta \boldsymbol{\sigma} - \mathbf{K} \mathbf{s} \quad (34)$$

where $\Delta \boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}} - \boldsymbol{\sigma}$ is an estimate error vector of all the robot dynamic parameters. Consider a candidate of the Lyapunov function as

$$\begin{aligned} V = \frac{1}{2} & \left(\mathbf{s}_o^T \mathbf{M}_o \mathbf{s}_o + \Delta \boldsymbol{\sigma}_o^T \boldsymbol{\Gamma}_o^{-1} \Delta \boldsymbol{\sigma}_o + \mathbf{s}^T \mathbf{M} \mathbf{s} \right. \\ & \left. + \Delta \boldsymbol{\sigma}^T \boldsymbol{\Gamma}^{-1} \Delta \boldsymbol{\sigma} + \boldsymbol{\eta}^T \boldsymbol{\Psi} \boldsymbol{\eta} \right) \end{aligned} \quad (35)$$

Since $\dot{\mathbf{M}}_o - 2\mathbf{C}_o$ and $\dot{\mathbf{M}} - 2\mathbf{C}$ are skew-symmetric, substituting Eqs. (33), (34), (19), and (32) into the time derivative of Eq. (35) along the solution of the error equation, gives

$$\dot{V} = -\mathbf{s}_o^T \mathbf{K}_o \mathbf{s}_o - \mathbf{s}^T \mathbf{K} \mathbf{s} - \Delta \mathbf{F}^T \boldsymbol{\Omega} \Delta \mathbf{F} \leq 0 \quad (36)$$

This shows that V is the Lyapunov function, hence \mathbf{s} , \mathbf{s}_o , $\boldsymbol{\eta}$, $\Delta \boldsymbol{\sigma}$, and $\Delta \boldsymbol{\sigma}_o$ are bounded. Since $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}_o$ are constant, $\hat{\boldsymbol{\sigma}}$ and $\hat{\boldsymbol{\sigma}}_o$ are bounded. From Eq. (32), $\Delta \mathbf{F}$ is bounded. Using (16) leads to the boundness of \mathbf{e}_o and $\dot{\mathbf{e}}_o$. These results yield to the boundness of \mathbf{r}_o , $\dot{\mathbf{r}}_o$, $\ddot{\mathbf{r}}_{or}$, and $\ddot{\mathbf{r}}_{or}$ because of the boundness of $\dot{\mathbf{r}}_o^d$ and $\ddot{\mathbf{r}}_o^d$. Hence, $\mathbf{Y}_o(\mathbf{r}_o, \dot{\mathbf{r}}_o, \ddot{\mathbf{r}}_{or}, \ddot{\mathbf{r}}_{or})$ and \mathbf{F}_o^d are bounded. From the supposition that the contact between arm end-effecters and object is maintained and \mathbf{f}_{int}^d is bounded, \mathbf{W} , \mathbf{r}_i , and \mathbf{F}^d are bounded. Hence, \mathbf{F}_i^d is bounded.

From (20)–(22), the following relations are obtained: $\dot{\mathbf{r}} = \mathbf{W}^T \dot{\mathbf{r}}_o$, $\dot{\mathbf{r}}^d = \mathbf{W}^T \dot{\mathbf{r}}_o^d$, and $\dot{\mathbf{e}} = \mathbf{W}^T \dot{\mathbf{e}}_o$. These show that $\dot{\mathbf{r}}$, $\dot{\mathbf{r}}^d$, and $\dot{\mathbf{e}}$ are bounded. The boundness of \mathbf{r}_o , $\dot{\mathbf{r}}_o$, \mathbf{r} , and $\dot{\mathbf{r}}$ yields that $\dot{\mathbf{W}}$ is bounded. Equation (23) leads to $\dot{\mathbf{r}}_r = \dot{\mathbf{r}}^d + \rho \mathbf{W}^T \mathbf{e}_o + \boldsymbol{\Omega} \Delta \mathbf{F} + \boldsymbol{\Psi} \boldsymbol{\eta}$, which shows the boundness of $\dot{\mathbf{r}}_r$.

The boundness of $\Delta \mathbf{F}$ brings to the boundness of \mathbf{F} , $\Delta \mathbf{F}_o$, and \mathbf{F}_o . Hence, Eq. (33) proves the boundness of $\dot{\mathbf{s}}_o$, the time derivatives of Eqs. (16) and (20) show the boundness of $\ddot{\mathbf{r}}_o$ and $\ddot{\mathbf{r}}$, respectively, and the boundness of \mathbf{u} , $\dot{\mathbf{r}}_r$, and $\dot{\mathbf{s}}$ are shown through Eqs. (10), (30), and (34). The time derivative of Eq. (32) is $\dot{\mathbf{s}} = \dot{\mathbf{W}}^T \mathbf{s}_o + \mathbf{W}^T \dot{\mathbf{s}}_o - \boldsymbol{\Omega} \Delta \dot{\mathbf{F}} - \boldsymbol{\Psi} \Delta \dot{\mathbf{F}}$, which shows the boundness of $\Delta \dot{\mathbf{F}}$.

Differentiating (36) with respect to time gives

$$\dot{V} = -2(\dot{\mathbf{s}}_o^T \mathbf{K}_o \mathbf{s}_o + \dot{\mathbf{s}}^T \mathbf{K} \mathbf{s} + \Delta \dot{\mathbf{F}}^T \boldsymbol{\Omega} \Delta \mathbf{F}). \quad (37)$$

\dot{V} is bounded because of the boundness of \mathbf{s} , $\dot{\mathbf{s}}$, and $\Delta \dot{\mathbf{F}}$. This means that \dot{V} is uniformly continuous. From the Lyapunov-Like Lemma, it is shown that $\dot{V} \rightarrow 0$ as $t \rightarrow \infty$. This implies that $\mathbf{s} \rightarrow 0$, $\mathbf{s}_o \rightarrow 0$, $\mathbf{e}_o \rightarrow 0$, $\dot{\mathbf{e}}_o \rightarrow 0$, and $\boldsymbol{\eta} \rightarrow 0$, as $t \rightarrow \infty$. Therefore, $\mathbf{r}_o \rightarrow \mathbf{r}_o^d$, $\dot{\mathbf{r}}_o \rightarrow \dot{\mathbf{r}}_o^d$, $\mathbf{F} \rightarrow \mathbf{F}^d$ and $\mathbf{F}_o \rightarrow \mathbf{F}_o^d$ as $t \rightarrow \infty$. Finally, It is shown using Eq. (20) that $\mathbf{r} \rightarrow \mathbf{r}^d$ and $\dot{\mathbf{r}} \rightarrow \dot{\mathbf{r}}^d$ as $t \rightarrow \infty$.

Remark: $\ddot{\mathbf{r}}_o$ and $\ddot{\mathbf{e}}_o$ are bounded. The convergence of \mathbf{F} and \mathbf{F}_o to \mathbf{F}^d and \mathbf{F}_o^d , respectively, means that the internal force applied the object converges to $(\mathbf{I} - \mathbf{W}^+ \mathbf{W}) \mathbf{f}_{int}^d$. If the persistently exciting condition of motion is satisfied, the estimates of the dynamic parameters of the object and the arms converge to the true values.

4. Adaptive Coordinated Control 2

If $\boldsymbol{\sigma}$ is known and $\boldsymbol{\sigma}_o$ is unknown, then the adaptive laws of the dynamic parameters of the robot arms are not needed. The integrated control law is given by

$$\mathbf{u} = \mathbf{Y}(\mathbf{r}, \dot{\mathbf{r}}, \ddot{\mathbf{r}}_r, \ddot{\mathbf{r}}_r) \boldsymbol{\sigma} + \mathbf{F}^d - \mathbf{K} \mathbf{s} \quad (38)$$

Then, the following theorem can be written.

Theorem 2: Consider a rigid object attached by k robot arms. For a system (12) using the integrated control law (38), in which the desired external force of the object is given by (14), the closed-loop system is asymptotically stable in the sense that

$$\begin{aligned} (1) \quad & \mathbf{r}_o \rightarrow \mathbf{r}_o^d, \quad \dot{\mathbf{r}}_o \rightarrow \dot{\mathbf{r}}_o^d, \text{ as } t \rightarrow \infty \\ (2) \quad & \mathbf{F} \rightarrow \mathbf{F}^d, \quad \mathbf{F}_o \rightarrow \mathbf{F}_o^d, \text{ as } t \rightarrow \infty \end{aligned}$$

The proof of Theorem 2 can be done similarly to that of Theorem 1 but using the following positive definite function

$$V = \frac{1}{2} \left(\mathbf{s}_o^T \mathbf{M}_o \mathbf{s}_o + \Delta \boldsymbol{\sigma}_o^T \boldsymbol{\Gamma}_o \Delta \boldsymbol{\sigma}_o + \mathbf{s}^T \mathbf{M} \mathbf{s} + \boldsymbol{\eta}^T \boldsymbol{\Psi} \boldsymbol{\eta} \right) \quad (39)$$

5. Coordinated Control

If $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}_o$ are known, then the parameter estimates of the object and robot arms are not needed. The desired external force is generated by

$$\begin{aligned} \mathbf{F}_o^d &= \mathbf{M}_o(\mathbf{r}_o) \ddot{\mathbf{r}}_{or} + \mathbf{C}_o(\mathbf{r}_o, \dot{\mathbf{r}}_o) \dot{\mathbf{r}}_{or} + \mathbf{g}_o(\mathbf{r}_o) \\ &= \mathbf{Y}_o(\mathbf{r}_o, \dot{\mathbf{r}}_o, \ddot{\mathbf{r}}_{or}, \ddot{\mathbf{r}}_{or}) \boldsymbol{\sigma}_o \end{aligned} \quad (40)$$

and the integrated control law is given by (38). Then, the following theorem may be written.

Theorem 3: Consider a rigid object attached by k robot arms. For a system (12) using the integrated control law (38), in which the desired external force of the object is given by (40), the closed-loop system is asymptotically stable in the sense that

- (1) $r_o \rightarrow r_o^d, \dot{r}_o \rightarrow \dot{r}_o^d$, as $t \rightarrow \infty$
- (2) $F \rightarrow F^d, F_o \rightarrow F_o^d$, as $t \rightarrow \infty$.

The proof of Theorem 3 can be done by using the following positive definite function

$$V = \frac{1}{2} (s_o^T M_o s_o + s^T M s + \eta^T \Psi \eta) \quad (41)$$

6. Experiment

Experiments using two robot arms were performed to show the effectiveness of the proposed control method in the case in which the dynamic parameters of the object and the robot arms are unknown. In the experiment, as shown in Fig. 3, two robot arms, the 1st robot of which, on the left side, has 6DOF, and the 2nd of which, on the right side, has 5DOF, are used as 3 DOF robot arms that grasp one object rigidly and carry it in a vertical plane. Link lengths from the 1st link to the 3rd link of the 1st robot arm are 103.9, 90, and 71 mm, respectively. Similarly, those of the 2nd robot arm are 80, 134.4, and 46.3 mm, respectively. Each arm is equipped with 6-axis force sensors (NANO sensors by BL.AUTOTEC, LTD.) at the end link to measure the contact force. The object is a rectangular solid made of aluminum, $30 \times 30 \times 20$ mm in size, and its mass is 50 g. Position vectors from the origin of Σ_o to the origin of Σ_i with respect to Σ_p are $p_{oc1} = (-22, 0, 0)^T$ and $p_{oc2} = (22, 0, 0)^T$. The desired trajectory of the object is a circle 30 mm in diameter, and in which the motion time is 3 sec. The magnitude of the desired internal force is set as $f_{int}^d = 0$ N. The controller gains are chosen as: $K_1 = K_2 = \text{diag}(500, 500, 5)$, $\rho = 100$, $\Gamma_o = \text{diag}(0.01, 0.001)$, and all the diagonal elements of Γ_1 and Γ_2 are 0.001. The initial values of the unknown dynamic parameters are set as zero. Coulomb friction and the damping friction coefficient at each joint are also estimated adaptively, and feedforward input using their esti-

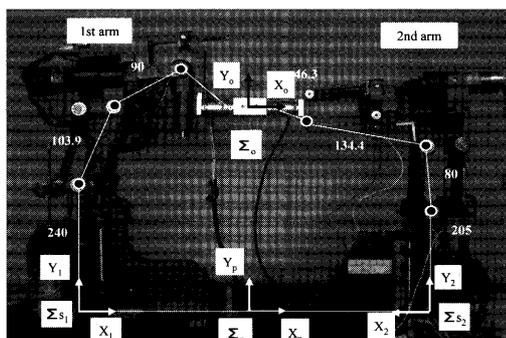
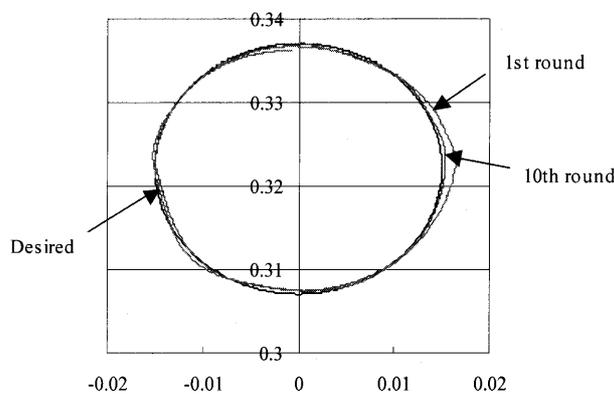


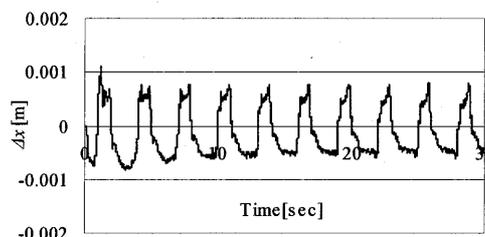
Fig. 3 Experimental system consisting of two robot arms

mates are added. The sampling cycle is 0.5 ms.

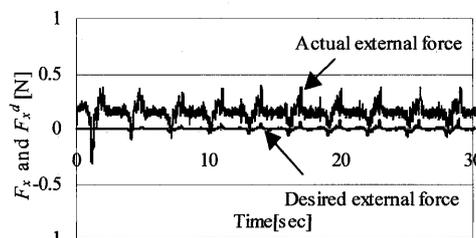
First, the adaptive control without using force sensor⁽¹⁵⁾ is applied. The experimental results of the trajectory of the object with respect to the task coordinate system are shown in Fig. 4, in which (a) is the trajectories



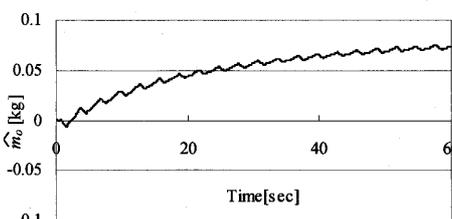
(a) Object trajectory



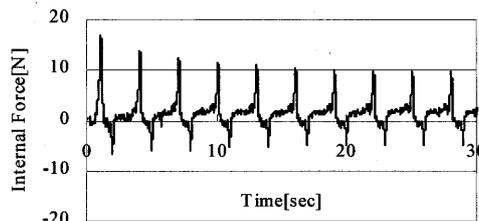
(b) x element of object position error



(c) x element of external force



(d) Estimated mass of object



(e) Internal force

Fig. 4 Adaptive control without force sensor

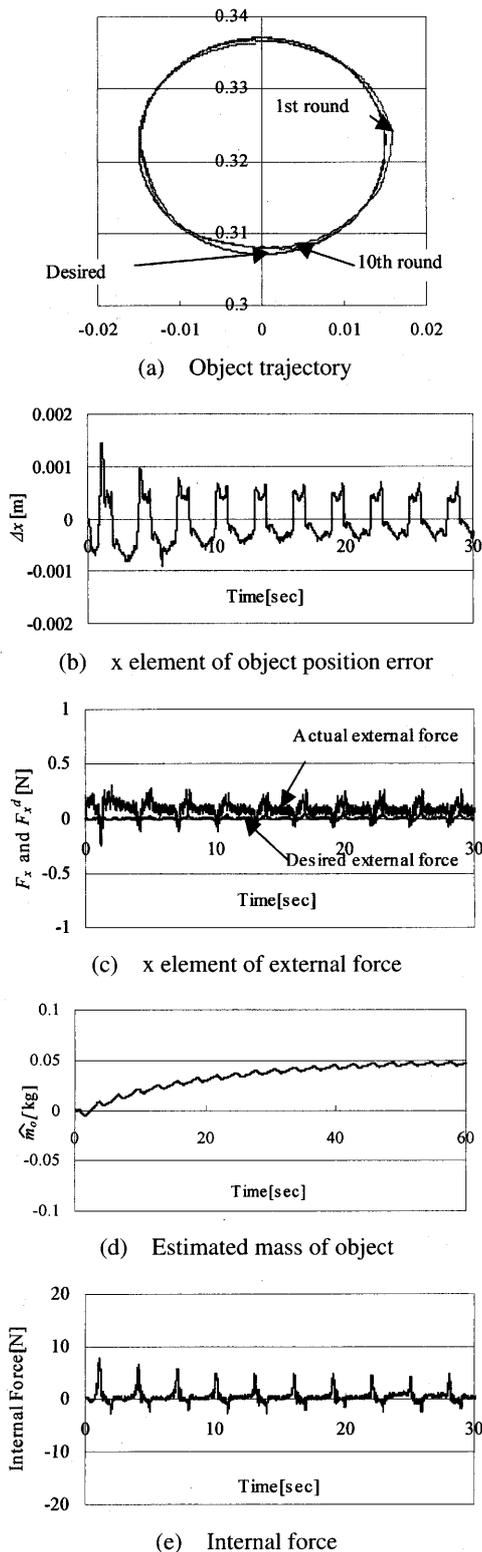


Fig. 5 Adaptive control with force sensor

of object, (b) is x element of position error, (c) is x element of external force acting on the object, (d) is estimated mass of object, and (e) is internal force acting on the object. These figures show that the position error, external force and internal force decrease by repetition of motion. Parameter estimates converge nearly to constant values,

which are not true values. There is a reason why the parameter estimates do not converge to true values, that is, the persistently exciting condition is not satisfied because of redundancy of the dynamic parameters.

Second, the proposed adaptive control with using force sensor is applied. The experimental results of the trajectory of the object with respect to the task coordinate system are shown in Fig. 5. These figures show that the position error, external force and internal force decrease by repetition of motion. The external force is larger than the desired external force, but it is smaller than the case of adaptive control without using force sensor. There are several reasons why the position errors do not converge to zero; the dynamics of the mechanism such as the flexibility of the joint is not modeled; the controller is not a continuous-time control system but a discrete-time control system whose accuracy of trajectory depends on the sampling cycle⁽¹⁷⁾. These results show that the adaptation in control is working successfully and effectively.

7. Conclusion

An adaptive coordinated control method for multiple robot arms handling a common object has been presented. In the control, the desired motion of the robot arms is generated by an estimated reference model of the object, which is evaluated using parameter estimates and the desired motion of the object. The motion of each robot arm is controlled independently and adaptively without communication among the arms. The asymptotic convergence of motion and contact force has been proved by the Lyapunov-like Lemma. Experimental comparisons between adaptive control with force sensor and one without force sensor for the two robot arms grasping a common object were shown. They showed the control objective was achieved successfully.

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