

## Simple Modelling and Strength Evaluation Methods for Bolt Joints Using Shell Elements and Beam Elements

(2nd Report, Strength Evaluation Method)\*

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### Abstract

The stiffness of an engineering structure with joints depends not only on the materials and dimensions of the structure but also on the stiffness of the fasteners that connect its components. To evaluate the mechanical behaviour of a complex structure with bolted joints, we first developed a simple finite element (FE) modelling technique that was simplified by using shell and beam elements and accounts for the effect of the stiffness of jointed plates and bolts. In evaluating the mechanical behaviour, we also need a method for evaluating the strength of bolt joints from the results of FE analysis. In this study, we have developed a strength evaluation method for the beam forces of FE analysis by considering the mechanics of bolt joints. This method can be used to evaluate static failure and fatigue failure of bolts and slip on clamped plates and bearing surfaces. We can easily evaluate the strength of bolt joints of industrial products with many bolted joints by using this method.

**Key words:** Bolted Joint, Finite Element Method, Stress Analysis

### 1. Introduction

Strength design guidelines of bolted joints had previously evaluated static collapse, fatigue failure, and slip between fastened plates by using stress calculation methods based on material mechanics <sup>(1)-(4)</sup>. However, it is difficult to directly apply these design guidelines to the structures of industrial products having many bolted joints, first because they require load condition of each joint and second because the load prediction method is not sufficiently accurate.

On the other hand, great progress has recently been made in Computer Aided Engineering, or CAE, and this progress has made this technique a practical way to evaluate bolted-joints strength by using finite element (FE) analysis. Papers have been published that precisely calculated the stress and force of bolted joints based on contact analysis of joint

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elements <sup>(5)</sup> and analyzed the loosening phenomenon of bolted joints by taking more complex contact conditions into account <sup>(6)</sup>. However, if an entire-structure FE model were to be made while taking contact conditions that had previously been reported into consideration, the number of elements would be too large. What is needed, therefore, is a simplified bolted-joints analysis model to enable industrial products structures to be accurately analyzed.

A simple bolted-joint analysis model has been proposed that connects two bolt holes with rigid bar elements <sup>(7)</sup>. Other simple models with equivalent springs and beams have been proposed to achieve greater accuracy <sup>(8)-(11)</sup>. In the former, however, bolted-joints stiffness is not considered. In the latter, deformation can be calculated with high accuracy, however, it is not clear as to which strength evaluation method for the model can be applied. What is needed, therefore, is to develop a simple and accurate bolted-joint model that can be applied to any structure with many bolted joints, as well as a strength evaluation method that can be integrated into the model.

In the first paper we published regarding this study, we reported that we had developed a modelling technique that simplifies the bolted-joint structures by using shell and beam elements <sup>(12)</sup>. This technique took the bolted-joints mechanism into consideration, and is highly accurate despite its simplicity.

In this paper, we propose a strength evaluation method for our simplified bolted-joint model that takes strength design guidelines <sup>(1)-(4)</sup> into account. This method can be applied to simplified bolted-joint models, for which no clear-cut strength evaluation method had previously been developed.

## 2. Bolted-Joint Strength Evaluation Method

### 2.1 Modelling method of bolted joints

Figure 1 shows the bolted-joint modelling method that we presented in our first paper <sup>(12)</sup>. This modelling method uses the equivalent beam element, the stiffness of which is determined as comprising the stiffness of the bolt and that of the pressure cone, and the equivalent pressure area, which is defined as being within the pressure cone of two plates. Figure 2 shows a schematic depiction of a bolted joint. In the case when two plates are joined with sufficient clamping force and the contact conditions between the plates are maintained under external force, the subjected external load is transmitted from one plate to the other through the pressure cone shown in the figure. We calculated the axial, bending, and torsional stiffness of the equivalent beam element of a bolted joint as described below.

According to VDI2230 (1977) <sup>(4)</sup>, the axial compliance of a bolt  $\delta_b$ , which represents the axial stiffness of a bolt, is calculated as

$$\delta_b = \frac{0.4d}{E_b A_n} + \frac{l_g}{E_b A_n} + \frac{l_s}{E_b A_3} + \frac{0.4d}{E_b A_n}, \quad (1)$$

in which  $d$  is the nominal diameter of the bolt,  $E_b$  is Young's modulus of the bolts,  $l_g$  is the length of the cylindrical body as shown in Fig. 2,  $l_s$  is the length of the free-loaded thread as shown in Fig. 2,  $A_n$  is the nominal cross section, and  $A_3$  is the cross section with a bolt thread diameter of  $d_3$ , respectively. According to VDI2230 (1977) <sup>(4)</sup>, pressure cone stiffness is obtained in terms of a tube with an outer diameter equal to the equivalent diameter  $D_{eq}$  and an inner diameter equal to the hole diameter  $D_i$ , as shown in Fig. 3. The equivalent diameter of the pressure cone  $D_{eq}$  and the equivalent cross-section of the pressure cone  $A_{eq}$  are expressed by Eqs. (2) and (3). The axial compliance of the pressure cone  $\delta_c$  is calculated as in Eq. (4).

$$D_{eq} = d_w + \frac{l_f}{10} \quad \left( \text{when } 3d_w < D_c \text{ and } l_f \leq 8d \right), \quad (2)$$

$$A_{eq} = \frac{\pi}{4}(D_{eq}^2 - D_i^2), \quad (3)$$

$$\delta_c = \frac{l_f}{E_c A_{eq}}, \quad (4)$$

in which  $E_c$  is Young's modulus of the plates. The load factor  $\Phi$  is defined by using the  $\delta_b$  of Eq. (1) and  $\delta_c$  of Eq. (4) as in the following equation:

$$\Phi = \frac{\delta_c}{\delta_b + \delta_c}. \quad (5)$$

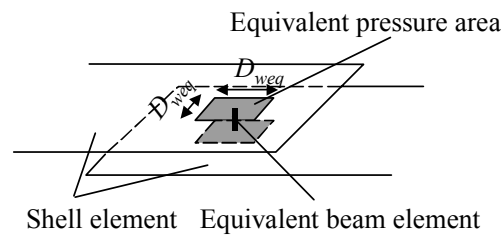


Fig. 1 Equivalent model of bolted joints

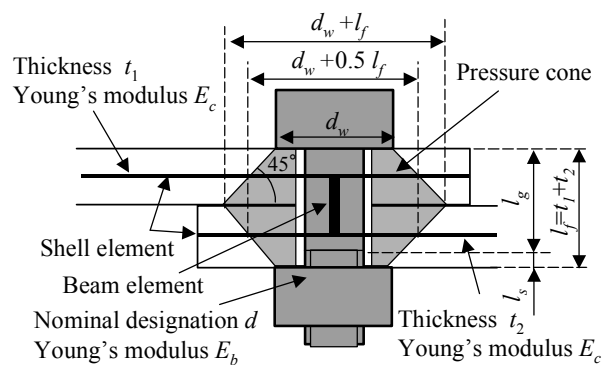


Fig. 2 Structure and model of bolted joints

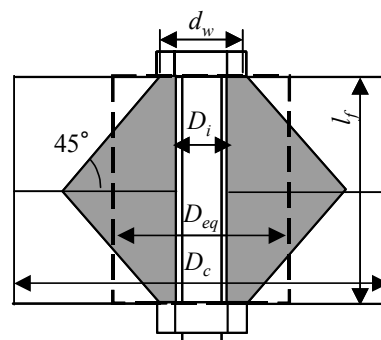


Fig. 3 Equivalent diameter of pressure cone

If Young's modulus of the equivalent beam is  $E_b$ , the cross section of equivalent beam  $A_{beq}$  and the equivalent diameter for the axial load  $d_{eq1}$  are obtained from Eq. (6):

$$A_{beq} = \frac{\pi}{4} d_{eq1}^2 = \frac{l_f}{E_b \delta_b \Phi}. \quad (6)$$

The bending compliance of the bolt  $\beta_b$  is a serial connection of cylinders similar to that shown in Eq. (1).

$$\beta_b = \frac{0.4d}{E_b I_n} + \frac{l_g}{E_b I_n} + \frac{l_s}{E_b I_3} + \frac{0.4d}{E_b I_n} \quad (7)$$

Here,  $I_n$  is the second moment of the nominal cross section, and  $I_3$  is the second moment of the bolt thread section. The bending stiffness of the pressure cone is also assumed to be a tube with an outer diameter equal to the equivalent diameter  $D_{eq}$  and an inner diameter equal to the hole diameter  $D_i$ , as shown in Eq. (8).

$$\beta_c = \frac{l_f}{E_c I_{eq}} = \frac{64l_f}{\pi E_b (D_{eq}^4 - D_i^4)} \quad (8)$$

The load factor for the bending moment,  $\Phi_b$ , is defined as in the following equation:

$$\Phi_b = \frac{\beta_c}{\beta_b + \beta_c}. \quad (9)$$

We obtain the second moment of the equivalent beam  $I_{beq}$  and the equivalent diameter for the bending stiffness  $d_{eq2}$  from Eq. (10):

$$I_{beq} = \frac{\pi}{64} d_{eq2}^4 = \frac{l_f}{E_b \beta_b \Phi_b}. \quad (10)$$

We considered the torsional stiffness of an equivalent beam to be similar to the bending stiffness. The polar second moment of the equivalent beam element is equal to twice the second moment of the equivalent beam element  $I_{beq}$ . The load factor for the torque, therefore, is identical to the load factor for the bending moment  $\Phi_b$ .

We set the equivalent pressure area to a square with a length of  $D_{weq} = d_w + 0.5l_f$ , and the shell element nodes within the equivalent pressure area are constrained to the tip node of the equivalent beam node. These boundary conditions are represented by the constraint equations, in which the tip node of the equivalent beam is the master node and the shell element nodes within the equivalent pressure area are slave nodes. These constraint equations can be simply defined by using the CERIG command in ANSYS®.

## 2. 2 Working loads of bolted joints

Bolted joints should be designed to maintain preloads at assembly and working loads. Working loads on each bolted joint had been calculated by dividing total working loads by the bolt number or had been estimated from the geometric equilibrium of force in strength evaluation guidelines<sup>(1)-(3)</sup>. These estimated working loads on each bolted joint therefore contained many errors. On the other hand, the modelling method of bolted joints that we described in our previous paper can easily calculate the working loads on each bolted joint as the output forces of the beam element. Since these output forces obtained with the model that takes the stiffness of bolted joints into consideration, they are more accurate than the working loads estimated by using strength evaluation guidelines.

Figure 4 shows a schematic depiction of output forces and moments of a beam element. The output forces are calculated on the coordinate system of the beam element. We can estimate the axial force  $W_a$ , shear force  $W_s$ , bending moment  $M$ , and torque  $T$  of the working loads from these output forces and moments as follows:

$$W_a = F_x, \quad (11)$$

$$W_s = \sqrt{F_y^2 + F_z^2}, \quad (12)$$

$$M = \max\left(\sqrt{M_{yi}^2 + M_{zi}^2}, \sqrt{M_{yj}^2 + M_{zj}^2}\right), \quad (13)$$

$$T = |M_x|. \quad (14)$$

Evaluating the bolted-joints strength mainly comprises evaluating [1] static strength, [2] fatigue strength and [3] slip. In addition to these primary evaluations, we evaluate [4] the pressure on the bearing surfaces and the preload change caused by embedding of the contact

surface and thermal deformation. These four evaluations are explained in the following sections.

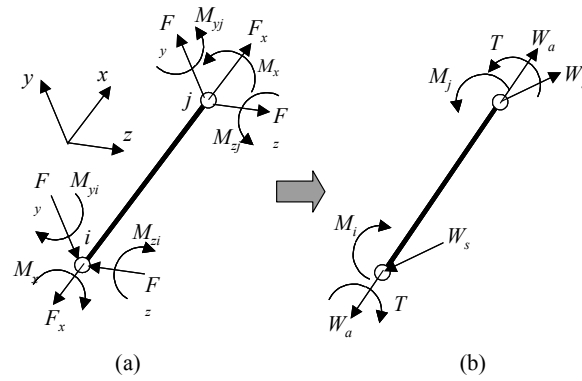


Fig. 4 Forces and moments of equivalent beam element of bolt

### 2.3 Static strength evaluation

When a bolt with a nominal diameter  $d$  is tightened with a torque  $T_f$ , the pretension  $F_f$  is calculated as in Eq. (15).

$$F_x = \frac{T_f}{Kd} \quad (15)$$

Here,  $K$  is called the "nut factor", which from Eq. (16) is calculated with the friction coefficient between nut and bolt threads,  $\mu_s$ , the friction coefficient in the head bearing area,  $\mu_w$  and the thread dimensions.

$$K = \frac{1}{2d} \left( \frac{P}{\pi} + \mu_s d_2 \sec \alpha' + \mu_w D_w \right) \quad (16)$$

Here,  $P$  is the thread pitch,  $d_2$  is the effective diameter of the thread,  $\alpha'$  is calculated as  $\tan \alpha' = \tan \alpha + \tan \beta$  from the half-angle of the threads  $\alpha$  and the lead angle of the threads  $\beta$ , and  $D_w$  is called the effective diameter of the head bearing surface, and it is obtained as following equation from the outer diameter of bearing area  $D_o$  and the hole diameter  $D_i$ :

$$D_w = \frac{2}{3} \frac{D_o^3 - D_i^3}{D_o^2 - D_i^2} \quad (17)$$

Since the friction coefficients vary widely and are hard to predict, the pretension scatter should be taken into consideration when bolts are tightened with the torque control method. If the scatter of the pretension is  $m\%$ , the maximum pretension  $F_{f\max}$  and the minimum pretension  $F_{f\min}$  are expressed with  $F_f$  of Eq. (15) as follows:

$$F_{f\max} = (1 + m/100)F_f, \quad (18)$$

$$F_{f\min} = (1 - m/100)F_f. \quad (19)$$

The scatter  $m$  is considered to be 15% in general.

From the above pretension values and the working forces as described in Section 2.2, the bolt stresses can be estimated. The stress on the thread section is calculated from the effective thread section  $A_s$ . The diameter of the effective thread section,  $d_s$ , is the average of the pitch diameter of the bolt thread,  $d_2$ , and the minor diameter of the bolt thread,  $d_3$ , i.e.:

$$d_s = \frac{d_2 + d_3}{2}. \quad (20)$$

In static strength evaluation, the maximum pretension  $F_{f\max}$  shown in Eq. (18) is used. The maximum axial stress  $\sigma_{f\max}$  on the effective thread section  $A_s$  under the pretension is obtained by:

$$\sigma_{f \max} = \frac{F_{f \max}}{A_s}. \quad (21)$$

The maximum shear stress  $\tau_{f \max}$  on the thread under the pretension is calculated by dividing the maximum torque  $T_{s \max}$  under the pretension by the torsional section modulus  $Z_{ps}$  with the diameter of the effective thread section expressed as in the following equation.

$$\tau_{f \max} = \frac{T_{s \max}}{Z_{ps}} = \frac{8F_{f \max}d_2}{\pi d_s^3} \left( \tan \beta + \frac{\mu_s}{\cos \alpha'} \right) \quad (22)$$

To prevent the bolt from yielding under the pretension, the equivalent stress of the thread  $\sigma_{e1}$ , which is calculated from  $\sigma_{f \max}$  of Eq. (21) and  $\tau_{f \max}$  of Eq. (22), should be smaller than the yield strength  $\sigma_{by}$  of the bolt material, taking the safety factor  $a_y$  into consideration, i.e.:

$$\sigma_{e1} = \sqrt{\sigma_{f \max}^2 + 3\tau_{f \max}^2} < \sigma_{by}/a_y. \quad (23)$$

Next, we consider how to evaluate the static strength under the working forces  $W_a$ ,  $W_s$ ,  $M$ , and  $T$  shown in Fig. 4(b). As previously mentioned, a bolted joint supports the working forces by dividing the forces between those of the bolt and those of the plates. In this case, the stresses caused by the shear force  $W_s$  and the bending moment  $M$  are so small that static failure caused by these working forces and moments do not need to be considered.

What needs to be evaluated, therefore, is the static strength by adding the stresses caused by the axial working force  $W_a$  and the working torque  $T$  to the stress under the pretension. When the bolt joint is subjected to the axial working force  $W_a$ , the bolt sustains the divided force  $\Phi W_a$  and the plates support the force  $(1-\Phi)W_a$ . Here,  $\Phi$  is the load factor defined by Eq. (5). The stress  $\sigma_n$  on the effective thread section under the axial working force  $W_a$  is obtained by the following equation:

$$\sigma_n = \frac{\Phi W_a}{A_s}. \quad (24)$$

Here, when the bolted joint is subjected to the working torque  $T$ , the bolt and the plates sustain the working torque together. The load factor of the torque is identical to the load factor of the bending moment,  $\Phi_b$ . Thus, the shear stress  $\tau_s$  on the effective thread section under the working torque,  $T$ , is calculated as

$$\tau_s = \frac{\Phi_b T}{Z_{ps}}. \quad (25)$$

To prevent the bolt from yielding under the working forces, the equivalent stress of the thread  $\sigma_{e2}$ , which is calculated from  $\sigma_{f \max}$  of Eq. (21),  $\tau_{f \max}$  of Eq. (22),  $\sigma_n$  of Eq. (24), and  $\tau_s$  of Eq. (25) should be smaller than the yield strength of the bolt material  $\sigma_{by}$ , taking the safety factor  $a_y$  into consideration, i.e.:

$$\sigma_{e2} = \sqrt{(\sigma_{f \max} + \sigma_n)^2 + 3(\tau_{f \max} + \tau_s)^2} < \sigma_{by}/a_y. \quad (26)$$

From the above static strength evaluation, we can determine the nominal diameter of the bolt by satisfying Eq. (23) and Eq. (26) under the pretension and the working forces, respectively.

## 2. 4 Fatigue strength evaluation

Fatigue failure in a thread is caused by the cyclic stress on the axial direction under the working forces. From the stresses under the working forces  $W_a$ ,  $W_s$ ,  $M$ , and  $T$  shown in Fig. 4 (b), we take the axial stress  $\sigma'_n$  under the axial force  $W_a$  and the bending stress  $\sigma'_b$  under the bending moment  $M$  into consideration. The axial stress  $\sigma'_n$  and the bending stress  $\sigma'_b$  are calculated on the cross section of the minor diameter  $d_3$ . The axial stress  $\sigma'_n$  under the axial force  $W_a$  is calculated by using the load factor  $\Phi$  defined by Eq. (5) as:

$$\sigma'_n = \frac{\Phi W_a}{A_3}, \quad (27)$$

in which  $A_3$  is the cross section of the minor diameter,  $d_3$ .

The bending stress  $\sigma'_b$  under the bending moment  $M$  is calculated by using the load factor  $\Phi_b$  defined by Eq. (9) as:

$$\sigma'_b = \frac{\Phi_b M}{Z_3}, \quad (28)$$

in which  $Z_3$  is the section modulus of the minor diameter,  $d_3$ .

Yamamoto et al. <sup>(1)</sup> found that the tread fatigue strength is almost constant when the pretension is sufficient. They also estimated the thread fatigue strength  $\sigma_{wk}$  by taking the stress concentration of the thread, the fatigue notch factor, and the fatigue strength of the bolt material into account. This estimated fatigue strength  $\sigma_{wk}$  is calculated with ISO strength classes and the nominal diameters <sup>(1)</sup>. To prevent the bolt from incurring fatigue failure, the thread stress amplitude  $\sigma_a$ , which is calculated from  $\sigma'_n$  of Eq. (27) and  $\sigma'_b$  of Eq. (28), should be smaller than this estimated fatigue strength  $\sigma_{wk}$  while taking the safety factor  $a_w$  into consideration.

$$\sigma_a = \max\left(\frac{|\sigma'_n + \sigma'_b|}{2}, \frac{|\sigma'_n - \sigma'_b|}{2}\right) < \frac{\sigma_{wk}}{a_w} \quad (29)$$

## 2. 5 Slip evaluation

When a bolted joint is subjected to working forces, it should be designed not to slip on the bearing surface between the bolt head and the plate, or between the nut and the plate, or on the surface between the two plates. The static friction forces on the bearing surfaces and on the surface between two plates, therefore, should be larger than the working forces. The static friction forces are calculated by multiplying the friction coefficients by the normal forces on these surfaces. The friction coefficient on the bearing surface is  $\mu_w$ , and that on the surface between the two plates is  $\mu_c$ . In the slip evaluation, the nominal forces on these surfaces are estimated by adding the axial additional bolt load by the axial force  $W_a$  and the minimum pretension  $F_{fmin}$  defined by Eq. (19).

The nominal force on the bearing surface is increased by  $\Phi W_a$  from the minimum pretension  $F_{fmin}$  under the axial force  $W_a$ . Here,  $\Phi$  is the load factor defined by Eq. (5). The static friction force  $F_w$  is calculated as

$$F_w = \mu_w (F_{fmin} + \Phi W_a). \quad (30)$$

The nominal force on the surface between the two plates is decreased by  $(1 - \Phi)W_a$  from the minimum pretension  $F_{fmin}$  under the axial force  $W_a$ . The static friction force  $F_c$  is calculated as:

$$F_c = \mu_c \{F_{fmin} - (1 - \Phi)W_a\}. \quad (31)$$

We compared these static friction forces with the shear force  $W_s$  and torque  $T$ . To prevent the bolt joint from slipping under the shear force  $W_s$ , the following inequalities should be satisfied while taking the safety factor  $a_s$  into consideration:

$$W_s < F_w / a_s, \quad (32)$$

$$W_s < F_c / a_s. \quad (33)$$

Further, to prevent the bolt joint from slipping on the bearing surface under torque  $T$ , the following inequality should be satisfied while taking the safety factor  $a_s$  into consideration:

$$T < \frac{F_w}{a_s} \times \frac{D_w}{2}. \quad (34)$$

Here,  $D_w$  is the effective radius of the head bearing surface defined by Eq. (17). The

pressure cone diameter is so large that the surface between the two plates might not slip under torque  $T$ . Therefore, only the slippage on the bearing surface should be evaluated under torque  $T$ .

## 2. 6 Evaluating pressure on bearing surfaces and the preload changes caused by embedding and thermal deformation

In this section, we evaluate the pressure on the bearing surfaces and preload changes caused by embedding on the contact surface and thermal deformation. Bearing surface pressure causes yield and creep deformation on the contact surfaces, and it loosens the bolted joints. The limit pressure on the bearing surface has been obtained for various materials of plates experimentally in some literature<sup>(1)-(4)</sup>. We compared the bearing surface pressures under the preload and under working forces to this limit pressure of the plates' materials,  $p_L$ .

$$\frac{F_{f\max}}{A_w} < p_L, \quad \frac{F_{f\max} + \Phi W_a}{A_w} < p_L \quad (35)$$

Here,  $A_w$  is the bearing surface area, which is calculated with the outer diameter of the bearing surface  $D_o$  and the hole diameter  $D_i$ .

$$A_w = \frac{\pi}{4} (D_o^2 - D_i^2) \quad (36)$$

Next, we evaluate the preload change caused by embedding on the contact surface. Embedding on the contact surface occurs when microscopic roughness on the contact surface is smoothened by yield deformation and it results decreasing the preload. The preload loss  $F_{z1}$  resulting from embedding is calculated from the permanent deformation on the bearing surface  $f_z$ , the load factor  $\Phi$  defined by Eq. (5), and the compliance of the plates  $\delta_c$  defined by Eq. (4) as:

$$F_{z1} = \frac{\Phi}{\delta_c} f_z. \quad (37)$$

Several methods for calculating the permanent deformation  $f_z$  on the bearing surface have been proposed in conjunction with experimental results<sup>(1)-(4)</sup>. Here, VDI2230 equation<sup>(4)</sup> is shown as Eq. (38), which is obtained from the nominal diameter  $d$  and the clamping length  $l_f$ . Since this equation (38) is calculated in terms of micrometer, the micrometer unit needs to be changed to the unit of Eq. (37), when Eq. (38) is substituted into Eq. (37).

$$f_z = 3.29 \left( \frac{l_f}{d} \right)^{0.34} (\mu m) \quad (38)$$

We evaluated the pretension change incurred by embedding on the contact surface while substituting the minimum pretension term  $F_{fmin}$  described in Section 2.5 into  $(F_{fmin} - F_{z1})$ .

Next, we considered the preload change caused by thermal deformation. When the thermal expansion coefficients are different between the bolt material and the plate materials, and the bolted joint temperatures are varied, the preload would change with thermal deformation. The modelling method of bolted joints by using shell and beam elements described in Section 2.1 can estimate the in-plane deformation by thermal expansion. However, shell elements cannot evaluate the thermal deformation in the thickness direction. Thus, the preload change caused by the thermal deformation in the thickness direction should be evaluated by the following method:

The thermal expansion coefficients of the bolt and the plates are determined as  $\alpha_b$  and  $\alpha_c$  respectively. The temperature rises in the bolt and the plates are represented by  $\Delta t_b$  and  $\Delta t_c$  respectively. The preload loss  $F_{z2}$  determined by the thermal deformation result is calculated from the load factor  $\Phi$  defined by Eq. (5), the compliance of the plates  $\delta_c$  defined by Eq. (4) and the clamping length  $l_f$  as:

$$F_{z2} = \frac{\Phi}{\delta_c} (\alpha_b \Delta t_b - \alpha_c \Delta t_c) l_f. \quad (39)$$

We can evaluate the pretension change resulting from the thermal deformation by substituting the term of the axial force change  $\Phi W_a$  in Sections 2.3, 2.4 and 2.5 and the pressure evaluation on the bearing surface described in Section 2.6 into  $(\Phi W_a - F_{z2})$ .

### 3. Application of the Developed Strength Evaluation Method

#### 3.1 Analysis model

We applied our strength evaluation method to the specific example shown in Fig. 5, which we also analyzed in our first paper <sup>(12)</sup>. Three channels were assembled into an H-shaped structure and tightened down with four M12 bolts at each of two intersections. The structure was fixed in place at the four ends and subjected to an out-of-plane load at the centre. Taking the symmetry into consideration, the analysis model was a half model as shown in Fig. 6. The analysis was carried out with ANSYS® 7.1. We made an FE model by using our modelling method for bolted joints described in Section 2.1. The x-axis of the coordinate system corresponded to the axial directions of bolts.

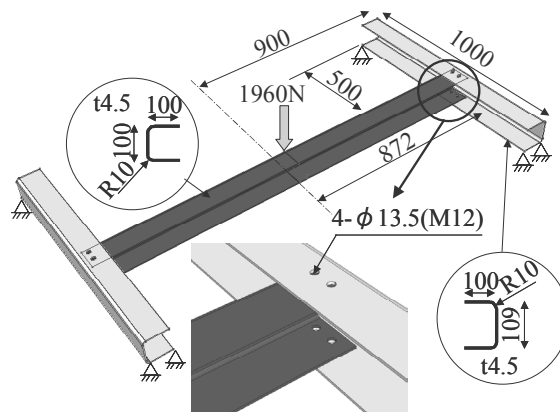


Fig. 5 Example of bolted joint structure

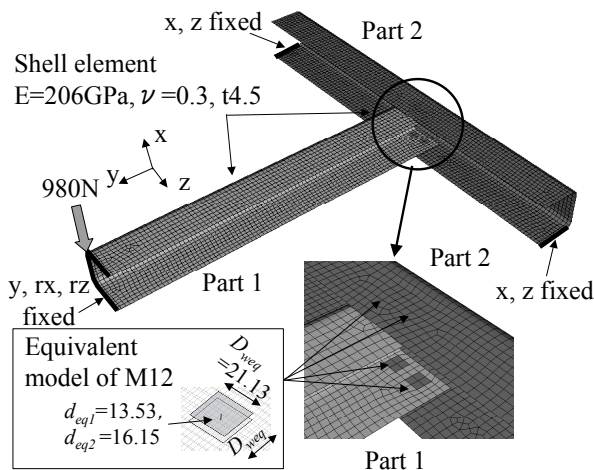


Fig. 6 Analysis model of example

#### 3.2 Strength evaluation

Figure 7 shows the results obtained for the working forces  $F_x$  and  $F_y$  loaded to four equivalent beam elements of bolted joints 1-4. Working force  $F_z$  and moments  $M_x$ ,  $M_y$  and  $M_z$  were obtained as well. These forces and moments are shown in Table 1. Taking the force directions into consideration, axial force  $W_a$ , shear force  $W_s$ , bending moment  $M$ , and torque  $T$  calculated by using Eq. (11)~(14) are also shown in Table 1.

As an example of strength evaluation, we evaluated bolt no. 1 under the forces shown in Table 1. Table 2 shows inputs with ISO class 4.8 and 8.8 bolts and with SS400 carbon steel plates, the results indicating evaluated static strength, fatigue strength, slip, bearing surface pressure, and preload change.

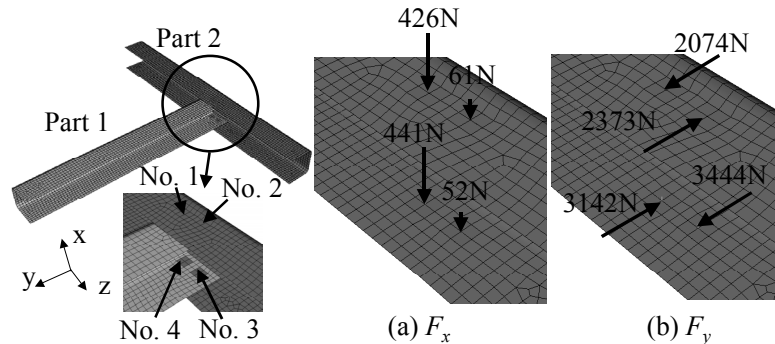


Fig. 7 Load distribution of analysis results

Table 1 Load distribution of bolts

No. of bolt	No. 1	No. 2	No. 3	No. 4
$F_x$ [N]	-426	-61	-52	-441
$F_y$ [N]	2074	-2377	3444	-3142
$F_z$ [N]	-159	21	29	109
$M_x$ [N·mm]	13131	9565	-15411	-17649
$M_y$ [N·mm]	-4579	1974	2159	-4662
$M_z$ [N·mm]	-12257	9533	12129	-14938
$W_a$ [N]	426	61	-52	-441
$W_s$ [N]	2080	2377	3444	3144
$T$ [N·mm]	13131	9565	15411	17649
$M$ [N·mm]	13084	9735	12320	15649

We determined the pretension as being 70% of the yield tightening torque  $T_{fy}$  with the friction coefficient  $\mu_w=0.15$  on the bearing surface written in JIS 1083. The friction coefficient  $\mu_s$  on the thread surface is equal to the friction coefficient  $\mu_w$  of 0.15 on the bearing surface. The nut factor is  $K=0.199$  according to JIS 1083. We calculated the pretension  $F_f$  from Eq. (15), and obtained the maximum and minimum pretensions with scatter  $m$  of 15% obtained by Eqs. (18) and (19). We estimated the preload loss obtained by embedding on the contact surface from Eqs. (37) and (38). We also estimated safety factors of static strength, fatigue strength, slip, and bearing surface pressure as ratios of the allowable and the calculated value.

If the bolt is an ISO class 4.8 bolt, the slip evaluation safety factor is smaller than 1.0 as shown in the grey cell in Table 2. This shows that the bolted joint cannot hold the shear force  $W_s$  and would slip between the two plates. If the bolt is an ISO class 8.8 bolt, the bolted joints could be tightened with higher pretension and the static friction force would be increased. The slip evaluation safety factor, therefore, would increase to greater than 1.0.

On the other hand, when the bolt is an ISO class 8.8 bolt, the safety factor of the bearing surface pressure is smaller than 1.0 as shown in the grey cell in Table 2. The bearing surface pressure is so high that the bearing surfaces on the two plates are significantly deformed plastically. Thus, the plate materials need to be changed to those having higher yield strength. Alternatively, wide-diameter washers need to be used to decrease the bearing surface pressure.

We also evaluated fatigue strength by calculating the stress amplitude from the axial

force  $W_a$  and the bending moment  $M$  as shown in Table 2. Preload changes caused by thermal deformation can be evaluated in this way, however we found that in this model such changes need not be evaluated because the bolt and plate materials are carbon steel types.

Table 2 Example of strength evaluation of No. 1 bolt

	Parameter name	Symbol	Dimension	Input and output	
Input	Nominal designation	$d$	[mm]	12	
	Thread pitch diameter	$d_2$	[mm]	10.863	
	Thread root diameter	$d_3$	[mm]	10.106	
	Hole diameter	$D_i$	[mm]	13.5	
	Outer bearing diameter	$d_w$	[mm]	16.63	
	Young's modulus of bolt	$E_b$	[GPa]	206	
	ISO grade			4.8	8.8
	Yield strength of bolt material	$\sigma_{by}$	[MPa]	340	640
	Fatigue strength of bolt	$\sigma_{wk}$	[MPa]	48	53
	Tightening torque	$T_f$	[N·mm]	38000	69000
	Pretension	$F_f$	[N]	16150	29325
	Maximum pretension	$F_{fmax}$	[N]	18573	33724
	Minimum pretension	$F_{fmin}$	[N]	13728	24927
	Young's modulus of plate	$E_c$	[GPa]	206	
	Thickness of plate 1	$t_1$	[mm]	4.5	
	Thickness of plate 2	$t_2$	[mm]	4.5	
	Friction coefficient of bearing surface	$\mu_w$		0.15	
	Friction coefficient of clamped plate	$\mu_c$		0.15	
Output	Stiffness calculation	Compliance of bolt	$\delta_b$	[mm/N]	9.567E-07
		Compliance of clamped plates	$\delta_c$	[mm/N]	4.448E-07
		Load factor	$\Phi$		0.3174
		Bending compliance of bolt	$\beta_b$	[1/N·mm]	1.311E-07
		Bending compliance of clamped plate	$\beta_c$	[1/N·mm]	1.454E-08
		Load factor for bending moment	$\Phi_b$		0.0998
	Preload loss	Permanent deformation of bearing surface	$f_z$	[μm]	2.9830
		Pretension loss	$F_{z1}$	[N]	2129
	Stress calculation	Axial stress due to pretension	$\sigma_{fmax}$	[MPa]	215 391
		Shear stress due to pretension	$\tau_{fmax}$	[MPa]	100 182
		Axial stress due to axial load $W_a$	$\sigma_n$	[MPa]	1.6
		Shear stress due to torque $T$	$\tau_s$	[MPa]	66.7
		Axial stress at thread root area due to axial load $W_a$	$\sigma'_n$	[MPa]	1.7
		Axial stress at thread root area due to bending moment $M$	$\sigma'_b$	[MPa]	12.9
		Equivalent stress due to pretension	$\sigma_{e1}$	[MPa]	276 502
		Equivalent stress due to pretension and working loads	$\sigma_{e2}$	[MPa]	284 509
		Stress amplitude	$\sigma_a$	[MPa]	7.3
	Evaluation for stress	Safety factor for equivalent stress due to pretension	$\sigma_{by}/\sigma_{e1}$		1.23 1.28
		Safety factor for equivalent stress due to pretension and working loads	$\sigma_{by}/\sigma_{e2}$		1.20 1.26
		Safety factor for fatigue strength	$\sigma_{wk}/\sigma_a$		6.59 7.27
	Evaluation for slip	Static frictional force on bearing surface	$F_w$	[N]	1760 3440
		Static frictional force on clamped plate	$F_c$	[N]	1696 3376
		Safety factor for slip on bearing surface by shear force $W_s$	$F_w/W_s$		0.85 1.65
		Safety factor for slip on clamped plate by shear force $W_s$	$F_c/W_s$		0.82 1.62
		Safety factor for slip on bearing surface by torque $T$	$F_w D_w/2/T$		1.01 1.98
	Evaluation for bearing surface pressure	Pressure due to pretension	$F_{fmax}/A_w$	[MPa]	251 455
		Pressure due to pretension and working loads	$(F_{fmax} + \Phi W_a)/A_w$	[MPa]	224 428
		Allowable pressure (for low carbon steel, SS400)	$p_L$	[MPa]	260
		Safety factor for pressure due to pretension	$p_L A_w/F_{fmax}$		1.04 0.57
		Safety factor for pressure due to pretension and working loads	$p_L A_w/(F_{fmax} + \Phi W_a)$		1.16 0.61

#### 4. Conclusion

We aimed to develop a strength evaluation method of bolted joint for the FE model that we described in our first paper<sup>(12)</sup> that simplifies the bolted-joint structures by using shell and beam elements. We obtained the following results in our work:

- 1) We proposed a reliability evaluation method for the bolted joints in the FE model that estimate the static and fatigue strength of bolt and their slip limit by calculating the forces and moments of equivalent beam and considering the mechanism of the bolted joints.
- 2) We applied our strength evaluation method to a specific example having many bolted joints and found that it was able to evaluate the bolt reliability.

Our method does not take the contact condition of bolted joints into consideration and, thus, it is unable to evaluate opening and slippage on the contact surface. However, when a structure many bolted joints is considered, our method provides sufficient information in most cases, while conventional modelling methods do not have a definite method of the strength evaluation.

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